Design Optimization of Printed Circuit Board Embedded Inductors through Genetic Algorithms with Verification by COMSOL

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Abstract: This paper describes the implementation of a complete design tool for design, analysis, optimization and production of PCB embedded inductors.

The papers shows how the LiveLink between MATLAB and COMSOL makes it possible to combine the scripting and calculation power of MATLAB with the simulation power of COMSOL in order to get an extremely efficient tool for inductor design.

The tool has been used to investigate PCB embedded spiral, solenoid and toroidal inductors. Due to the fact that the spirals are axisymmetric they can be simulated in 2D, which speeds up the simulation significantly. This is not possible for the solenoid and toroid, hence complete 3D structure has been made and simulated for these structures.

Keywords: Inductor, Resistance, Optimization, LiveLink to MATLAB, axisymmetric.

1. Introduction

Passive energy storing elements constitute the major part of modern switch-mode power supplies (SMPS), both regarding size and price. The physical and numerical size of these, scales directly with the inverse of the switching frequency. The constant strive for smaller and cheaper power supplies has therefore let to constantly increasing switching frequencies ever since the development of the SMPS in the early 70's.

Recent research combines radio frequency (RF) circuits and power electronics to design SMPSs with switching frequencies in the Very High Frequency range (VHF, 30-300MHz) [1]-[2]. This is a dramatic increase from the .1-5 MHz which traditional SMPS topologies are limited to due to switching losses.

Several benefits arise from the increased switching frequency; higher power density, decreased cost, reduced weight and faster transient response [3]-[4]. The increased frequency however also leads to some new challenges [5]-[6]. One of these challenges is the magnetic components.

The core materials that are normally used to get high coupling in transformers and high inductance in inductors have high core losses at these frequencies. New core materials or ways of designing the magnetic components therefore has to be developed.

The inductance needed at these frequencies are very low (~10-200nH) and the skin depth is 4-12 μm. Hence embedding these as air core inductors in the Printed Circuit Board (PCB), and thereby reduce the size and price of these even further, becomes a viable solution [7]-[8].

In order to keep the efficiency high at these highly elevated frequencies soft switching is essential, as hard switching would lead to breakdown of the semiconductors. A key parameter to achieve this is accurate inductance value. Formulas for these are all based on approximations leading to small deviations. As the inductance cannot be fine tuned once the PCB has been manufactured, Finite Element Analysis (FEA) becomes a very attractive way to insure that the circuit is correctly designed the first time.

2. System level

In order to design and produce the inductors a MATLAB program with a GUI has been designed, see figure 1. The program takes the production parameters and the desired inductance and size as input and uses a genetic algorithm and formulas to find parameters for the initial design.

Then COMSOL is used to investigate the current distribution and magnetic fields and to verify the inductance and resistance.

Once the design has been verified in COMSOL and any error has been corrected, a PCB file is generated directly from the same interface.

3. MATLAB optimization

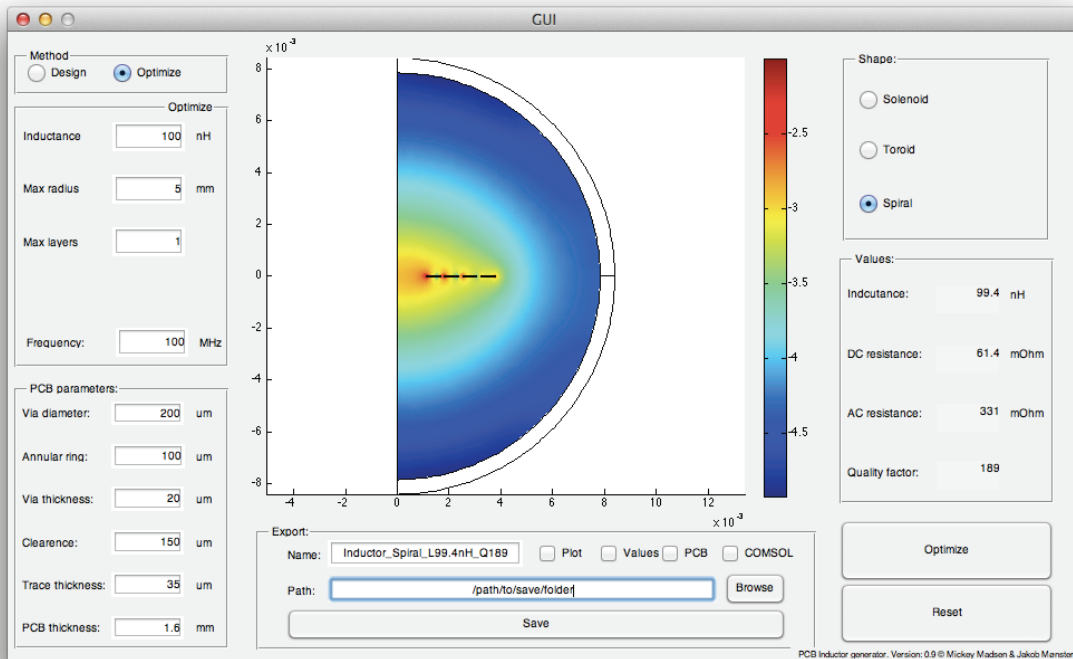The optimization routine is used to find the best inductor, within a given area. Best is defined

Figure 1. GUI of the MATLAB program, with the extracted simulation results from COMSOL via LiveLink for MATLAB.

as the required inductance the user needs, with the highest Q factor and with the lowest area, in that preferred order. The optimization routine is based on a genetic algorithm already implemented in MATLAB. The optimization parameters are the desired inductance, a maximum area and the Q factor. The Q factor is the relationship between the inductance and resistance, given by:

$$Q = \frac{\omega L}{R}$$

The inductor should have the highest possible Q factor and the desired inductance. This is achieved by having a cost function that the genetic algorithm minimizes. The genetic algorithm takes any number of inputs, with possibility to define the variable as an integer. Each variable is setup so a closer value to the desired will lower the cost function, and each variable is weighted according to the importance of that specific parameter. The cost function used is:

$$Cost = a \cdot \left| \frac{L}{L_{desired}} - 1 \right| + b \cdot \frac{1}{Q} + c \cdot A$$

where $L_{desired}$ is the desired inductance and A is the area of the inductor. The weight factors used are shown in Table 1.

Table 1. The weight factors used for the cost function.

a	b	c
100	10,000	1

The overall MATLAB program with the optimization routine is described by the flowchart shown in Figure 2.

4. COMSOL:

To verify that the calculations used to derive the optimum structure and evaluate that the formulas in overall are usable the MATLAB program is used to interface to COMSOL via COMSOL LiveLink for MATLAB [9].

The AC/DC module in COMSOL is used to simulate the three possible inductor structures. The AC/DC module gives possibility for extracting the inductance and resistance, which

are the desired parameters [10]. A precondition for using the AC/DC module is that the length of the structure l, is much less than the wavelength of the frequency at which the system is simulated. That is:

$$l < {}^1\!/_{10} \cdot \lambda$$

At VHF frequencies the wavelength is still in the range of a few meters, why the AC/DC module is a viable approximation to the system, as the structures are very small.
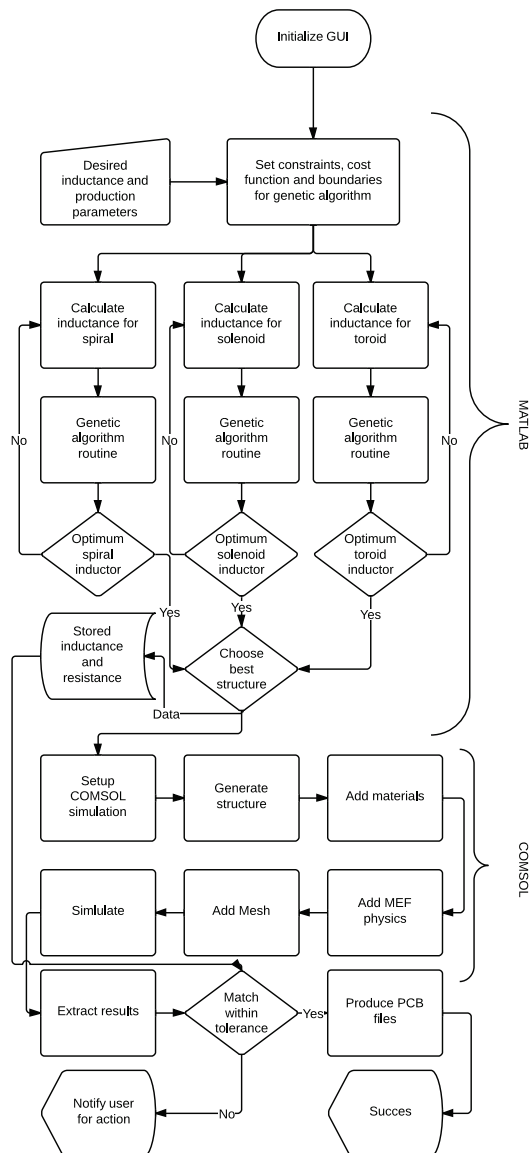

Figure 2. Flowchart of the MATLAB program, with the optimization routine and the COMSOL routine

4.1 Design steps

The COMSOL model is set up in five steps. Each are described here in detail. The program can simulate three different structures: A solenoid, a toroid and a spiral. The spiral is simulated in 2D to achieve faster simulation time. Examples of the three different structures are shown in Figure 3-5.

LiveLink for MATLAB gives access to all the normal features of COMSOL with the scripting capabilities of MATLAB. In general five steps are used in the interface.

4.2 Step 1 – Generating the structure

The structures are made by implementing the layers of the PCB in a z-shifted plane, and then extruding them. Cylinders are added as vias, and then subtracting smaller cylinders inside the other for the holes in the vias. This gives a very exact representation of the full 3D inductor structures [11]. There is a short loop for the 2 connectors in the inductors, and between is made a gap. This is used when setting up the physics. At last the entire inductor is made into a union. During the generation of the structure, several selections are made, both with boxes and explicit. These are automatically updated if more structures are added in the process.

The spiral is only modeled in an axisymmetric 2D model with infinite elements [12] to lower the simulation time. The two other structures are (unfortunately) not possible to split into an symmetry.

The PCB's dielectric material is not added to the model, as it does not influence the inductances and resistance of structure, and only will increase simulation time. If the parasitic capacitance or resonance frequency of the inductors is to be extracted from the simulation, this has to be added as it do influences the capacitance.
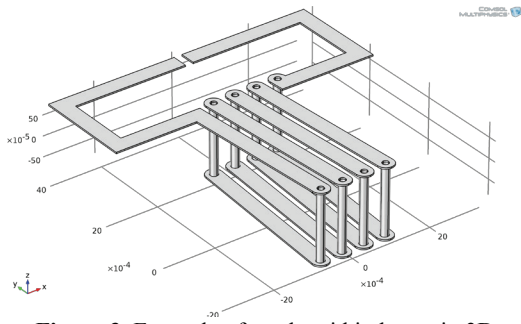
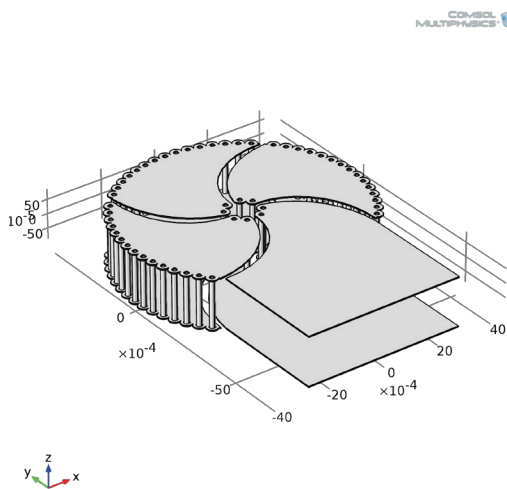**Figure 3.** Example of a solenoid inductor in 3D.

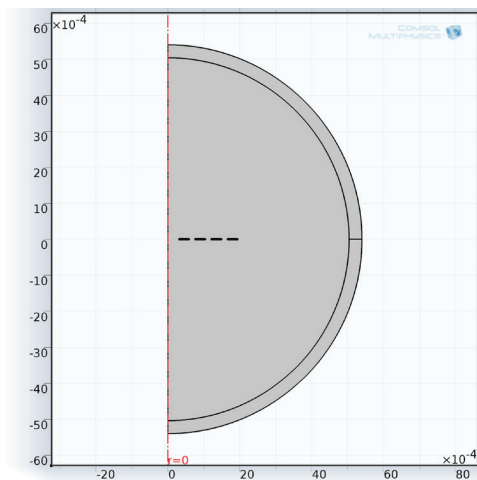**Figure 4.** Example of a toroid inductor in 3D.

Figure 5. Example of a spiral inductor in 2D.

4.3 Step 2 – Setting up the physics

The physics is based on the Magnetic Fields (MF) with a frequency domain study. The physic is set to work at the entire system expect the gap created between the terminals. This will introduce a small error but smaller than if the connectors were taken to the exterior of the bounding box. The surroundings are modeled as air, and the structure as copper. As the magnetic field is not guided by a core, there is added an infinite element to ensure the entire field is simulated within the bounding box's boundaries.

For exciting the inductor a "Single-turn coil" is used [13], with a voltage input. Furthermore the physics is set to linear, as this decreases the simulation time dramatically, and does not change the results.

4.4 Step 3 - Meshing

As there is great difference between the overall size of the structure and the holes in the vias and the height of the copper traces, which have an even smaller skin-depth, the mesh must be customized to the structure. The structure is split in three. The inductor structure uses "Free Tetrahedrals" with the size set to "Extra fine". Boundary layers is used to get the correct current density in the copper tracks,. The air is also meshed with "Free Tetrahedrals", but with the size set to "Fine". The infinity domain is swept with a simple distribution of 3 elements. This configuration gives a mesh quality of approximately 0.7.

4.5 Step 4 – Study setup

The study is set up to use a frequency domain. The standard stationary solver is changed to use a SOR solver instead of the Multigrid, as the system is set to linear.

4.6 Step 5 – Extracting parameters

The inductance and ac-resistance are added to a table from a global evaluation. The table is then imported into MATLAB together with a plot of the magnetic field with the LiveLink for MATLAB functions "mphtable" and "mphplot".

5. Verification

In order to verify the formulas used to calculate the inductance and resistance and to ensure that the implementation of the models in COMSOL is a good approximation a comparison of calculated, simulated and measured values are

performed. The extensive comparison is only made for spirals, as the simulation time if done for solenoids and toroids would accumulate to weeks.

The simulations are shown for two set of spiral structures: one with one layer and a 150um trace shown in Figure 6 and one set with a 2 layer structure and 300um trace in Figure 7. The number of turns is swept from 1 to 15 and 1 to 10 respectively.

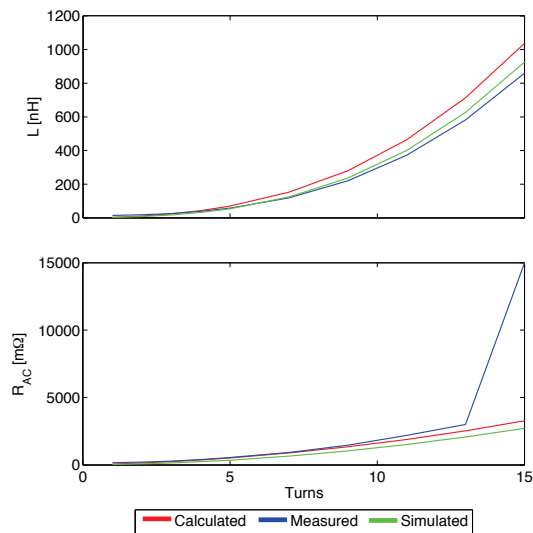The comparison shows that there in general is a good agreement between calculated, simulated

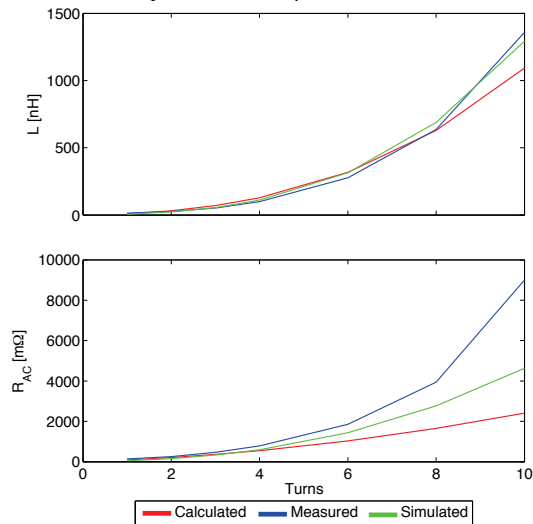**Figure 6.** Inductance and resistance of single layer spirals with 150μm trace width.

Figure 7. Inductance and resistance of two layer spirals with 300μm trace width.

and measured inductances, but with higher accuracy for the simulated values.

The same is the case for the ac-resistance. In both sets there is a large increase in the measured resistance when the structures become large. This is the start of the first resonance, and this isn't part of neither the simulation nor the calculation.

In Figure 8 and 9 is shown a cutout of the magnetic field of a solenoid and a toroid. The simulation shows that solenoid and toroid structures encapsulate the magnetic field, resulting in a much weaker external field than seen for spirals. In the solenoid all the magnetic flux goes through the center of the structure and returns along the outer edges of the structure. For the toroid the major part of the field is running inside the toroidal structure and only a small part of the field (caused by the single turn) runs outside.
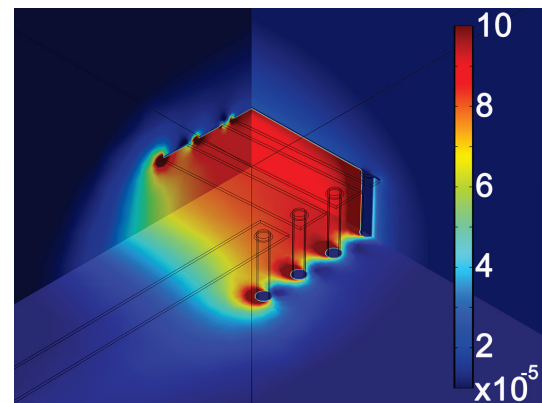

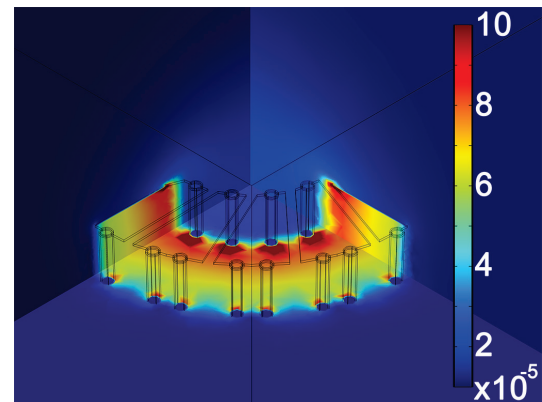**Figure 8.** Magnitude of the B-field in a solenoid inductor (Multislice).

Figure 9. Magnitude of the B-field in a toroid inductor (Multislice).

6. Conclusion

It has been demonstrated how the LiveLink between COMSOL and MATLAB can be used to design, analyze and optimize PCB embedded inductors. The scripting and calculation power of MATLAB combined with the simulation power of COMSOL can serve as an extremely efficient tool for inductor design.

A genetic algorithm has been used to design the optimal inductors based on the available equations. Then COMSOL has been used to verify the calculated inductance and resistance and investigate the current distribution and the magnetic fields. This analysis is very useful for finding new ways to design inductors in order to make new shapes with even better performance.

Ones the inductor has been investigated and verified to give the desired impedances the PCB files can be generated directly from the same MATLAB GUI giving a very simple and yet accurate design tool.

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