Validation of Space Charge Laminar Flow in Diodes

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Introduction: Diode is a building block of ion sources and accelerators. Its design includes: a) an analytic solution for a closed anode system; b) a method to treat small holes in the anode. Nonlinear solver is here validated with case 'a'. The detailed understanding of large size beams is still an issue, and is here studied with moving mesh on the beam.

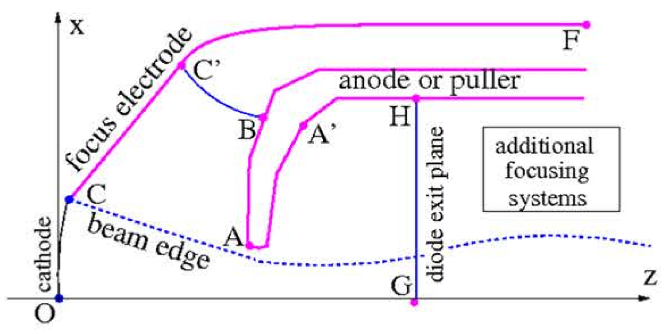

Figure 1. Typical diode geometry (open anode)

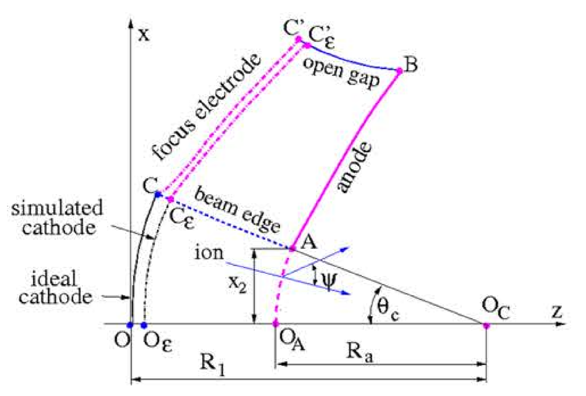

Figure 2. Closed anode simulation geometry

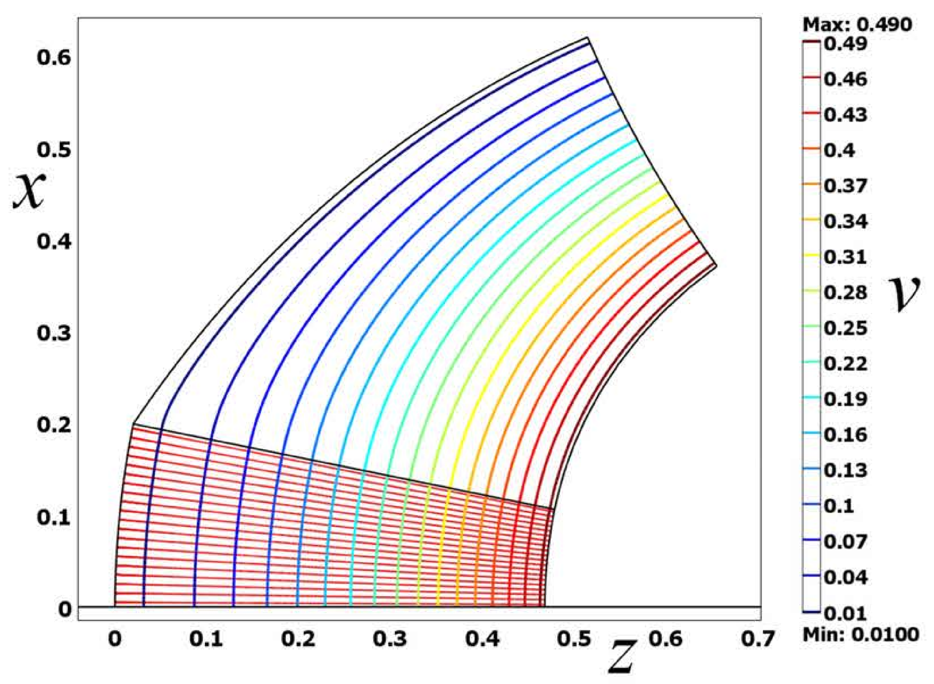

Figure 3. Closed anode simulation; anode axis intercept $z_a = 0.468\, R_1$

Computational Methods:

Poisson eq. for scaled potential v

$$\triangle_z v = \frac{4C(z,x)}{9\sqrt{\sqrt{v^2+(c_2)^2}}} \frac{j_e(\ell)}{R_1^2}$$

$$v = -q(\phi - \phi_C)/K_{E0} \qquad K_{E0} = |q|\,(|q| j_a {R_1}^2/k_0)^{2/3} \qquad k_0 = \frac{4\epsilon_0}{9}\sqrt{\frac{2|q|}{m}}$$

Cut off c_2 helps initialization of v, later $c_2 = 0$

Beam compression factor C

$$C(s,\ell) = \mathcal{G}(s,\ell)/\mathcal{G}(0,\ell)$$

with 'flow line density' G defined by | **dx** | = dX / G where

+spatial (moving frame) **x** = (z,x) or $z = z + ix$

+reference frame coordinates (Z,X) ; for metal walls in open anode case and everywhere in the closed anode case, they are defined as

Z+i X = $w \equiv s + i\ell = -\log\left(1 - \frac{z}{R_1}\right)$ (simple flow)

Design rules: potential for closed anode

$$v_r = s^{4/3}\left(1 - \tfrac{2}{15}s + \tfrac{11}{450}s^2 - \tfrac{437}{111375}s^3 + O(s^4)\right)$$

small hole anode: prediction of exit angle $\chi = \ell\left[-1 + \frac{v_{,s}(s_a)}{2v(s_a)}\right]$

Motion equations, maps and ALE

$$z_{,\lambda\lambda} = v_{,z} \quad , \quad x_{,\lambda\lambda} = v_{,x} \qquad \lambda \text{ is a scaled time}$$

For beam region, mapping from (Z,X) into (z,x) is determined by leapfrog integration of motion eq. and its interpolation

$$(z,x) = (z_M(Z;\ell_i), x_M(Z;\ell_i)) \text{ where } \ell_i = X$$

$$\ell_i = (i - \tfrac{1}{2})\theta_c/N \qquad \text{beam edge } \ell_i = \theta_c$$

Inside vacuum region, mesh may move freely

Maps influence C(z,x), so Poisson eq. solution v needs to be iteratively updated

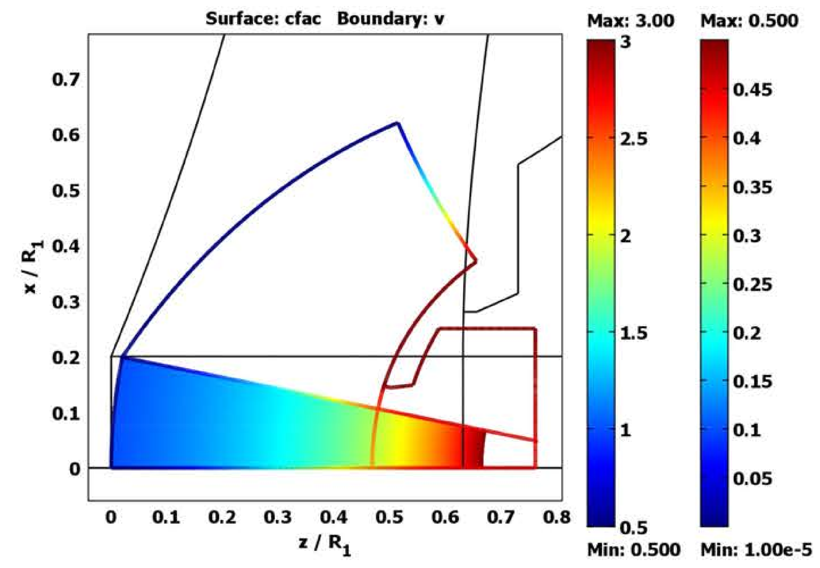

Figure 5. Beam compression C(z,x) before iterations

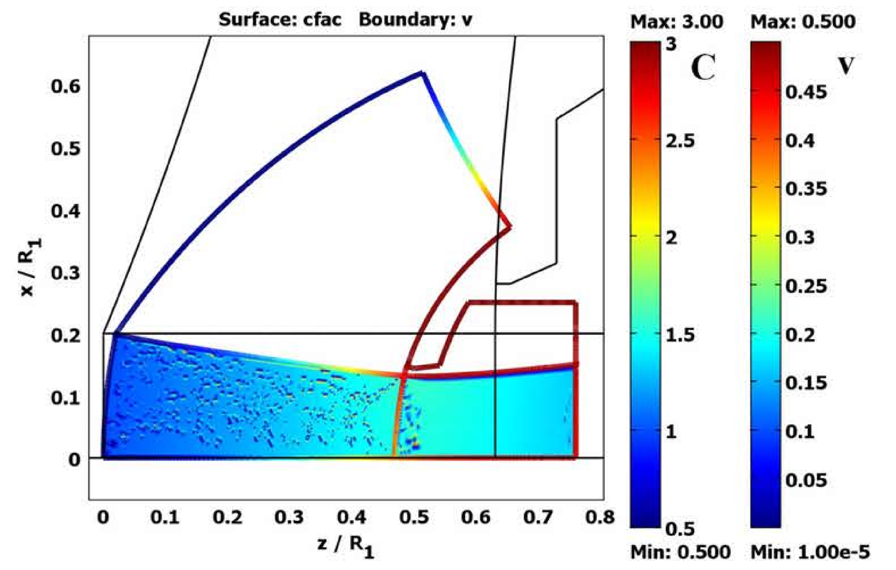

Figure 6. Beam compression C(z,x) at first iteration

Results: for closed anode: theory - code agreement is 4.5 digits; theory predicts mesh size needed for codes. For open anode: code converges in 3 iterations; large anode lens negative effects are confirmed (0.1 rad beam spread)

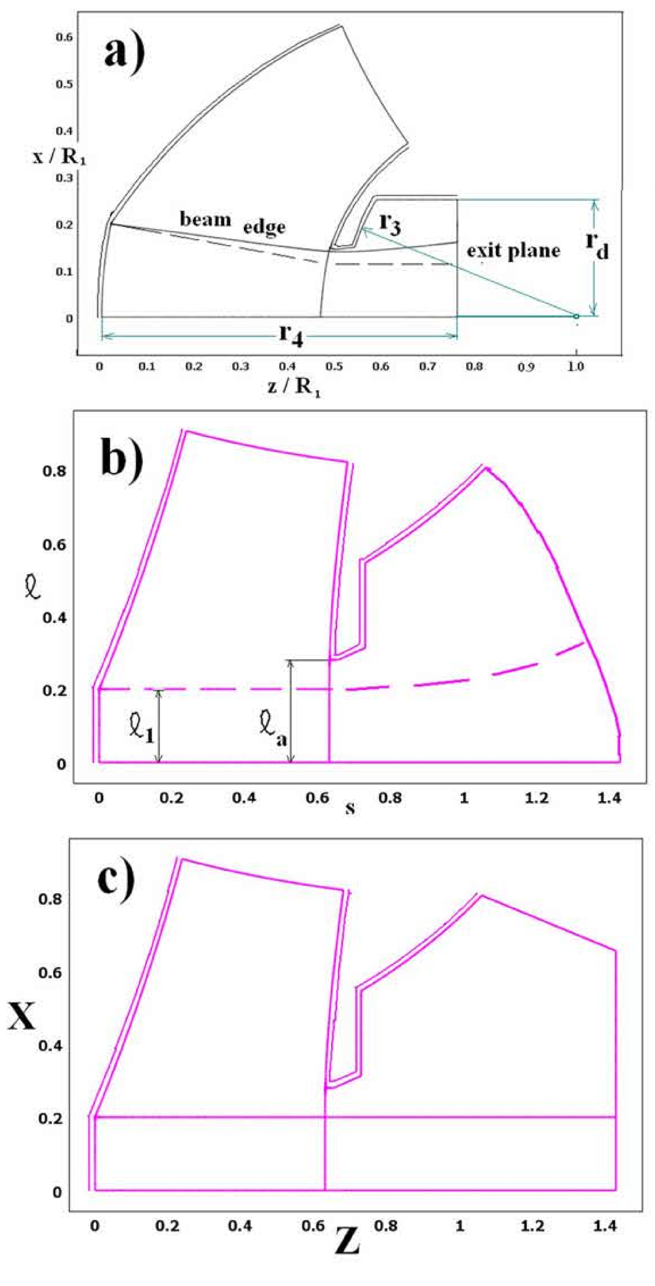

Figure 4. From spatial geometry (a) to reference frame (c) for ALE mode 'ale'

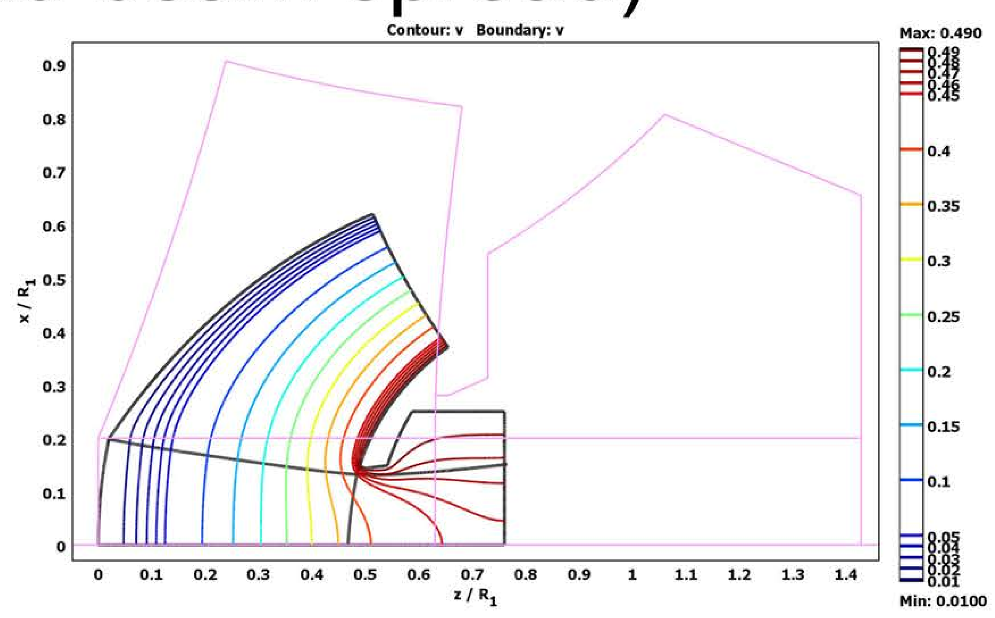

Figure 7. Contour lines of v in spatial frame (thick lines); reference frame (thin lines)

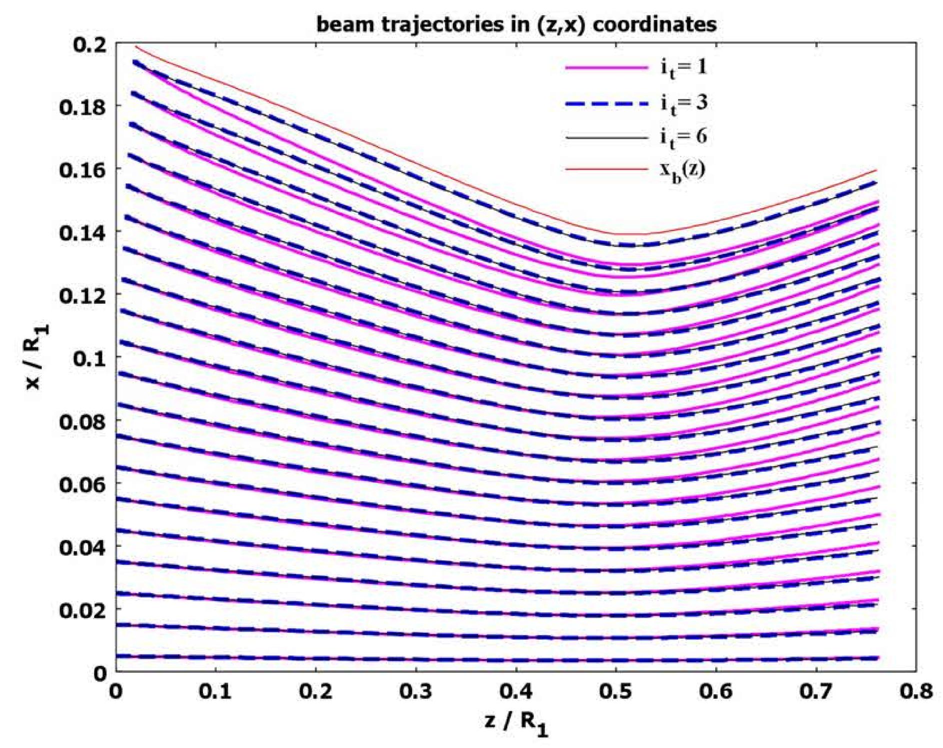

Figure 8. Test particles flow at several iteration i_t; the final beam edge $x_b(z)$ is also shown

Conclusion: the moving mesh is a powerful tool to model a laminar beam, as compared to PIC. A robust and rapidly convergent ALE-based Poisson-trajectory solver is here introduced and it allows cross validation with theory (especially for anode lens).

Some References:

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3. I. Langmuir and K. Blodgett, Phys Rev., 22 , 347 (1923).
4. G. R. Brewer "High-intensity electron guns" in Focusing of Charged Particles (ed. A. Septier, Academic Press, Orlando, 1967) vol. 2, p 23-72

***) Acronyms: ALE arbitrary Lagrangian-Eulerian; PIC particle in cell**