Magnetic Control of Deformation of a Ferrofluid Droplet in Simple Shear Flow

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INTRODUCTION: A detailed investigation of the effect of uniform magnetic field on the deformation of the ferrofluid droplet in a two dimensional (2D) simple shear flow by means of numerical simulation is presented here. In this case, the magnetic field is applied perpendicular to the flow field domain.

COMPUTATIONAL METHODS: The conservative level set method is used to track the dynamic interface of the droplet where the level set function is advected by the velocity field[1,2]:

$$\frac{d\phi}{dt} + \mathbf{u}\cdot\nabla\phi = \gamma\nabla\cdot\left(\varepsilon\nabla\phi - \phi(1-\phi)\frac{\nabla\phi}{|\nabla\phi|}\right)$$

Being treated as a single phase flow, the different properties of the flow domain are related to ϕ through the following equations:

$$\rho = \rho_c + (\rho_d - \rho_c)\phi;\quad \eta = \eta_c + (\eta_d - \eta_c)\phi$$

$$\mu = \mu_c + (\mu_d - \mu_c)\phi;\quad \chi = \chi_c + (\chi_d - \chi_c)\phi$$

The flow field under the effect of uniform magnetic field can be governed by the continuity and momentum equations:

$$\nabla\cdot\boldsymbol{u} = 0$$

$$\rho\left(\frac{\delta\mathbf{u}}{\delta t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \nabla\cdot\boldsymbol{\tau} + \mathbf{F}_\sigma + \mathbf{F}_m$$

where, the surface tension force, $\mathbf{F}_\sigma$ can be defined as:

$$\mathbf{F}_\sigma = \nabla\cdot[\sigma\{\mathbf{I} + (-\mathbf{n}\mathbf{n}^T)\}\delta]$$

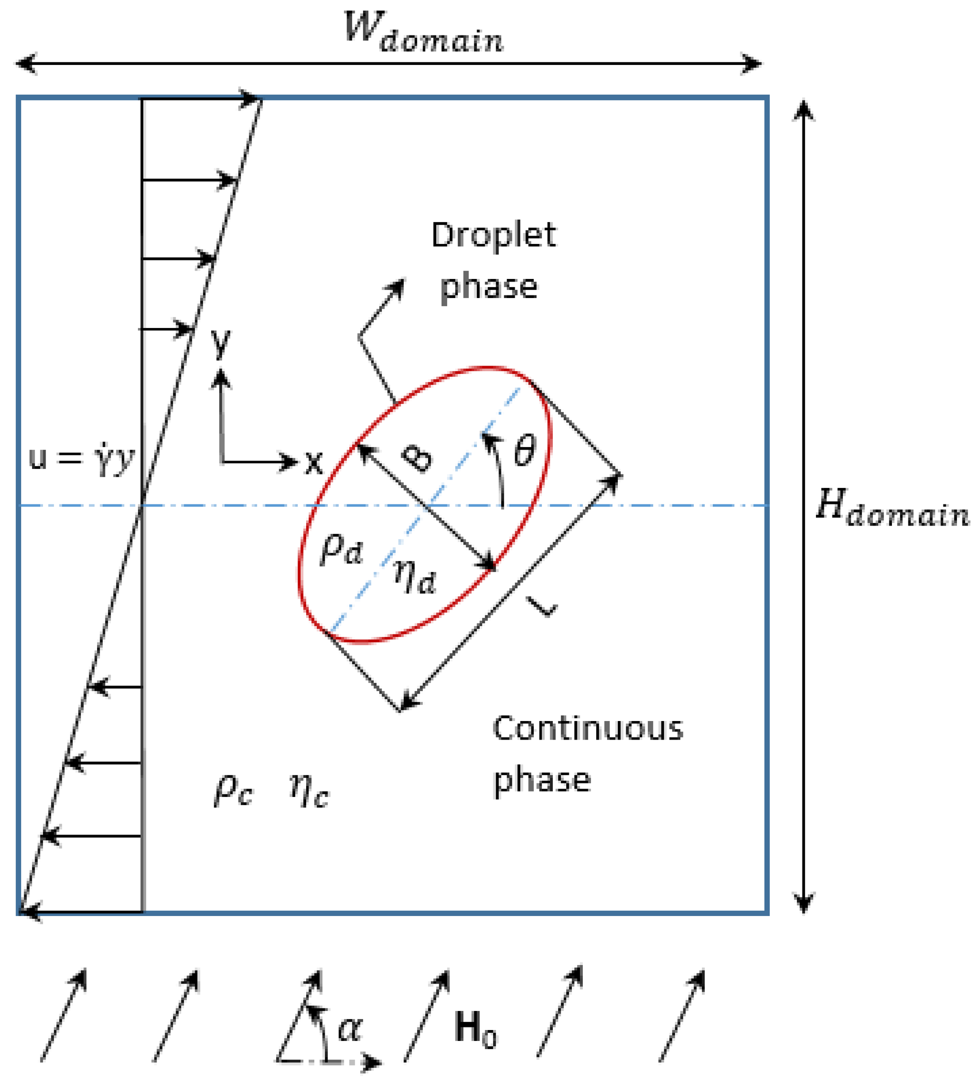
and magnetic force, $\mathbf{F}_m$ can be calculated as:

$$\mathbf{F}_m = \nabla\cdot\boldsymbol{\tau}_m = \nabla\cdot\left(\mu\mathbf{H}\mathbf{H}^T - \frac{\mu}{2}H^2\mathbf{I}\right)$$

The magneto-static Maxwell equation can be written as:

$$\nabla\cdot\mathbf{B} = 0,\ \nabla\times\mathbf{H} = 0,\ \nabla\cdot(\mu\nabla\varphi) = 0$$

$$\mathbf{M} = \chi\mathbf{H}\ \text{ and }\ \mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}) = \mu_0(1+\chi)\mathbf{H}$$



Dimensionless Groups:

- $\mathrm{Re} = \frac{\rho_c R_0^2 \dot{\gamma}}{\eta_c}$
- $\mathrm{Ca} = \frac{\eta_c R_0 \dot{\gamma}}{\sigma}$
- $\mathrm{Bo_m} = \frac{R_0 \mu_0 H_0^2}{2\sigma}$

Figure 1. Schematic illustration of a ferrofluid droplet suspended in another viscous fluid in a simple shear flow under a uniform magnetic field, $\mathbf{H}_0$.

RESULTS: The effect of different shear flow rates and magnetic field strengths on the deformation of the ferrofluid droplet deformation

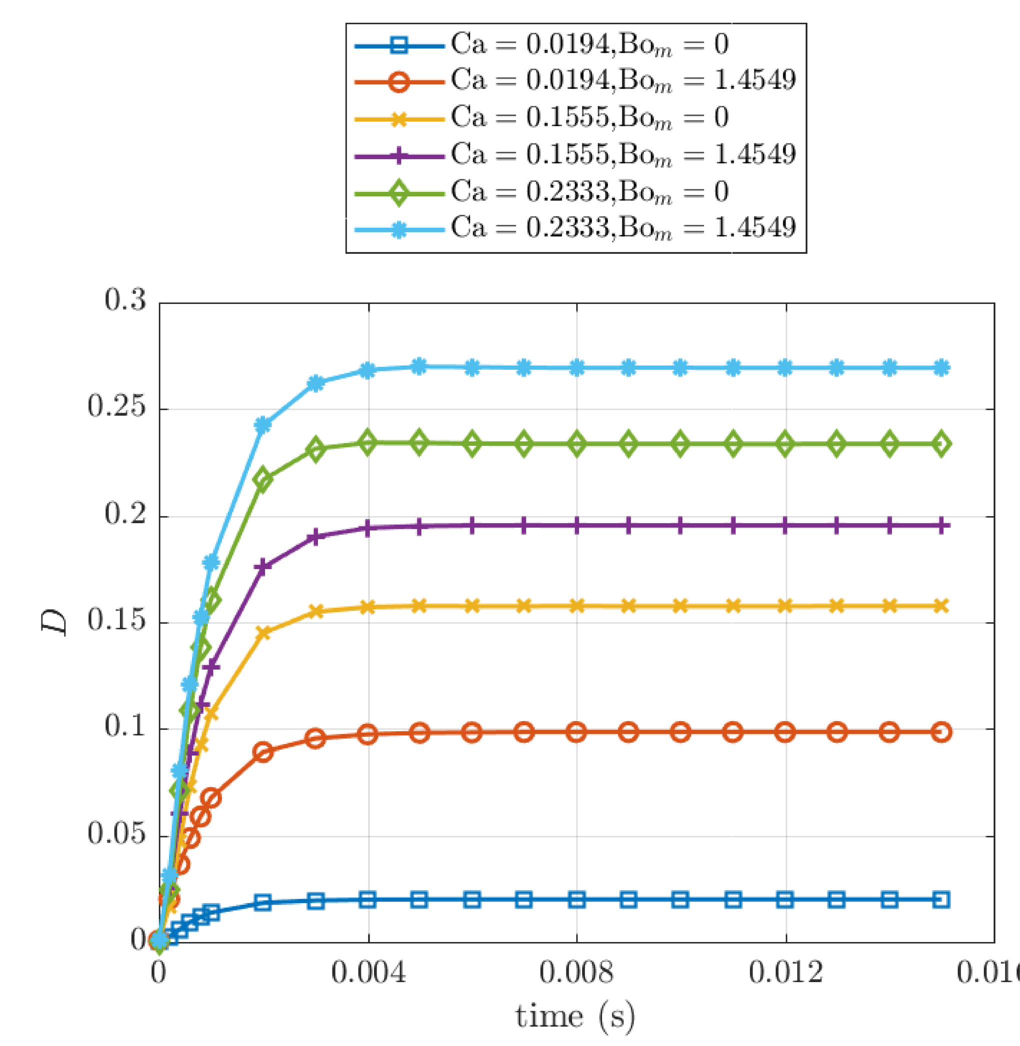


Figure 2. Effect of a perpendicular magnetic field on the deformation, D of the ferrofluid droplet against time.

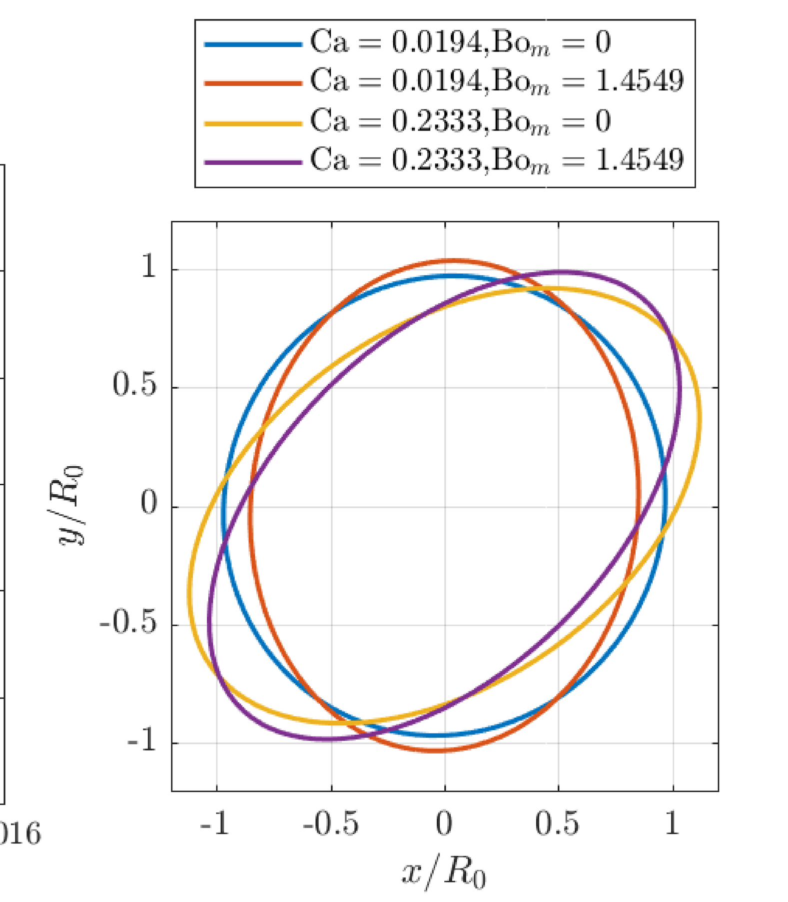


Figure 3. Outline of the droplet shape at steady state

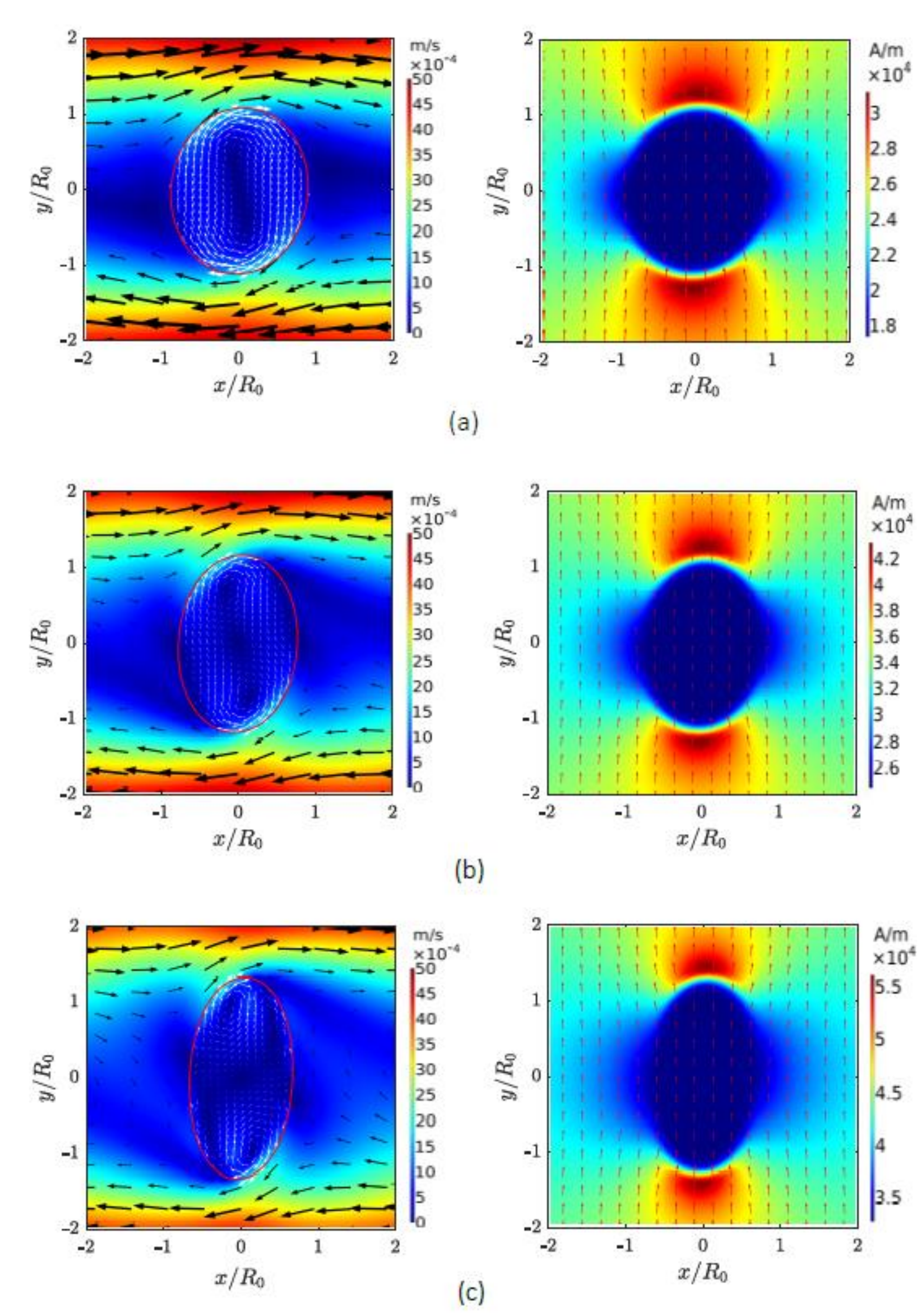


Figure 4. Velocity and magnetic field for Ca = 0.0194, Re = 0.0015 at α = 90°. (a) $\mathrm{Bo_m}$ = 1.4549, (b) $\mathrm{Bo_m}$ = 2.8515, (c) $\mathrm{Bo_m}$ = 4.7138.

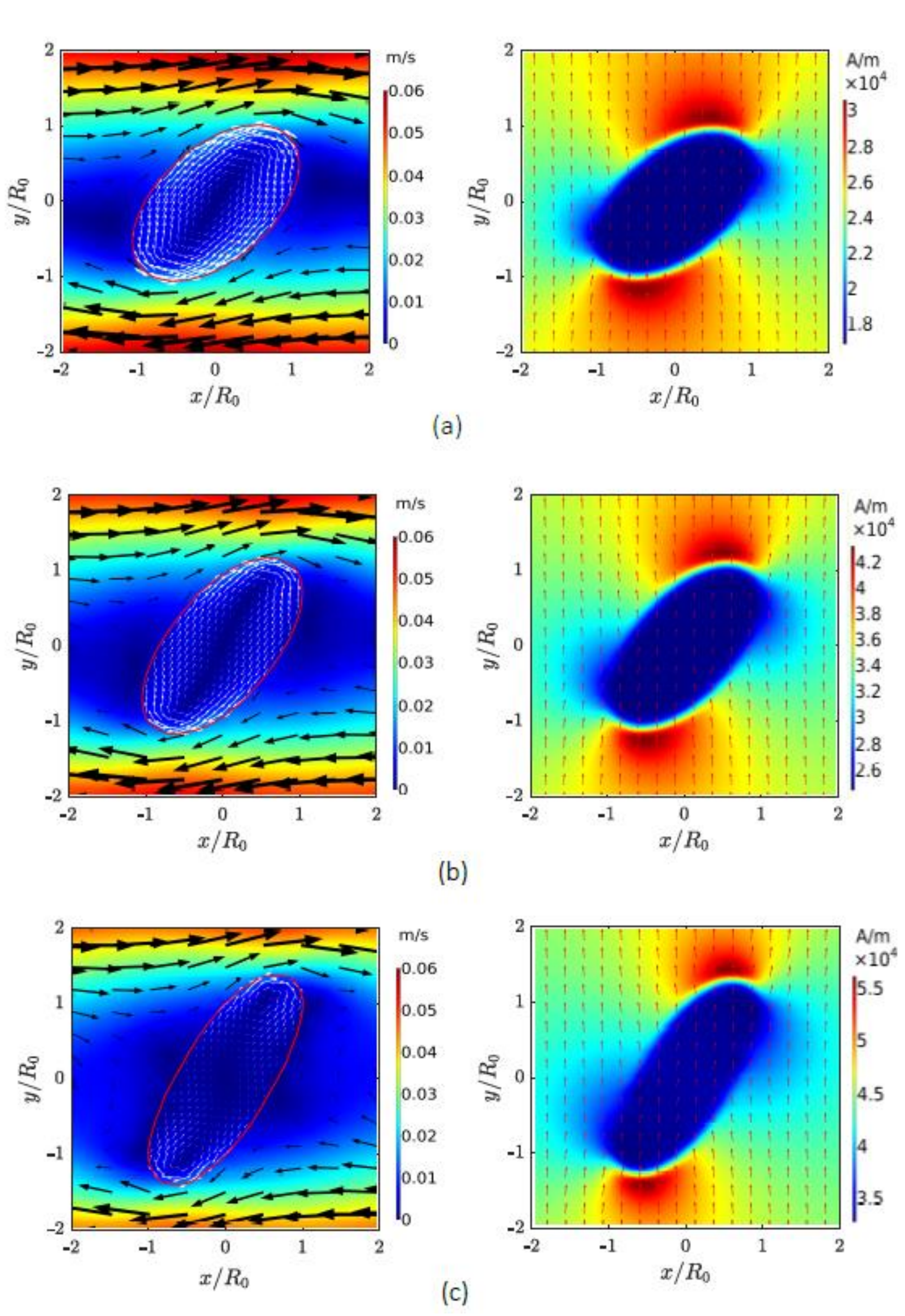


Figure 5. Velocity and magnetic field for Ca = 0.2333, Re = 0.018 at α = 90°. (a) $\mathrm{Bo_m}$ = 1.4549, (b) $\mathrm{Bo_m}$ = 2.8515, (c) $\mathrm{Bo_m}$ = 4.7138.

CONCLUSIONS: At a low shear rate (Ca ≅ 0.02), the deformation of the ferrofluid droplet is controlled by the magnetic field effect due to its dominant nature over shear flow effect while for the high shear rate (Ca ≅ 0.25), the deformation of the ferrofluid droplet is predominantly determined by the shear flow although the magnetic field has a considerable effect at higher strengths.

References:

1. COMSOL, "CFD Module Application Library Manual." .
2. E. Olsson and G. Kreiss, "A conservative level set method for two phase flow," *J. Comput. Phys.*, vol. 210, no. 1, pp. 225–246, 2005.