Numerical and Experimental Study of Water Drop Movement Subjected to an Air Stream in Porous Medium

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Abstract: Considering a liquid drop in relative movement with respect to the air flow at uniform velocity, the liquid will be driven to the surface by the viscous friction. Internal vortices will appear inside the drop. This problem has already been studied in fluid mechanics and is well known as a classic problem.

The idea of the present work is to resume the same analysis in a porous medium within an approximately spherical zone of porous medium saturated with liquid (drop) surrounded by a gas flow.

We have simulated the problem of two-phase flow of a water drop in a porous medium using COMSOL [1], [2], where we have validated the model by experimental data obtained from Magnetic Resonance Imaging (MRI) method. The effects of capillary pressure and application of different rules have been studied.

Keywords: porous media, two-phase flow, MRI, capillary pressure model.

1. Introduction

The goal of this work is to implement a two-phase flow model in porous media to describe the movement of an isolated spheroidal liquid zone subjected to an air stream inside a porous medium.

Given the scale and resolution required for the flow model, it seemed appropriate to perform the implementation using the standard finite element framework provided in COMSOL Multiphysics.

The model is based on the movement of a water drop when it subjected to an air stream at uniform velocity inside a porous medium.

The implementation is compared with experimental data obtained from Magnetic Resonance Imaging (MRI) method.

MRI is a powerful tool for the study, visualization and quantification of fluids in porous media. So this technique is clearly recommended to study the global movement of a liquid drop in porous media.

2. Two-phase flow model in porous media

Two-phase flow in porous media follows separate equations for the wetting and non-wetting fluids. Conservation of mass for each phase is stated by:

$$\begin{cases} \dfrac{\partial}{\partial t}(\varphi\rho_w S_w) + \nabla.(\rho_w V_w) = \rho_w q_w \\ \dfrac{\partial}{\partial t}(\varphi\rho_{nw} S_{nw}) + \nabla.(\rho_{nw} V_{nw}) = \rho_{nw} q_{nw} \end{cases} \quad (1)$$

Each phase has its own density ρ, saturation S, velocity V and source term q. S_w and S_{nw} respectively are the wetting and non-wetting phase saturations. φ is the porosity.

Darcy's law for two-phase systems can be defined by:

$$V_\alpha = -K\frac{k_{r\alpha}}{\mu_\alpha}(\nabla P_\alpha - \rho_\alpha g) \quad \alpha = w, nw \quad (2)$$

with K the intrinsic permeability and $k_{r\alpha}$ the relative permeability of phase α.

Inserting Eq. 2 into Eq. 1 gives:

$$\begin{cases} \dfrac{\partial}{\partial t}(\varphi\rho_w S_w) + \nabla.\left(-K\rho_w \dfrac{k_{rw}}{\mu_w}(\nabla P_w + \rho_w g)\right) \\ = \rho_w q_w \\ \dfrac{\partial}{\partial t}(\varphi\rho_{nw} S_{nw}) + \nabla.\left(-K\rho_{nw} \dfrac{k_{rnw}}{\mu_{nw}}(\nabla P_{nw} + \rho_{nw} g)\right) \\ = \rho_{nw} q_{nw} \end{cases} \quad (3)$$

It is assumed that fluids are incompressible and neglecting the effects of gravity then:

$$\begin{cases} \frac{\partial}{\partial t}(\varphi S_w)+\nabla.\left(-K\frac{k_{rw}}{\mu_w}\nabla P_w\right)=q_w \\ \frac{\partial}{\partial t}(\varphi S_{nw})+\nabla.\left(-K\frac{k_{rnw}}{\mu_{nw}}\nabla P_{nw}\right)=q_{nw} \end{cases} \quad (4)$$

The fact that the two fluids jointly fill the pore space implies the relation.

$$S_w+S_{nw}=1 \quad (5)$$

Due to the curvature and surface tension of the interface between the two phases, the pressure in the wetting fluid is less than that in the non-wetting fluid. The pressure difference is given by the capillary pressure:

$$P_c=P_{nw}-P_w \quad (6)$$

The following semi-empirical relationship is used to describe and quantify capillary pressure:

$$P_{nw}-P_w=P_c(S_w) \quad (7)$$

The following Brooks and Corey capillary pressure model is used in the present work:

$$P_c(S_w)=P_t S_e^{-\left(\frac{1}{\theta}\right)} \quad (8)$$

θ : Brooks-Corey parameter

P_t : Inlet or outlet threshold pressure

The effective saturation S_e is determined by:

$$S_e=\frac{S_w-S_{rw}}{1-S_{rw}-S_{rn}} \quad (9)$$

S_{rw} : Irreducible wetting phase saturation

S_{rn} : Residual non-wetting phase saturation

Consequently, we can express the capillary pressure gradient as follows:

$$\nabla P_c=\frac{dP_c}{dS_w}\nabla S_w \quad (10)$$

$$\frac{dP_c}{dS_w}(S_w)=-\frac{P_t}{\theta(1-S_{rw}-S_{rn})}S_e^{-\left(\frac{1}{\theta}+1\right)} \quad (11)$$

The Brooks-Corey model for relative permeabilities in two–phase system is given by the formulas:

$$k_{rw}=S_e^{\frac{2}{\theta}+3} \quad (12)$$

$$k_{rnw}=(1-S_e)^2\left(1-S_e^{\frac{2+\theta}{\theta}}\right) \quad (13)$$

By rewriting equations 4, 5, 6 for S_w and P_{nw}, we obtain:

$$\begin{cases} -\nabla\cdot(k\lambda\nabla P_{nw})+\nabla\cdot\left(K\lambda_w\frac{dP_c}{dS_w}\nabla S_w\right) \\ =q_w+q_{nw} \\ \varphi\frac{\partial S_w}{\partial t}+\nabla\cdot\left(K\lambda_w\frac{dP_c}{dS_w}\nabla S_w\right)- \\ \nabla\cdot(K\lambda_w\nabla P_{nw})=q_w \end{cases} \quad (14)$$

$$\lambda_w=\frac{k_{rw}}{\mu_w} \quad , \quad \lambda=\frac{k_{rw}}{\mu_w}+\frac{k_{rnw}}{\mu_{nw}} \quad (15)$$

The system of equations 14 for non-wetting pressure and wetting saturation can be rewritten in matrix form as follows:

$$\begin{pmatrix}0&0\\0&\phi\end{pmatrix}\frac{\partial}{\partial t}\begin{pmatrix}P_{nw}\\S_w\end{pmatrix}+\nabla.\left[-\begin{pmatrix}K\lambda & -K\lambda_w\frac{dP_c}{dS_w}\\ K\lambda_w & -K\lambda_w\frac{dP_c}{dS_w}\end{pmatrix}\right]$$

$$\nabla\begin{pmatrix}P_{nw}\\S_w\end{pmatrix}=\begin{pmatrix}q_w+q_{nw}\\q_w\end{pmatrix} \quad (16)$$

Standard COMSOL equation in PDE form is:

$$e_a\frac{\partial^2 u}{\partial t^2}+d_a\frac{\partial u}{\partial t}+\nabla\cdot(-c\nabla u-\alpha u+\gamma)$$
$$+\beta\cdot u+u=f \quad (17)$$

Equation 14 can be translated in straightforward manner to the standard COMSOL notation as follows:

$$u\equiv\begin{pmatrix}p_{nw}\\S_w\end{pmatrix}$$

$$d_a\equiv\begin{pmatrix}0&0\\0&\varphi\end{pmatrix}$$

$$c\equiv\begin{pmatrix}K\lambda & -K\lambda_w\frac{dp_c}{dS_w}\\ K\lambda_w & -K\lambda_w\frac{dp_c}{dS_w}\end{pmatrix} \quad (18)$$

$$f\equiv\begin{pmatrix}q_w+q_{nw}\\q_w\end{pmatrix}$$

$$e_a,\alpha,\gamma,\beta\equiv 0$$

3. Regularization of the capillary pressure

For small saturation, capillary pressure tends to infinity, so we have used threshold saturation to regularize it. When saturation is less than the threshold value, we use a linearization of the capillary pressure from the threshold value:

$$S_{thresh}=0.01$$

for $S_{thresh} < S_e < 1$

$$P_c = P_t . S_e^{-\frac{1}{\theta}} \quad (19)$$

for $S_e < S_{thresh}$

$$m = \frac{dP_c}{dS_w} = -\frac{P_t}{\theta} S_{thresh}^{-\left(\frac{1}{\theta}+1\right)}$$

$$P_c\big|_{S_{thresh}} = P_t . S_{thresh}^{-\frac{1}{\theta}}$$

$$\Rightarrow P_c = P_c\big|_{Sthresh} + m(S_e - S_{thresh})$$

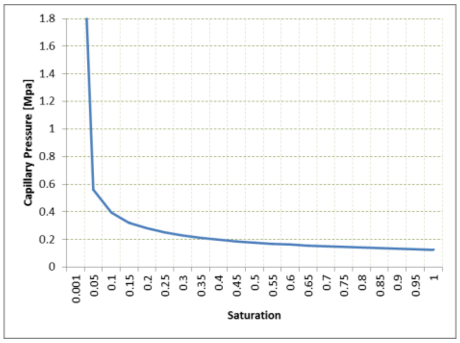


Figure 1: Capillary Pressure - Saturation relationship

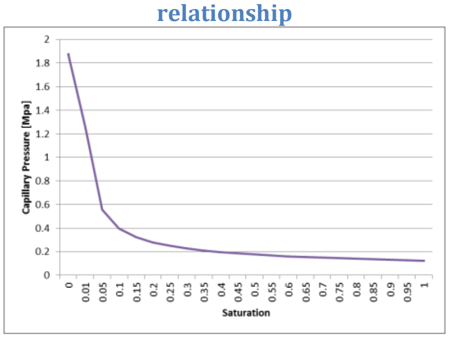


Figure 2: Capillary Pressure - Saturation relationship after regularization

4. Water drop model

We have considered the water drop in porous media as a spherical wet zone like figure 3.

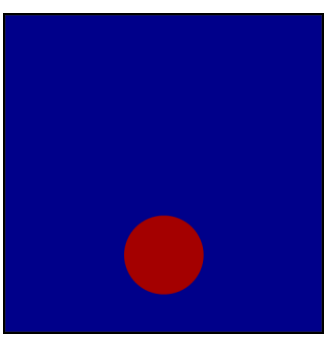


Figure 3: Porous medium and water drop as a saturated wet zone

We imposed the following initial conditions:

$$P(t_0) = P_0 = 1\,\text{bar}$$
$$S_w(t_0) = S_{w0} = 1 \quad (20)$$
$$S_{nw}(t_0) = S_{nw0} = 0$$

and boundary conditions:

$$\Delta P = 0.2\,\text{bar} \quad S_{in,out} = 0 \quad (21)$$

Table 1: Properties of fluids and porous domain

Property	Value	Description
μ_w	1e-3 (Pa.s)	Viscosity of water
μ_{nw}	1.81e-5 (Pa.s)	Viscosity of air
K	2.65E-11 (m^2)	Permeability
φ	0.4	Porosity
S_{rw}, S_{rn}	0	Residual water and air saturation
θ	2	Brooks-Corey parameter
P_t	0.125 (MPa)	Threshold pressure
q_w, q_{nw}	0	Volumetric flux source term

5. MRI experiments

Porous media have fragile structures; in order to analyse their physical properties it is sometimes necessary to use non-destructive and non-invasive techniques of measurement.

In this regard, magnetic resonance imaging (MRI) is an invaluable tool for the studying, visualizing and quantifying fluids in porous media. So we have used this technique to study the global movement of a liquid drop in porous media.

Although the use of beads beds allows a good approximation of porous media to the first order, geological porous media are mostly siliciclastic, therefore hydrophilic. The use of small glass beads (borosilicate) is well suited for this application. Various tests were carried out on columns of different types of beads (borosilicate, polystyrene) and different sizes (100 µm to 1 mm).

Polystyrene beads with diameter 150-250 µm have finally been chosen to minimize the gravity effects, while maintaining a consistent diameter for good visualization by MRI.

In order to investigate the movement of the water drop, we designed a device which can be inserted into the imaging probe (fig. 4). This device can be filled with different size of beads and should be able to transfer air flow at different pressures and flow rates.

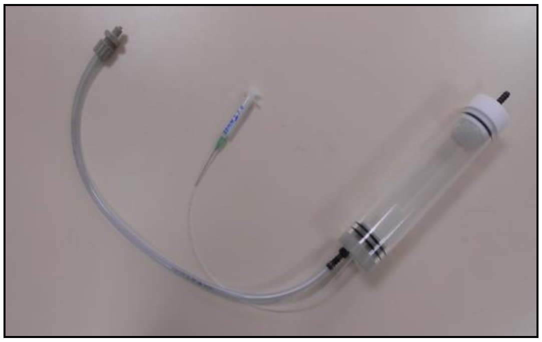


Figure 4: Designed device for MRI experiments

6. Comparison

In figure 5, we present a comparison between MRI experimental results and COMSOL numerical results in 6 time steps.
We can observe:

- In the first step, as initial condition, there are two separated zones: a wet zone saturated by water and a non-wet zone saturated by gas. The wet zone is like a sphere in the experiment.
- After starting air flow, the spherical drop stretches at the head of the wet zone.
- This stretch continues to form a long and axisymmetric filament.
- This stretching ends with a complete dispersion of the wet zone through the entire porous medium (spray).

The experimental and numerical results are not exactly the same but very similar. The main differences are due to:

- o MRI experiments are not real time; we have taken images in several times, so we have missed what was happening between images; but in COMSOL model-ling it is real time and everything is recorded.
- o In the same way, the inlet gas flow in MRI experiments is not continuous unlike COMSOL modelling.
- o The numerical model is 2D Cartesian.

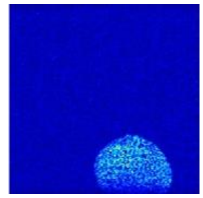
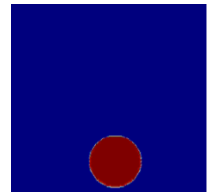
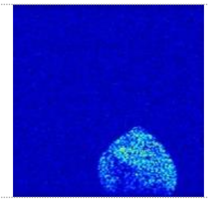
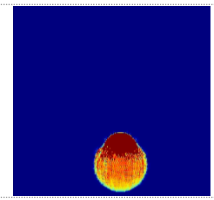
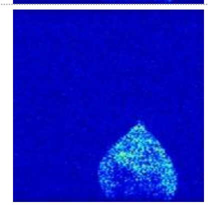
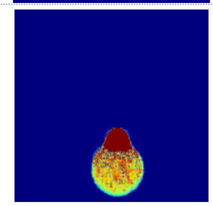
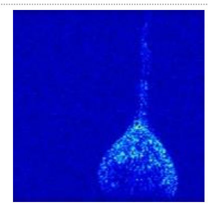
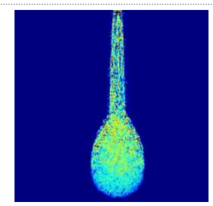
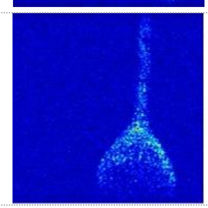
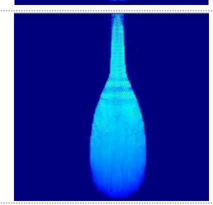
Time stage	Experimental (MRI)	COMSOL model
I		
II		
III		
IV		
V		

Figure 5: Comparison between experimental data (MRI) and numerical modeling of movement of a water drop (saturated zone) in an air flow through a porous media.

Reference

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