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Investigation of Stability of Current Transfer to Thermionic Cathodes

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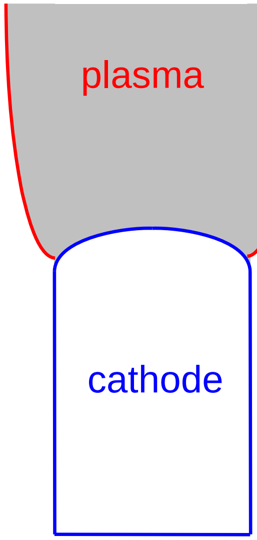
Introduction



Diffuse mode

Spot mode

Cathode of an arc discharge in argon. W, R = 0.75 mm, p = 4.5 bar, I = 2.5 A.
From S. Lichtenberg *et al* 2002.

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Introduction

- The diffuse mode is favorable for operation of cathodes of high-pressure arc devices, however it is difficult to be realized.

- Solutions describing the diffuse mode and different spot modes have been obtained and analyzed in detail.

- This information is not yet sufficient for engineering practice: one needs also to have information on **stability** of each of these modes in some or other particular conditions.

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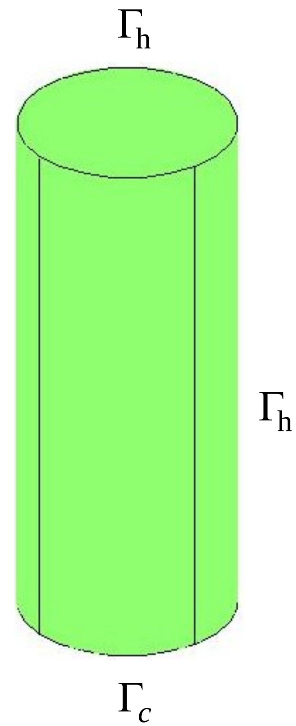
Equations and boundary conditions

- Non-stationary equation of heat conduction

$$\rho\, c_p(T)\frac{\partial T}{\partial t} = \nabla\cdot\left[\kappa(T)\,\nabla T\right]$$



- Boundary conditions

$$\Gamma_c : T = T_c$$

$$\Gamma_\mathrm{h} : \kappa(T)\frac{\partial T}{\partial n} = q(T,U)$$

- A given value of the arc current

$$I = \int_{\Gamma_h} j(T,U)\,dS$$

U: near-cathode voltage

Known functions: obtained from equations describing the current transfer through the near-cathode plasma layer.

The stationary problem admits multiple solutions describing different modes of current transfer!

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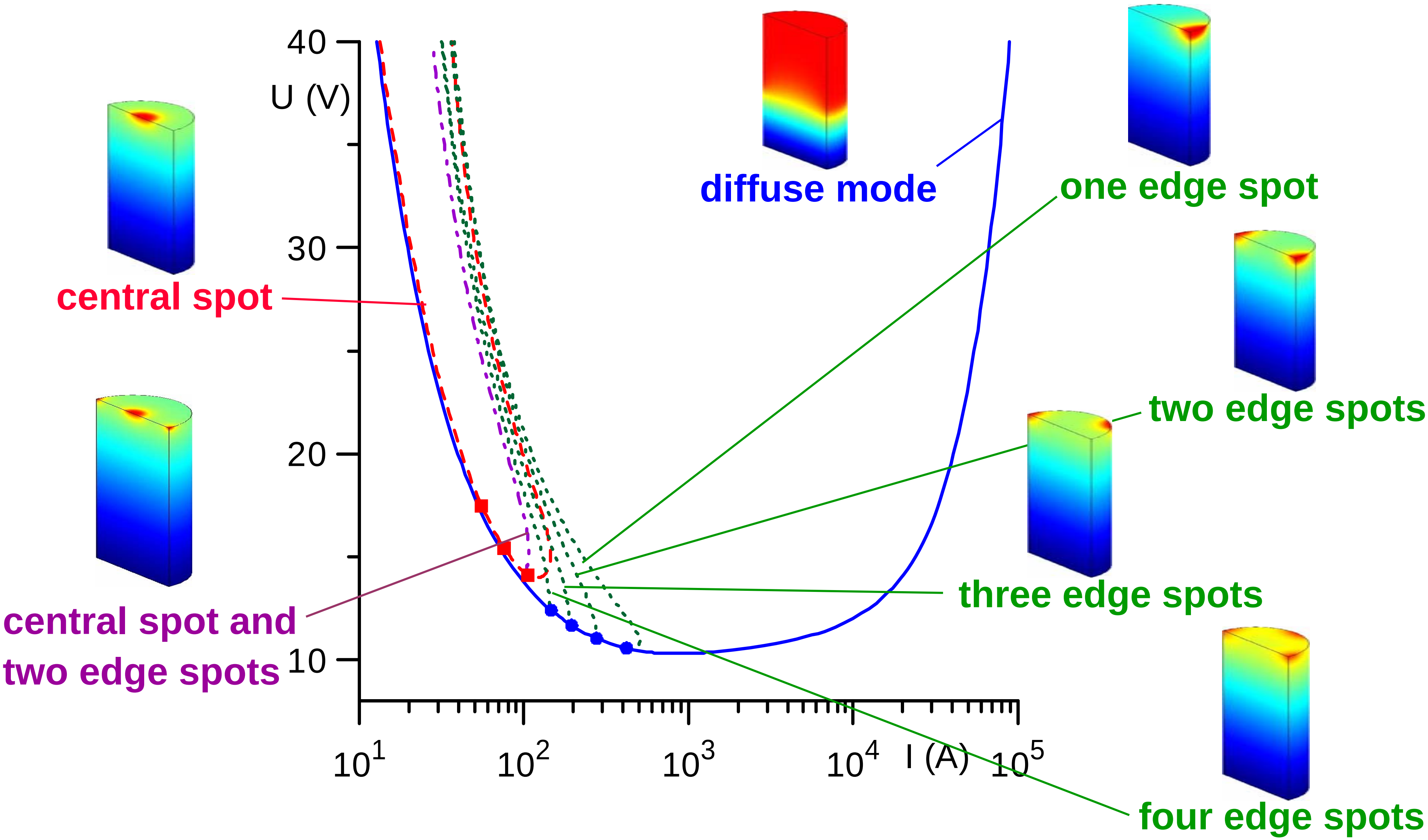
Multiple steady-state solutions



W, R = 2 mm, h = 10 mm, Ar, 1 bar. ■, •: bifurcation points.

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Formalism of the linear stability theory

Superposition of a steady-state solution and of a perturbation

$$T(\vec{r},t) = T_0(\vec{r}) + e^{\lambda t} T_1(\vec{r}) + \ldots$$

$$U(t) = U_0 + e^{\lambda t} U_1 + \ldots$$

$$I(t) = I_0 + e^{\lambda t} I_1 + \ldots$$

all $\lambda \leq 0$: the state is stable

At least one $\lambda > 0$: the state is unstable

Eigenvalue problem for perturbations

$$\rho c_p (T_0) \lambda T_1 = \nabla \cdot \left(\frac{d\kappa}{dT}(T_0) T_1 \nabla T_0 + \kappa(T_0) \nabla T_1 \right)$$

$$\Gamma_c : T_1 = 0$$

$$\Gamma_\mathrm{h} : \frac{d\kappa}{dT}(T_0) T_1 \frac{\partial T_0}{\partial n} + \kappa(T_0) \frac{\partial T_1}{\partial n} = \frac{\partial q}{\partial T}(T_0, U_0) T_1 + \frac{\partial q}{\partial U}(T_0, U_0) U_1$$

$$0 = \int_{\Gamma_h} \left(\frac{\partial j}{\partial T}(T_0, U_0) T_1 + \frac{\partial j}{\partial U}(T_0, U_0) U_1 \right) dS$$

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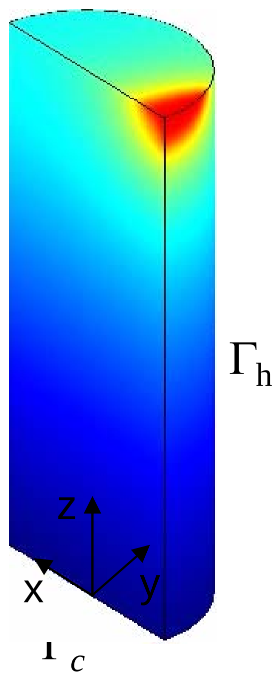
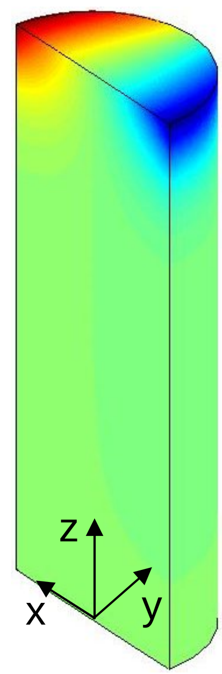
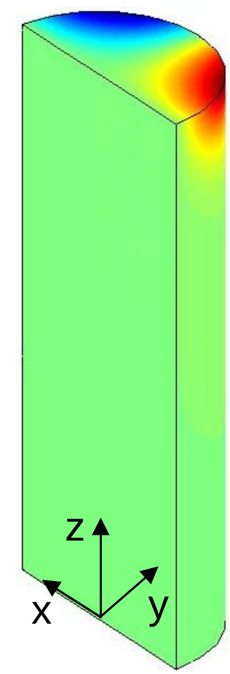
Even and odd perturbations



$$y = 0 : \frac{\partial T_0}{\partial y} = 0$$



$$y = 0 : \frac{\partial T_1}{\partial y} = 0$$



$$y = 0 : T_1 = 0$$

Steady-state solution is even

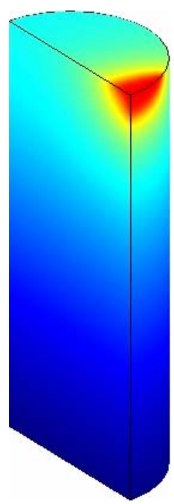
Even perturbations

Odd perturbations

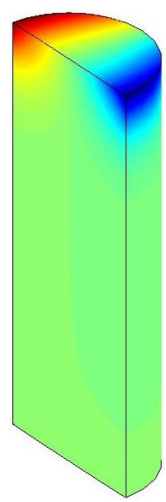
Stability: COMSOL straight

Heat transfer application mode → Stationary solver for the steady-state solutions

Heat transfer application mode → Eigenvalue solver for the perturbations



$$y = 0 : \frac{\partial T_0}{\partial y} = 0$$



$$y = 0 : \frac{\partial T_1}{\partial y} = 0$$

Problem: Only even perturbations are calculated!

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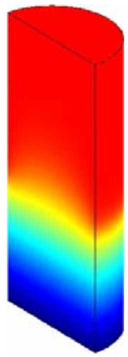
Stability: COMSOL straight

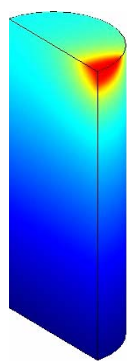
- Axially symmetric steady-state solutions



Even perturbation
$=$
Odd perturbation

Same eigenvalue:
Complete spectrum

- 3D steady-state solutions



Even perturbation
$\neq$
Odd perturbation

Different eigenvalues:
Incomplete spectrum!

Stability: combined approach

A combined approach: to use explicitly the linear stability theory and two modes of COMSOL

Steady-state solution: Heat transfer application mode, Stationary solver

+

Perturbations: PDE mode, Eigenvalue solver

$$y = 0 : \frac{\partial T_1}{\partial y} = 0 \longrightarrow \text{Even perturbations}$$

$$y = 0 : T_1 = 0 \longrightarrow \text{Odd perturbations}$$

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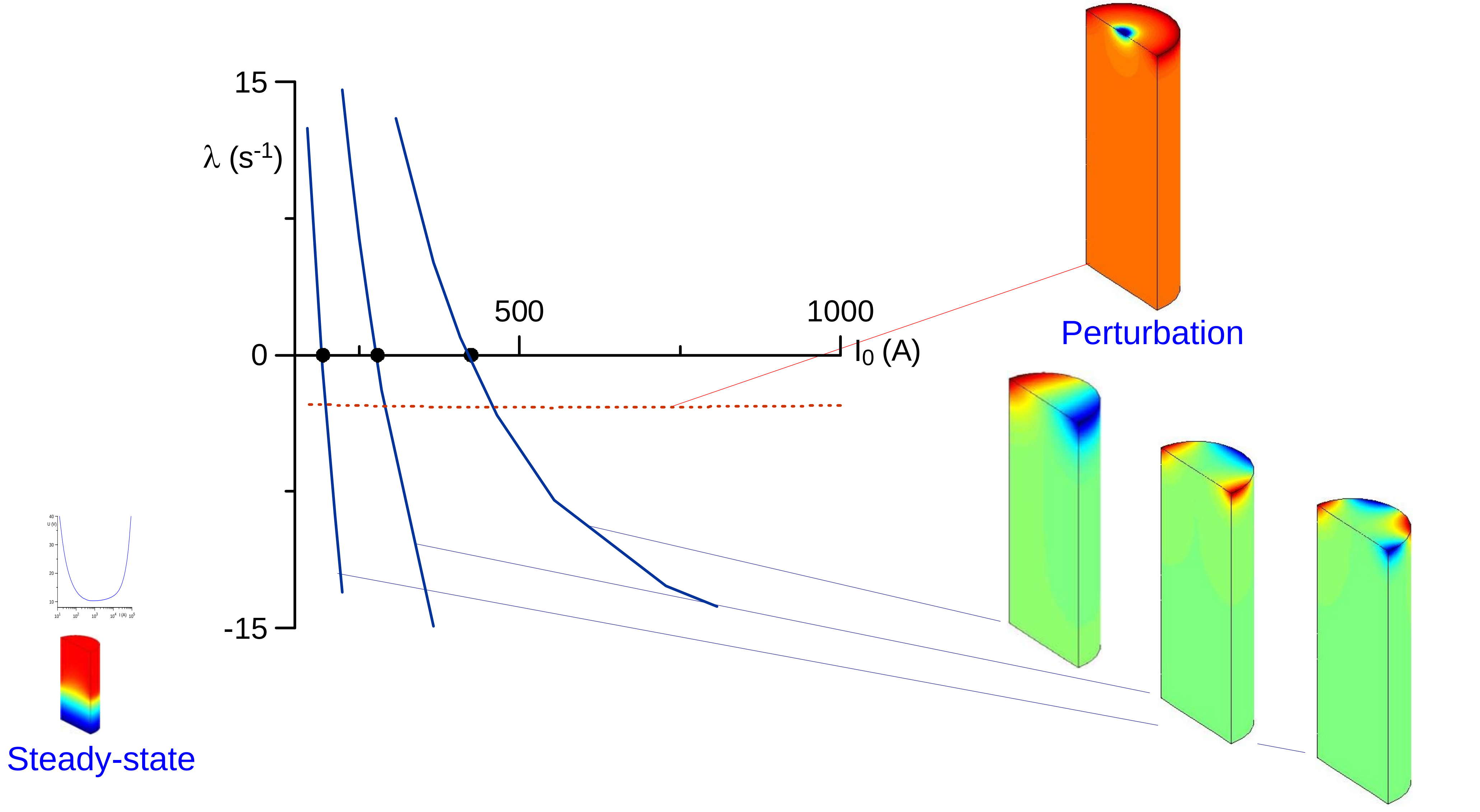
Numerical results: examples



W, R = 2 mm, h = 10 mm, Ar, 1 bar. •: bifurcation points.

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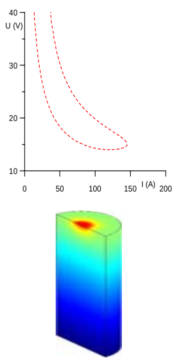
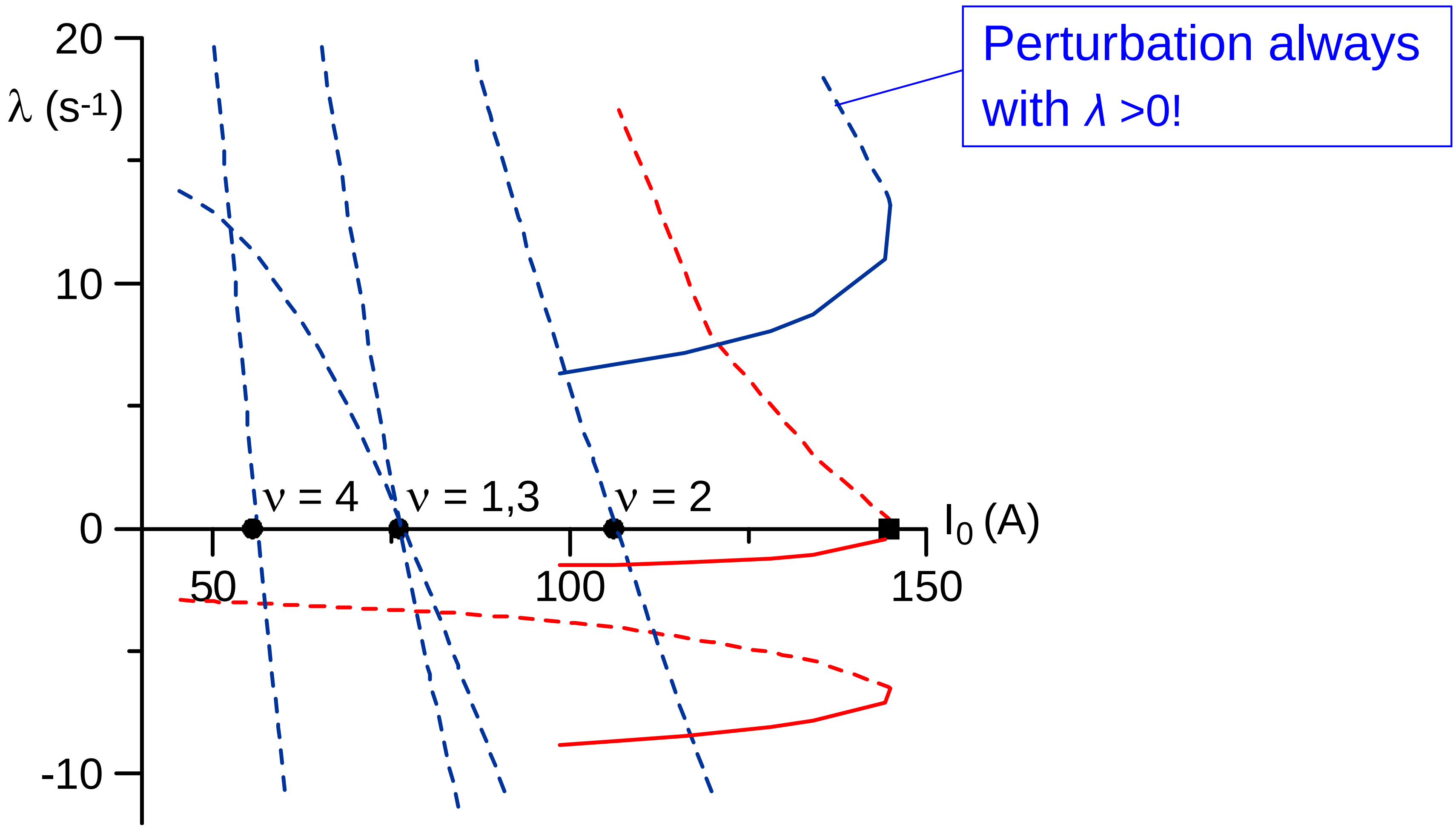
Numerical results: examples





W, R = 2 mm, h = 10 mm, Ar, 1 bar. •: bifurcation points. ■: turning point.

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Numerical results of stability of 3D spot modes

υ	*T*	**Even** perturbations	**Odd** perturbations
1	2π	+ $\rightarrow$ -	0
2	2π	+	+
	π	+ $\rightarrow$ -	0
3	2π	+, +	+, +
	$2\pi/3$		0
4	2π	+, +	+, +
	π	+	+
	$\pi/4$		0

W, R = 2 mm, h = 10 mm, Ar, 1 bar.
υ: number of spots at the edge of the front surface of the cathode. *T*: period.

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Summary of results of stability of 3D spot modes

Perturbations	
Even	**Odd**
Can change sign of their increment along 3D steady-state spot modes.	Do not change sign of their increment along 3D steady-state spot modes.

Perturbations of a steady-state mode with v spots at the edge of the front surface of the cathode are periodic with respect to the azimuthal angle with periods between 2π and $2\pi/v$.

A state with v spots at the edge of the front surface of the cathode is:

- unstable against v modes of even perturbations with period exceeding $2\pi/v$ in the region between the bifurcation point and the turning point;

- unstable against $v-1$ modes of even perturbations with period exceeding $2\pi/v$ in the region after the turning point or if the mode is supercritical;

- neutrally stable against one mode of odd perturbations with the period of $2\pi/v$;

- unstable against $v-1$ modes of odd perturbations with period exceeding $2\pi/v$;

- stable against all the others modes of perturbations with such periods.

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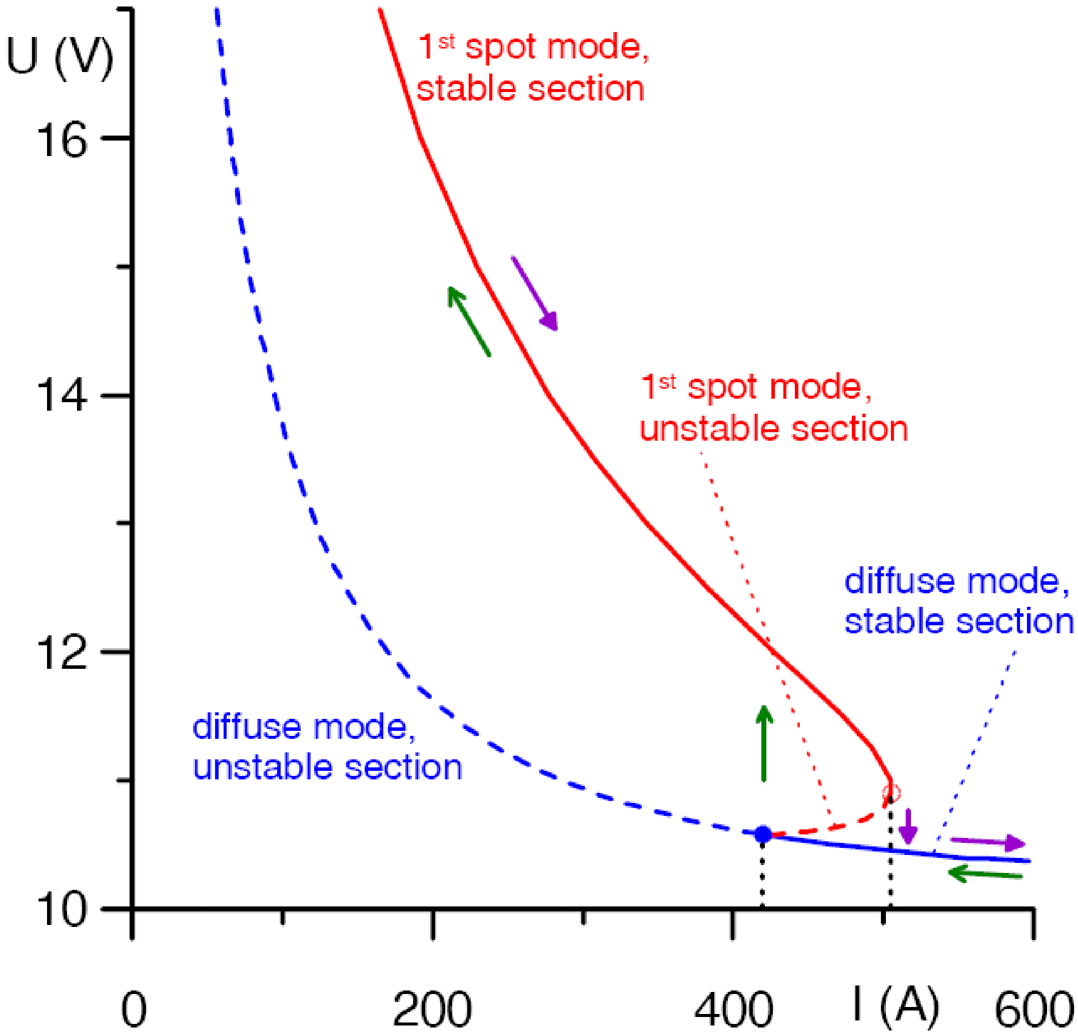
Application of the numerical results



W, R = 2 mm, h = 10 mm, Ar, 1 bar.

- Modes with one spot at the center or with multiple spots are always unstable.

- The only modes that can be stable are the diffuse mode and the high-voltage branch of the first 3D spot mode.

- The transition between these two modes is non-stationary and accompanied by hysteresis.

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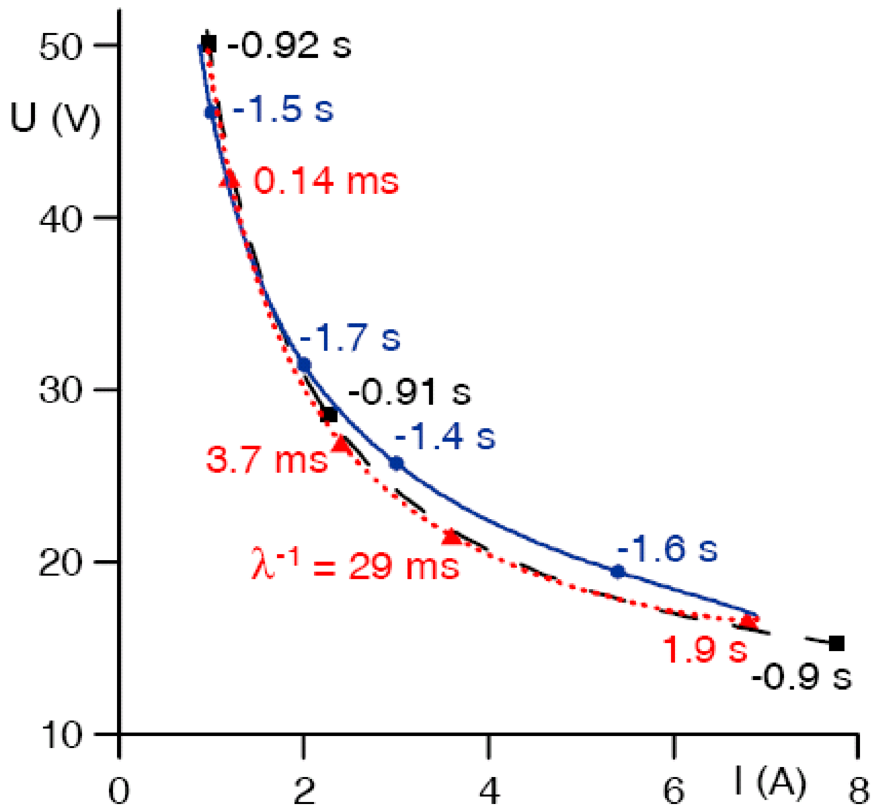
Stability of current transfer in experimental conditions



- In this experiment, both the diffuse mode and the high/voltage branch of the first 3D spot mode are stable in the whole range investigated (1A - 6A).

- => No reproducible diffuse-spot transition!

W, *R* = 0.75 mm, *h* = 20 mm,
rounding 100 μm, Ar, 2.6 bar.

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Conclusions

- A general pattern of stability of the different modes of current transfer has been established.

- This pattern conforms to trends observed in the experiment:

  - the diffuse-spot transition on arc cathodes is a monotonic process;

  - patterns with more than one spot are not normally observed;

  - the diffuse mode is observed at high currents and the mode with a spot at the edge of the cathode at low currents;

  - the transition between the diffuse mode and the mode with a spot at the edge is non-stationary and is accompanied by hysteresis;

  - this transition is difficult to be reproduced in the experiment.

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