Forces on parallel three-phase AC-conductors during a phase to ground fault

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Introduction

We consider the problem of calculating forces on high current solid conductors, as is present in various types of electrical installations e.g. in substations [1]. An example of such an installation with three parallel conductors is shown in Figure 1. The conductor forces are important for the design of the station, in particular for the conductor geometry and mechanical support.

The largest electromagnetic forces are experienced during short circuit fault conditions. In this paper we considered the most common fault which is a ground fault in one of the three phases. The full calculation includes a treatment of the skin effect due to the AC-currents and the relatively large cross sections of the conductors. This means that the non-homogenous current densities are calculated in all conductors. Then forces on each of the conductors are calculated and studied as function of short circuit current and geometry.

Figure 1. ABB static var compensator outside Dallas, US. The three long parallel conductors in the left part of the photo shows an example that have motivated the conductor setup considered in this project. Picture by ABB.

Theory

The relationship between the current and the magnetic field, is given by Ampère's law:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}. \quad (1)$$

Here, ***H*** is the magnetic field strength and ***J*** the current density. From this relation, it is clear that an alternating current will produce an alternating magnetic field. This alternating magnetic field, in turn, induces eddy currents in the conductor which enforces the current near the surface of the conductor and counteracts it near the center of the conductor. As a result, the majority of charge transport occurs close to the surface of the conductor. This is known as the skin effect. As the frequency of the current increases, the skin effect becomes more pronounced. The distance from the surface where the current density has decreased to $e^{-1} \approx 0.37$ of the value at the surface of the conductor is known as the skin depth, δ, and can be calculated as $\delta = \sqrt{\frac{2\rho}{\omega\mu}}$ for cylindrical conductors and angular frequencies $\omega \ll 1/\rho\epsilon$. Here, ρ is the resistivity of the conductor, μ the magnetic permeability, and ϵ the electric permittivity.

Forces on one of two similar line-like conductors separated by a distance a is given by the Ampère's force law:

$$\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{a}, \quad (2)$$

which is about $F=12$ N/m ($L=1$ m) for $I_1=1.0$ kA, $I_2=30$ kA, $\mu_0=1.257\cdot10^{-6}$ H/m, and $a= 0.5$ m. To set this force in perspective, a comparison with the force on one conductor (aluminum) with radius 50 mm (hollow with a wall thickness of 10mm) due to gravity, gives $F_G=76$ N/m. However, the forces due to AC are pulsating bending forces, which can be critical for the design of the supporting insulators.

A phase to ground fault drastically increases the current along one of the phases. In addition, there is often a large asymmetry in the current directly after the fault occurs depending where on the voltage curve the fault happens. This asymmetry decays after some time. Both effects are included when modeling the fault current in the middle conductor (i.e. I_2) according to:

$$I_1 = I_{1,0}\sin\left(2\pi f t - \frac{2\pi}{3}\right), \quad (3)$$

$$\begin{aligned} I_2 = {} & I_{2,0}\sin(2\pi f t) + \left(I_{short} - I_{2,0}\right) \\ & \cdot step(t - t_0)\left(\sin(2\pi f t) + e^{\frac{t-t_0}{\tau}}\right), \quad (4) \end{aligned}$$

$$I_3 = I_{3,0}\sin\left(2\pi f t + \frac{2\pi}{3}\right). \quad (5)$$

Above *step(·)* is a smoothed Heaviside function with width one time-period *T=1/f*. In Figure 6 we have plotted the above currents for the following parameter values: $I_{j,0} = 1.0\ kA\ (j = 1,2,3)$, $I_{short} = 30\ kA, t_0 = 0.08\ s, f = 50\ Hz, \tau = 0.02\ s.$ Note that all the currents are specified as peak values rather than rms.

Simulation method

We have used the Magnetic Fields interface within the AC/DC Module [2]. The equations to be solved are those defined by Ampère's law in Comsol:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}, \quad (6)$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}, \quad (7)$$

$$\boldsymbol{J} = \sigma \boldsymbol{E}. \quad (8)$$

Above, $\boldsymbol{H}$ is the magnetic field strength, $\boldsymbol{J}$ the current density, $\boldsymbol{B}$ the magnetic flux density, $\boldsymbol{A}$ the magnetic vector potential, σ the electrical conductivity, and $\boldsymbol{E}$ the electric field strength.

Focus here are on 2D-geometries, assuming long straight conductors perpendicular to the 2D plane, although also some 3D simulations have been carried out to compare with the 2D results for current densities and as initial tests of more complicated conductor geometries.

For the simulations of the skin effect in a single copper conductor we use a geometry consisting of a circle with radius r = 30 mm. The region with r < 10 mm represents the conductor while the region with r > 10 mm represents the surrounding air. The region with r > 20 mm is defined as an infinite element domain. To obtain the internal inductance, the simulation was carried out without the surrounding air. We use a physics controlled mesh calibrated for general physics with element size set to fine in the wire and normal in the surrounding air. Additionally, we apply eight boundary layers to the boundary of the wire to better capture the variation in current density near the surface.

For the simulations of the short-circuit forces between three parallel wires, we use three circular wires with radius r_{wire} = 50 mm separated by 0.5 m, measured from the center of the wires. The wires are enclosed in a circular domain with radius r = 1.25 m, of which the outermost 0.25 m is defined as an infinite element domain. We again use a physics controlled mesh calibrated for general physics. The element size in the air domain is set to normal and the element size in the wires is set to fine. Eight boundary layers are applied to each wire domain, in analogy with the previous mesh. The resulting mesh is shown in Figure 4. The currents are implemented using the single conductor mode in the coil functionality of Comsol multiphysics.

Assuming stationary AC-currents, *frequency studies* (mainly for 50 Hz) of the forces have been carried out. In addition, the inverse discrete Fourier transform have been applied to create time-series to compare to full *time dependent* simulations.

After such numerical tests were satisfactory, we introduced more realistic models, see Equation (3)-(5), for the currents during a phase to ground short circuit fault and carried out full *time dependent* simulations of the forces. Forces on each part of the conductors can be calculated from the Lorentz force. To obtain the total forces on each conductor, surface integrals were then set up. Alternatively the forces can be calculated via the Maxwell stress tensor with the conductor's circumference defining the path for the corresponding line integral in Greens' formula. After comparing both approaches in Comsol, and finding no significant difference in the results, we used the latter method. The surface stress is given by the expression

$$\boldsymbol{n_1} T_2 = -\frac{1}{2}(\boldsymbol{H} \cdot \boldsymbol{B})\boldsymbol{n_1} + (\boldsymbol{n_1} \cdot \boldsymbol{H})\boldsymbol{B}^T. \quad (9)$$

Here, $\boldsymbol{n_1}$ is the boundary normal pointing away from the conductor and T_2 the stress tensor of air [3], and the force per unit length is obtained by integrating around the circumference of the wires.

To improve convergence we assign a conductivity of 100 S/m to the surrounding air. Finally, we developed a Comsol *Simulation App* containing the functionality for calculating forces discussed above, see Figure 8.

Simulation results

Skin effect: Here we first focused on the current density in one conductor. We have investigated the skin effect in a cylindrical AC-conductor for various frequencies and compared the numerical results for the current density and resistance to well-known analytic models [4]. These results are shown in Figure 2 and Figure 3.

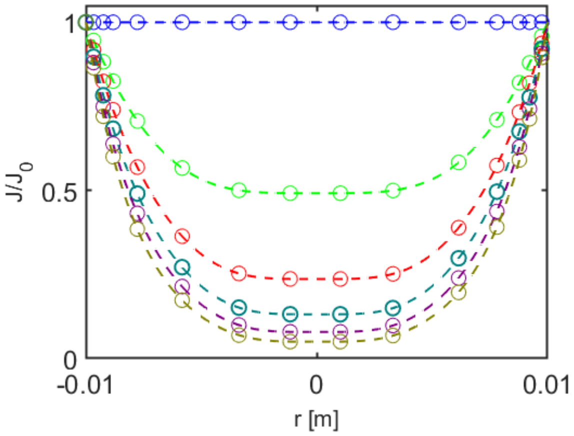


Figure 2. Normalized current densities for AC of different frequencies. The top (straight) line is for $f=0$, corresponding to DC, which have a homogenous current density. Lower curves are for higher frequencies in steps of $200\ Hz$, i.e. the lowest curve is for $f=1.0\ kHz$. Dashed curves are from analytic formulae [4], while open circles shows numerical results from Comsol.

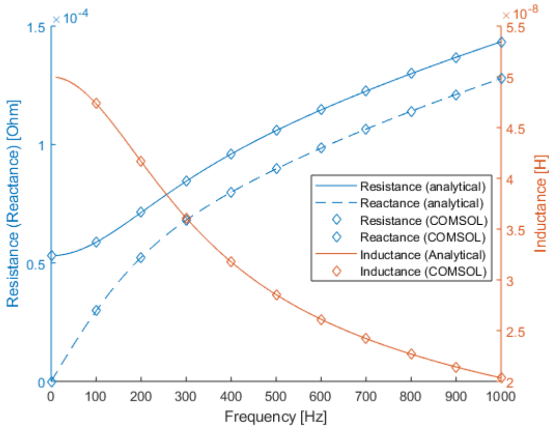


Figure 3. Resistance, reactance and inductance corresponding to Figure 2. The material used for the conductor here is copper.

Forces from stationary AC: The well-known case of forces between two parallel DC conductors, which up until 19th of May 2019 have been the basis for defining the SI-unit ampere [5], can be simulated in a public Comsol model [3]. We extended the model to include three conductors with 3-phase AC, see Figure 4. The currents are defined in Equation (3)-(5) with $I_{2,0}=I_{short}$. We have compared forces calculated from a *time dependent study* with stationary AC with $I_{1,0}= I_{3,0}=1.0\ kA$, $I_{short}=30\ kA$, and $f=50\ Hz$, with forces obtained from a (inverse) Fourier transform of a corresponding *frequency study* for the same frequency. Obtained time-series for forces of stationary AC were sinusoidal as expected, see Figure 5.

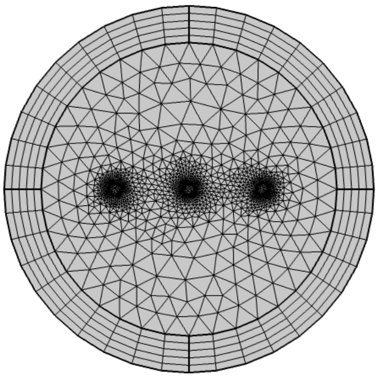


Figure 4. Example mesh for a 2D cross section of three AC-conductors.

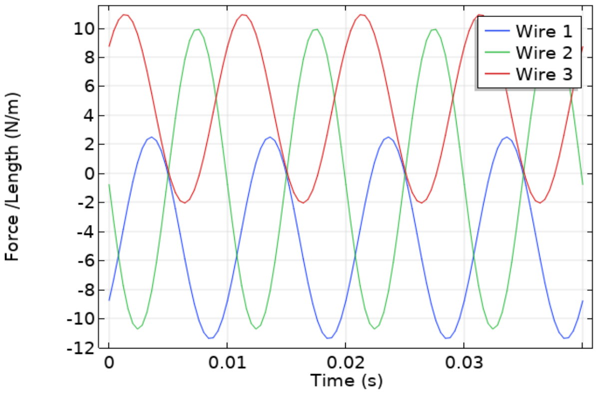


Figure 5. Time series of forces during stationary AC. As an order of magnitude comparison for the amplitude of (e.g.) Wire 2, see the value for two DC conductors under Equation (2).

Forces at short circuits: With the detailed numerical benchmarking reported in the two examples above, we finally simulated the time evolution for forces during a realistic phase to ground fault.

Introducing the realistic short circuit currents of Equation (3)-(5) results in complicated time-series for the forces, see Figure 6 and 7. Those in turn will form important input to mechanical engineers for the design of e.g. substations (Figure 1).

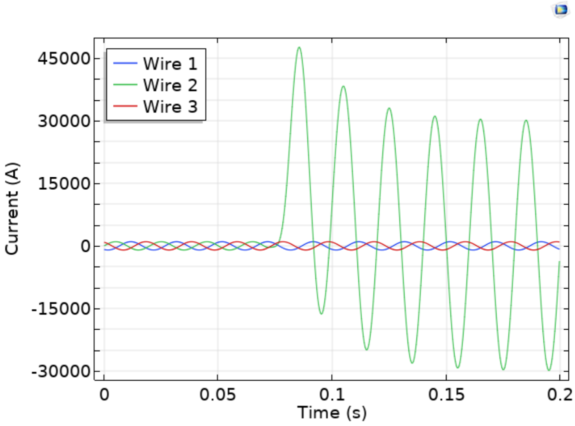


Figure 6. Time series of currents in the three conductors. The current in wire 2 is increased from 1 kA to 30 kA between t = 0.07 s and t = 0.09 s with an offset according to Equation (3).

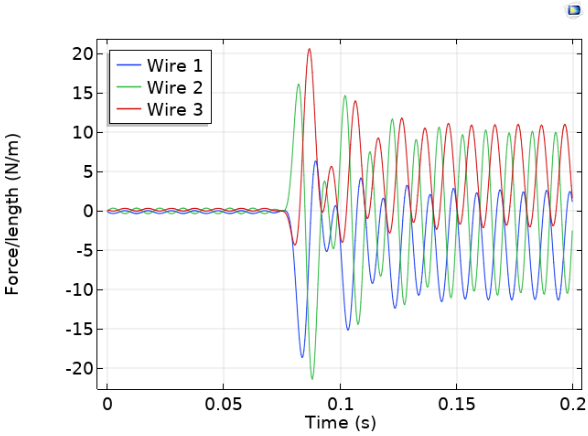
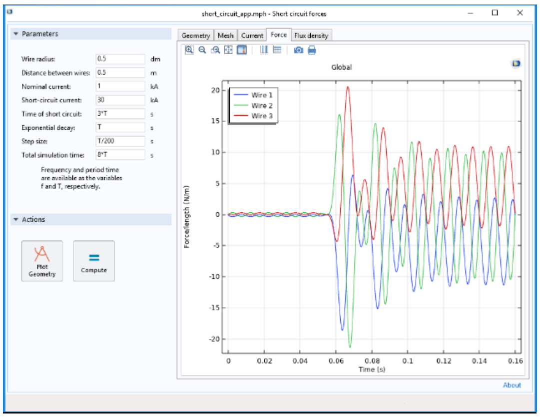


Figure 7. Short circuit forces corresponding to the currents in Figure 6.

After those initial tests we set up a geometry, containing parameters for all positions and distances to be included in a Comsol *Simulation App*, to treat three AC conductors in parallel, see an example of the layout in Figure 8.



Wire radius: 0.5 dm
Distance between wires: 0.5 m
Nominal current: 1 kA
Short-circuit current: 30 kA
Time of short circuit: 3*T s
Exponential decay: T s
Step size: T/200 s
Total simulation time: 8*T s

Frequency and period time are available as the variables f and T, respectively.

Figure 8. User interface of a *Simulation App* created to calculate forces on conductors for phase to ground faults.

Conclusions

We have initially shown that a Comsol implementation using the AC/DC Module reproduces a well-known example for the skin effect. After successfully comparing the outcome from *frequency studies* and *time dependent studies* for stationary AC, we are confident in the results for forces calculated from a *time dependent study* for a non-trivial short circuit current during a phase to ground fault. Finally, we have created a corresponding *Simulation App* to be useful within the electric power industry.

References

[1] IEEE Standard C37.12-2008 - IEEE Guide for Bus Design in Air Insulated Substations.

[2] We are using *Comsol 5.4* equipped with the *AC/DC Module*.

[3] www.comsol.se/model/electromagnetic-forces-on-parallel-current-carrying-wires-131

[4] Jordan, Edward Conrad (1968), *Electromagnetic Waves and Radiating Systems,* Prentice Hall, ISBN 978-0-13-249995-8.

[5] https://en.wikipedia.org/wiki/2019_redefinition_of_SI_base_units#Ampere