COMSOL® Analysis for Duct Acoustic

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Introduction

- Thermoacosutic or combustion instabilities constitute a major issue in several types of applications from aerospace propulsion systems to boilers and radiant heaters.
- They cause structural damaging, hardware melting, high noise, and overall systems failure.
- NASA Jet Propulsion Laboratory's experiment on 1956. "*A Mechanism for High-Frequency Oscillation in Ramjet Combustors and Afterburners*" [ref.1]
- The goal is preform Computational Aeroacoustics "CAA" simulation via COMSOL® Multiphysics, to predict the frequencies and mode shapes of the excited instabilities.

Figure 1: NASA space shuttle was powered by solid rocket propellants.

1- *Don E. Rogers and Frank E. Marble, "A mechanism for high-frequency oscillation in ramjet combustors and afterburners", the American Rocket Society, 1956.*

Thermoacoustic Instabilities

- When heat release oscillations and pressure acoustic waves are in phase, they are considered to be “coupled”. This causes the acoustic pressure mode to get excited and amplified.
- In other words, the heat release from the combustion must be released when the acoustic pressure wave near or at its maximum amplitude (antinode).
- Pressure acoustic oscillations effect dynamic processes, hence the feedback loop is created.
- These instabilities are generally characterized by two distinct frequencies which are: low pressure frequency (rumble), and high pressure frequency (screech).
- Mathematical expression: *Rayleigh’s criterion*

$$\int_0^\tau \int_0^V p'(x,t)\; q' dvdt \geq \int_o^\tau \int_0^V \Phi(x,t)\; dvdt \quad \text{(eq.1)}$$

$$\begin{aligned} p' &= pressure\ oscillation \\ q' &= heat\ release\ oscillation \\ \Phi &= energy\ losses \\ \tau &= oscillation\ period \\ V &= combustor\ volume \end{aligned}$$

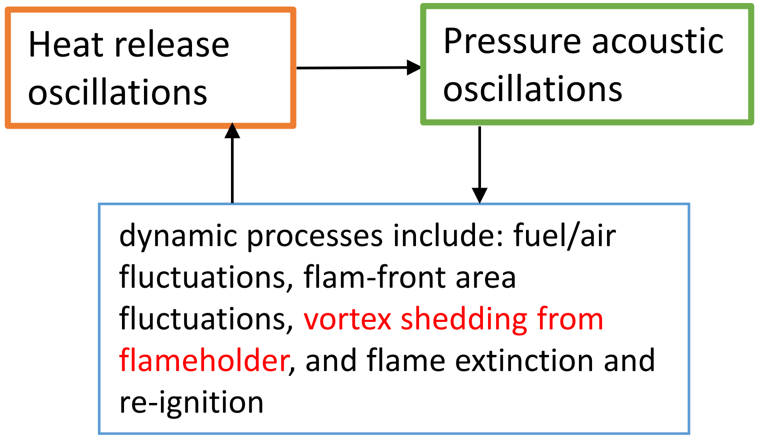


Figure 2: Basic feedback loop responsible for the instabilities.

Duct Acoustic and Boundary Conditions

- Pressure standing waves :
  - node (minimum frequency amplitude) at open boundary
  - antinode (maximum frequency amplitude) at closed boundary
- The longitudinal standing waves inside a duct with simple geometry can be calculated by using the following equations:

  1- Close-close & open-open boundaries:

  - the fundamental longitudinal mode is ½ wavelength.

$$f(any\ \#\ of\ harmonic) = \frac{any\ \#\ of\ harmonic *\ C}{2*L} \quad \text{(eq.2)}$$

c = speed of sound = $\sqrt{R\,T\,\gamma}$

L = length of the duct

  2- Close-open boundaries:

  - the fundamental longitudinal mode is ¼ wavelength

$$f(any\ \#\ of\ harmonic) = \frac{any\ \#\ of\ harmonic *\ C}{4*L} \quad \text{(eq.3)}$$

- Pressure transverse modes of a 2D rectangular duct can be calculated via this equation:

$$fm,n = \left(\frac{c}{2\pi}\right)\sqrt{(1-M^2)\left\{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{W}\right)^2\right\}} \quad \text{(eq.4)}$$

$H = hight\ of\ the\ duct$

$W = width\ of\ the\ duct$

$M = Mach\ number$

$(m,n) = modes\ of\ order$

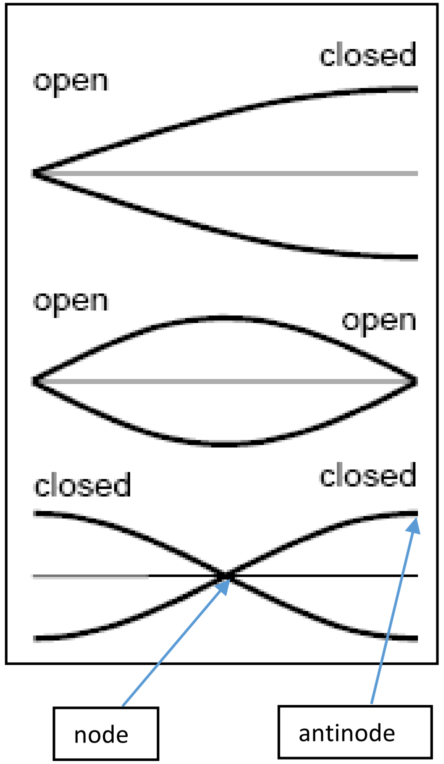


Figure 3: pressure standing waves behavior.

Experimental Duct and Conditions

- The experiment was carried out in a small combustion duct of rectangular cross section.
- The combustor was 1 in. by 4 in. rectangular cross section and extended 24 ½ in. in length beyond the end of the wedge-shaped flameholder (3 ½ in. long), as shown diagram below.
- The average temperature was 250 F = 394.26 K.
- Atmospheric pressure = 1 atm = 101.325 kPa.
- The pressure gage was located at the center of the duct, 11 in. away from the inlet.

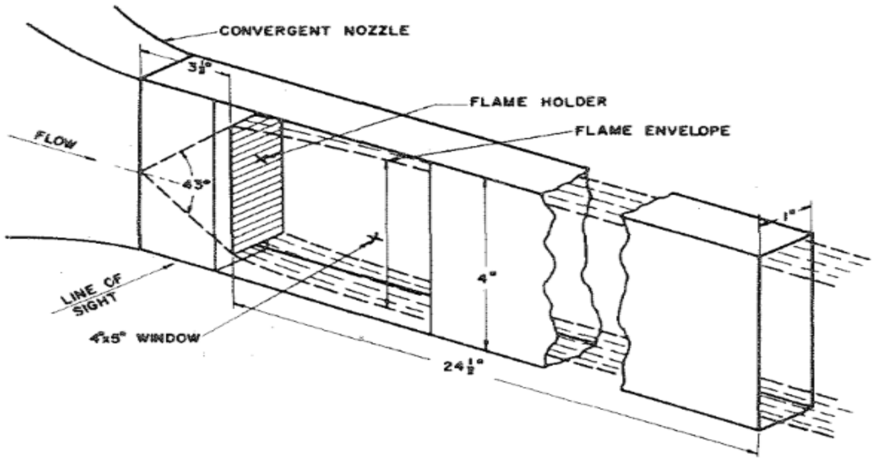


Figure 4: Schematic diagram of the combustion duct [ref.1].

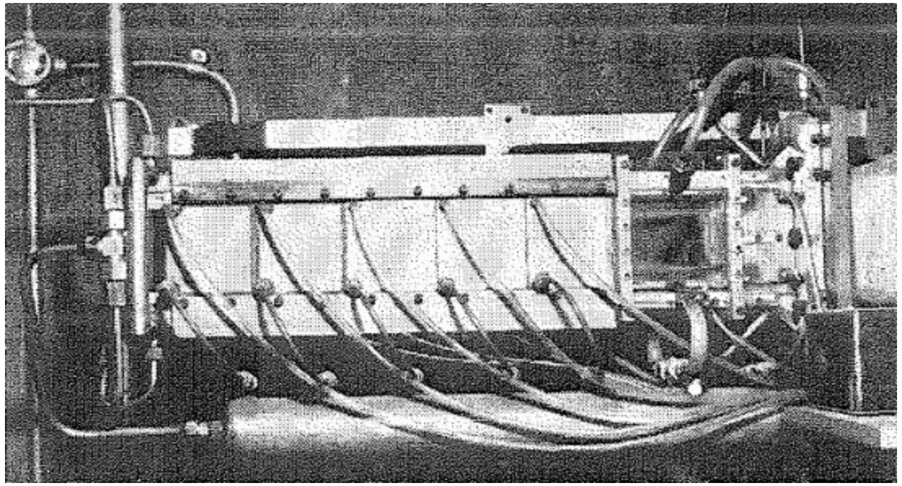


Figure 5: Side view of the combustion chamber at NASA Jet Propulsion lab [ref.1].

COMSOL Simulation

- In order predict the frequencies and mode shapes of the excited instabilities:
  - Geometric characteristics of the system
  - Average temperature distribution
- Pressure Acoustic, Frequency Domain was used.
  - the interface solves the Helmholtz equation

$$\nabla * \left(-\frac{1}{\rho_c}(\nabla p_t - q_d\right) - \frac{k_{eq}^2\, p_t}{\rho_c} = Q_m \quad \text{(eq.5)}$$

$$p_t = p + p_b$$

Pressure solved for

$$k_{eq}^2 = \left(\frac{\omega}{c_c}\right)^2$$

$$\omega = 2\pi f$$

- The assumption made is that the duct is "acoustically" closed why?
  1- at the inlet we have converging nozzle with contraction ratio of 28/1.
  2- the end of the duct is connected to an exhaust, which does not contribute on the acoustic field.
  3- calculation and simulations values matched the experimental value.
- No-flow simulation.

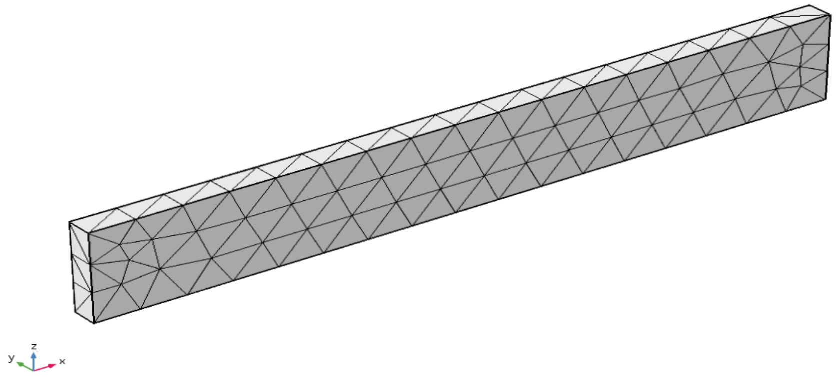


Figure 10: fine free tetrahedral mesh

Case 1	**Close-close**
Fund Longitudinal	279.817 Hz
Case 2	**Close-open**
Fund Longitudinal	139.909 Hz

Table 1: calculated values of the fundamental longitudinal mode by using equations 2 and 3.

Experimental Results

- The low-frequency oscillation was about 280 cps.
  - corresponds to the fundamental longitudinal mode (x-axis)
- Of particular interest the high-frequency oscillation mode that was about 3800 cps.
  - corresponds to an antisymmetric transverse mode across the 4 in. dimension of the duct
  - ***it is accompanied by vortices shed having the same frequency.***

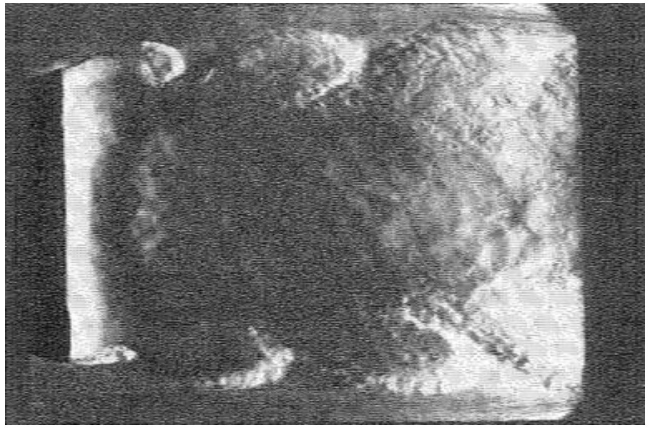


Figure 6: The vortex shedding accompanied with the high-frequency oscillations "screech". [ref.1]

- The driving mechanism:

The transverse velocity waves associated with transverse pressure standing wave → Vortex formed and moved into the area behind the flameholder (hot zone) → Vortex contains combustible materials. → The combustion of these material in the hot zone generates pressure waves (heat release oscillation) → Pressure waves produced excites one of the natural acoustic mode of the duct, if they are in phase.

- In the present case the vortex is off center in the duct, therefore the antisymmetric transverse mode is excited.

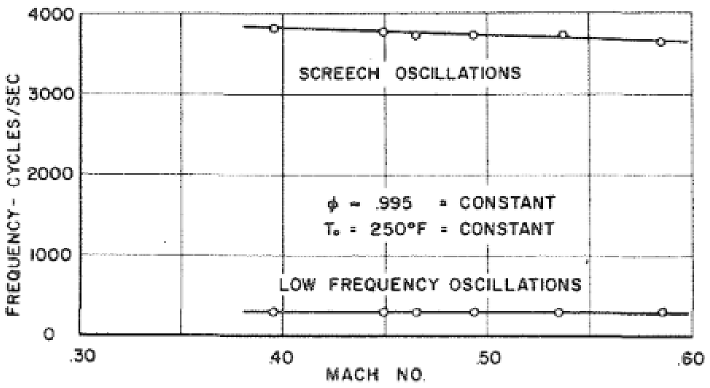


Figure 7: both modes are unaffected by flow

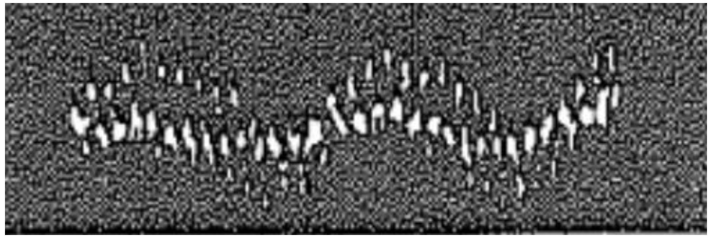


Figure 8: the shape of two modes were measured experimentally.

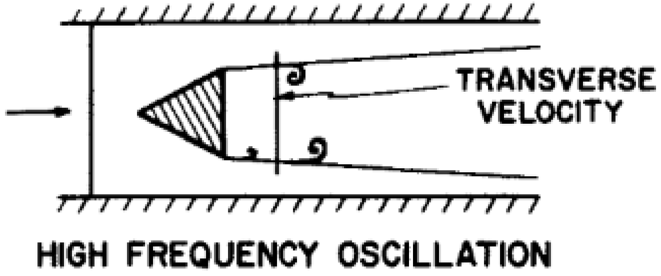


Figure 9: Vortex formation due to transverse velocity waves.

COMSOL Results

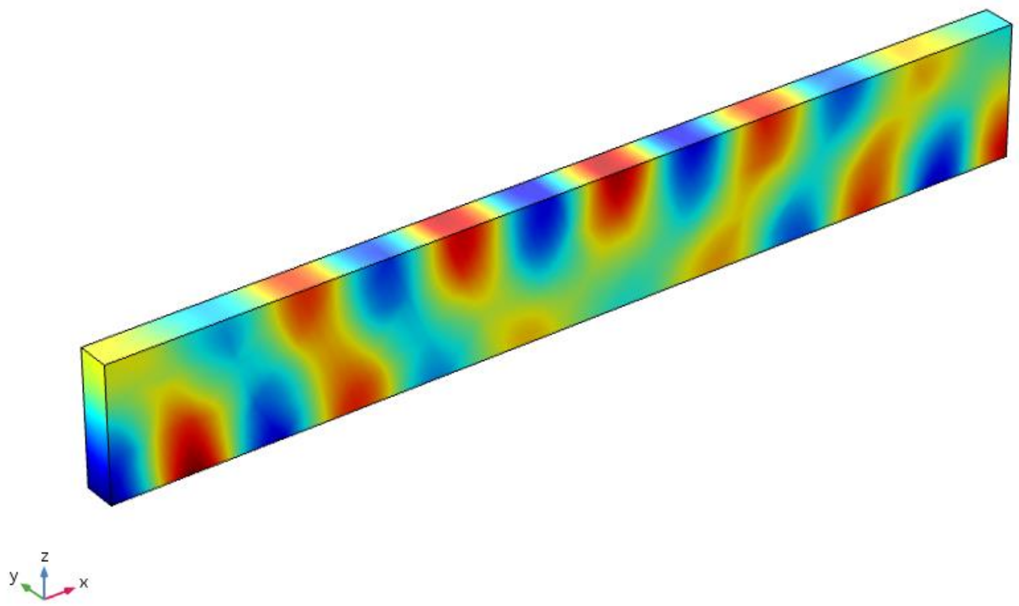


Figure 11: 3D plot of the high-frequency oscillation, which is around 3684 Hz. (antisymmetric transverse mode along the z-axis).

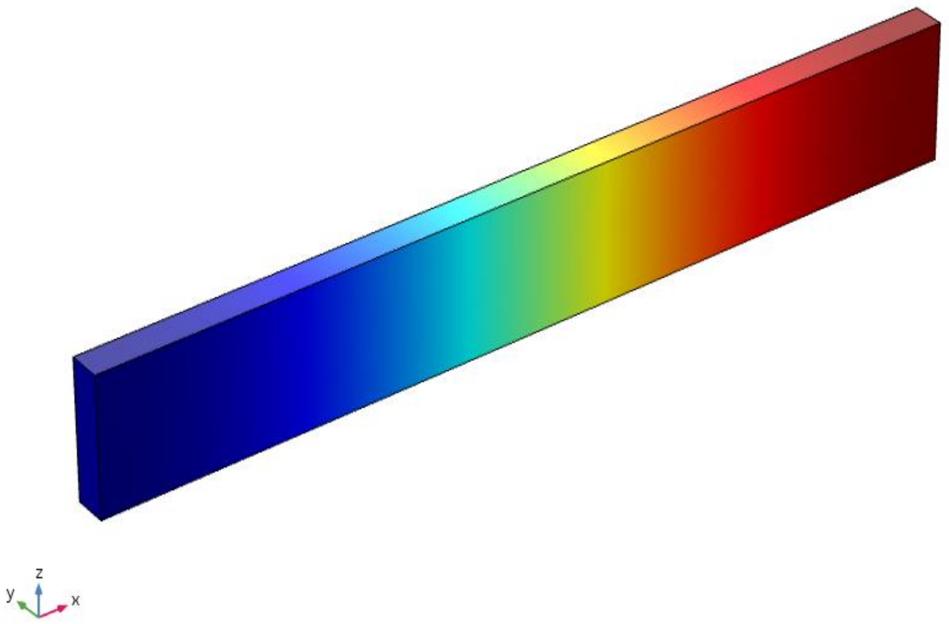
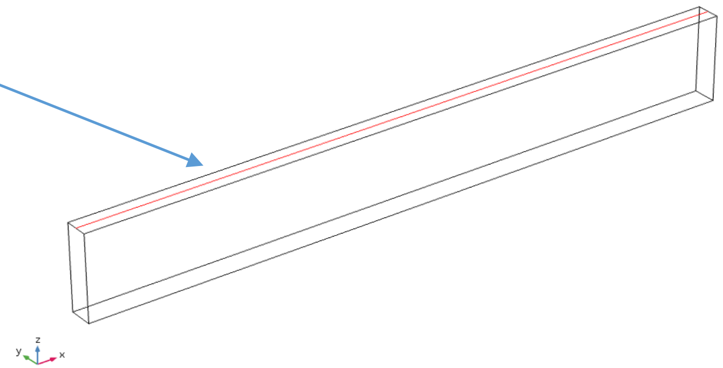


Figure 12: 3D plot of the low-frequency oscillation, which is around 279.57 Hz (the fundamental longitudinal mode of the duct).

Figure 13: a line was draw along the x-axis to find the behavior of the mods in the longitudinal direction.



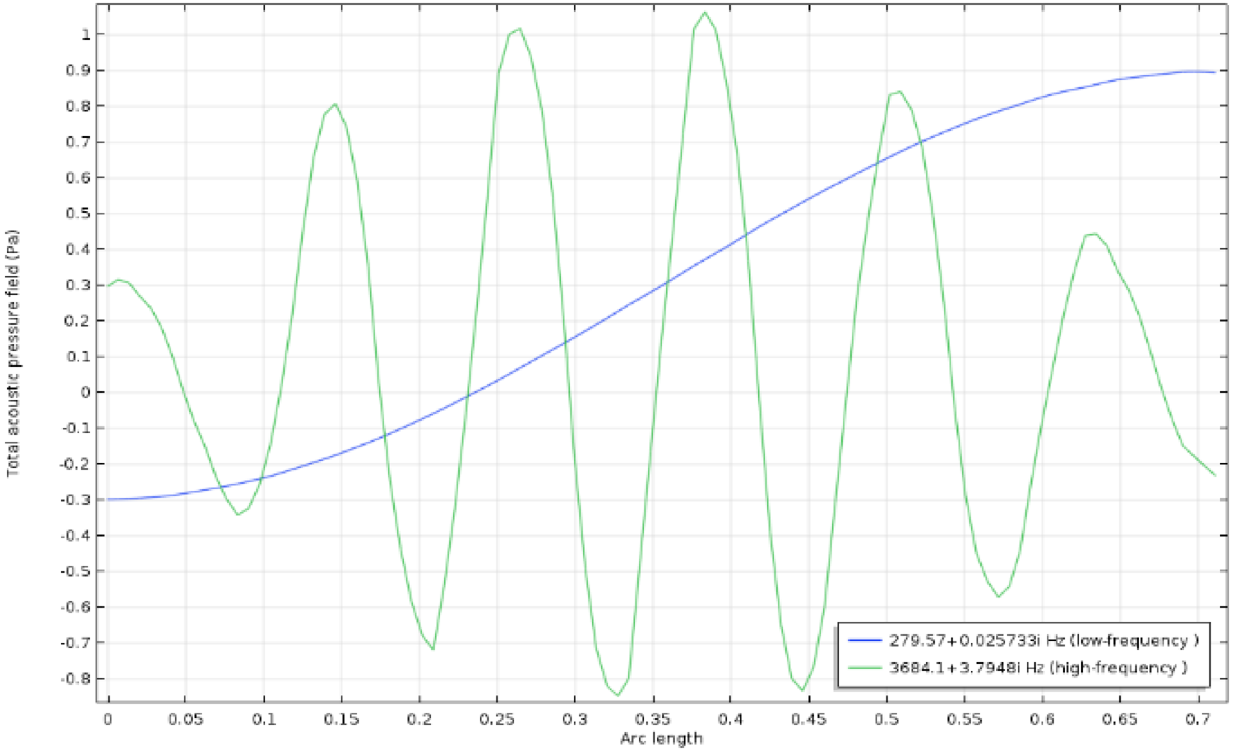


Figure 14: The acoustic pressure variations of the two modes along the duct.

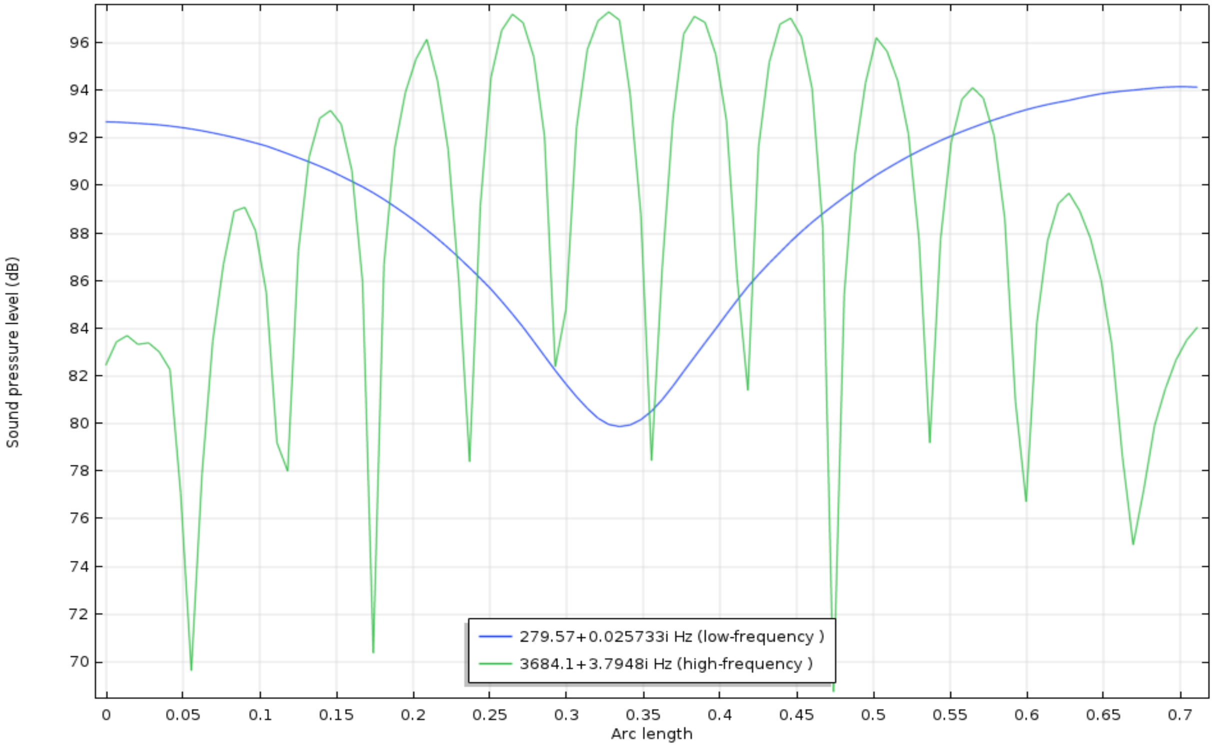
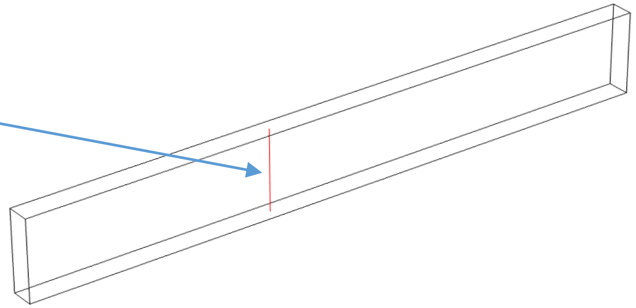


Figure 15: the SPL of the two modes along the duct (x-axis).

Figure 16: this line represent the gage pressure, which was located normal to the surface in the z-axis and 11 in. away from the inlet along the x-axis .



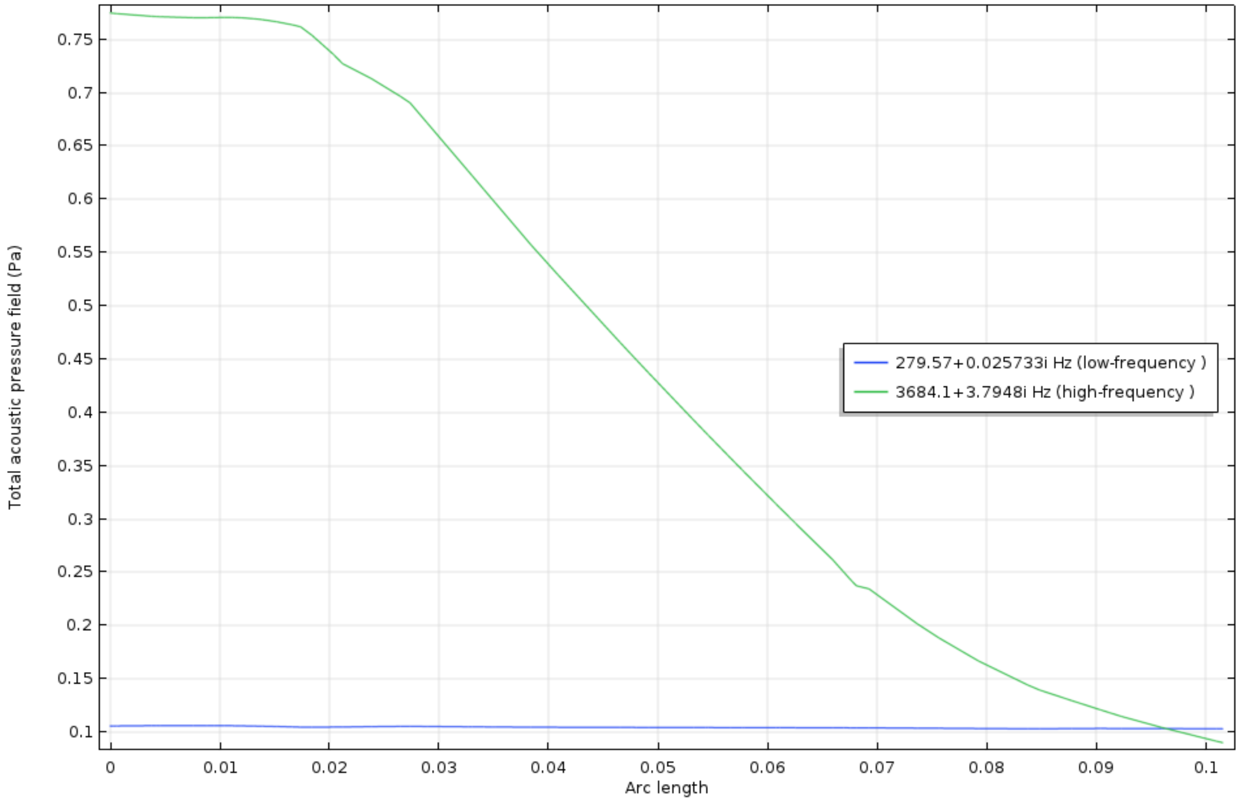


Figure 17 : The acoustic pressure filed of the modes along the z-axis, where the pressure gage was located.

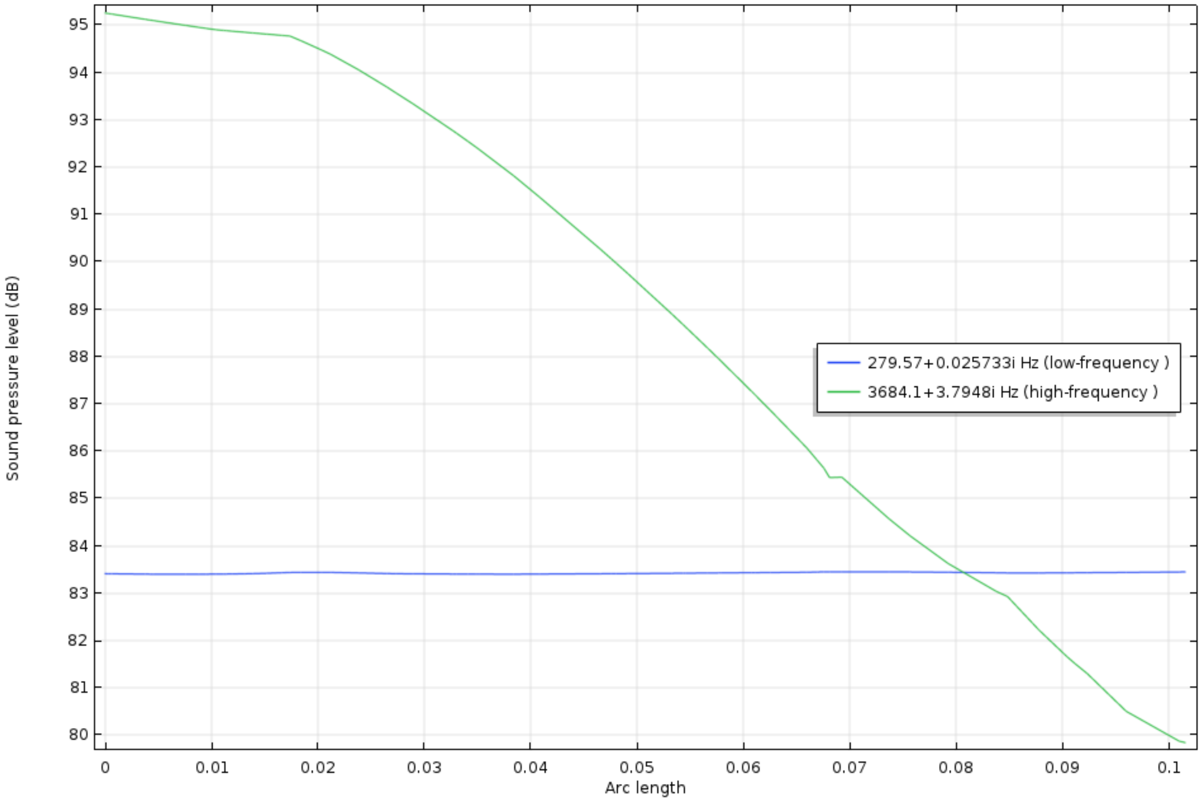
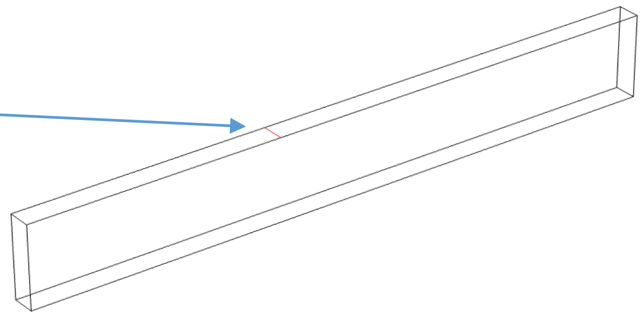


Figure 18: the SPL of modes along the z-axis. The SPL of the screech frequency is about 96 dB.

Figure 19: a third line was drawn along the y-axis to gain more information about the behavior of the modes. At the same location of the pressure gage 11 in. away from the inlet.



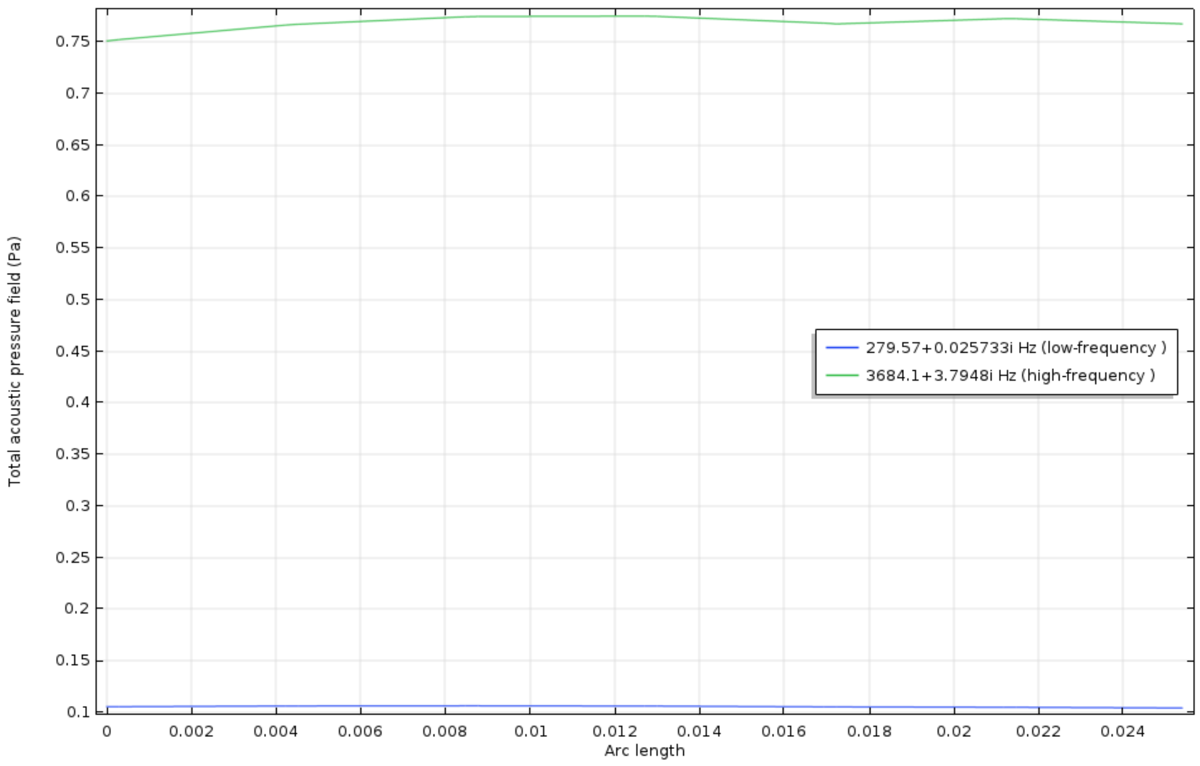


Figure 20: the acoustic pressure field of the modes along y-axis.

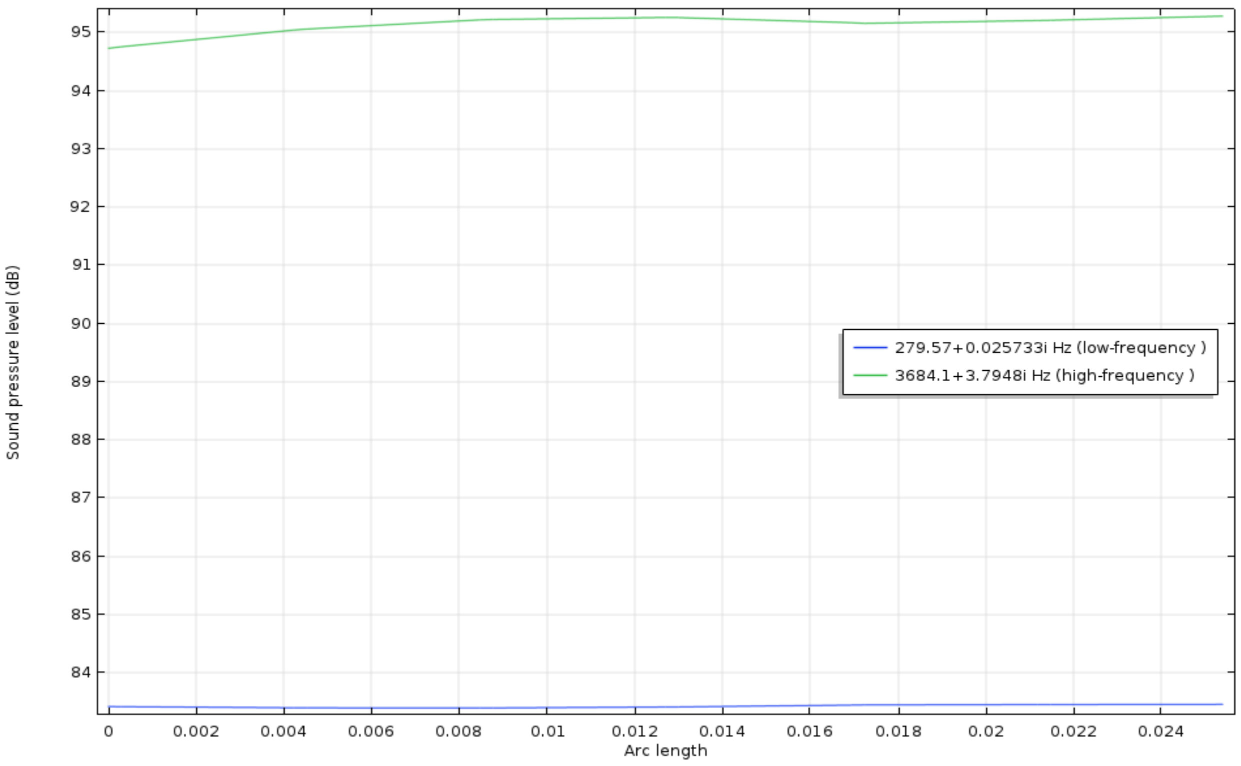


Figure 21: the SPL of the modes along y-axis.

Observations and Validations

- The simulations values were in a great agreement with the experimental values as shown in table2.
- Both showed that low-frequency oscillation was the fundamental longitudinal mode and the high-frequency oscillation was the fundamental antisymmetric transvers mode along the z-axis.
- From COMSOL results we can see that the high-frequency mode is rapidly changing in the longitudinal direction (x-axis). While the low-frequency has no interference along z-axis.
- Both modes are constant along the y-axis.
- The low-frequency mode will probably be excited at the inlet or outlet. The high-frequency will be excited at several location of the central region of the duct.
- COMSOL simulation validated that the oscillation frequencies are not effected by flow.
- Both modes have ½ wavelength pattern but in different directions.

	Low-frequency oscillation	High-frequency oscillation
Experimental results	285 cps	3800 cps
COMSOL results	279.57 Hz	3684 Hz

Table 2: comparison between experimental and COMSOL results.

COMSOL® App for Designing Afterburner Ducts

- COMSOL® CAA analysis provide more information about the behavior of excited instabilities inside the duct.
- COMSOL helps to estimate the location of the excitation.
- COMSOL Analysis can help to analyze the impacts of geometry and location of flameholders and fuel injectors on instabilities:

  1- By easily varying the dimensions of the duct.
  2- Suggesting the proper location of the flameholder and fuel injectros.

- Future work: Application software to design afterburners

  -saves time and reduces cost.

Figure 22: An afterburner under operation

THANK YOU!