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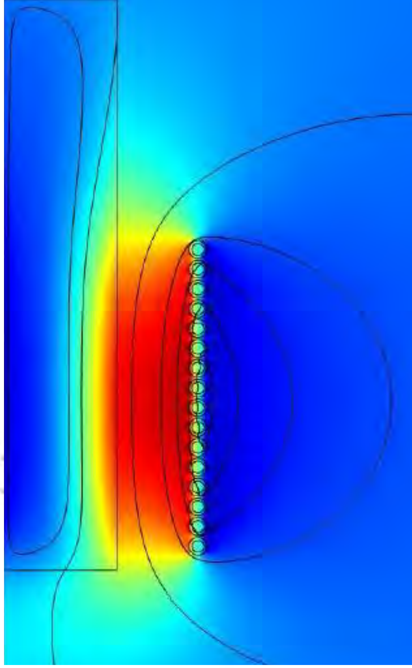


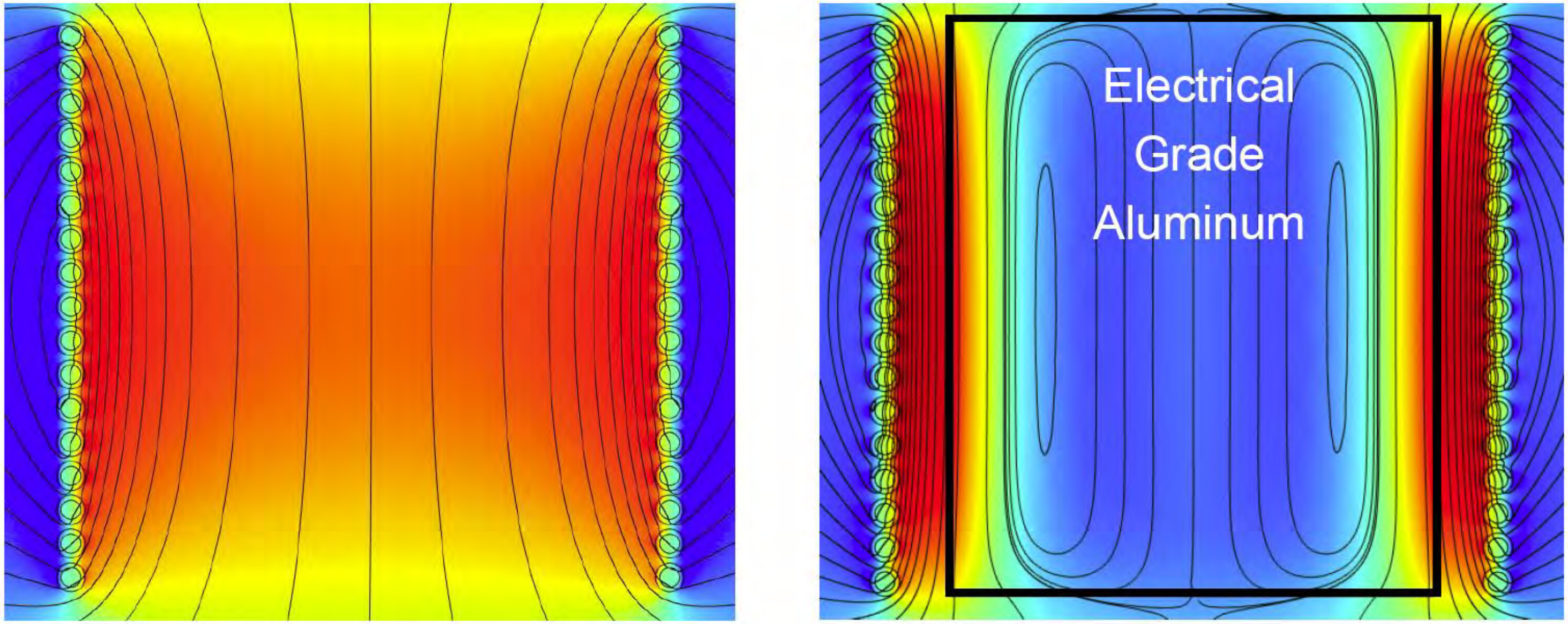
Analytical and Experimental Validation of Electromagnetic Simulations Using COMSOL®, re Inductance, Induction Heating and Magnetic Fields

Presented by M.W. Kennedy

Co-Authors: S. Akhtar, J.A. Bakken, R.E. Aune

Excerpt from the Proceedings of the 2011 COMSOL Conference in Stuttgart

Impact of work piece on magnetic field distribution and relative strength, 50 Hz



Infinite coil:

$$\left|\overrightarrow{B_\infty}\right| = \frac{\mu_o \mu_r N_c I_c}{l_c}$$

Short coil:

$$\left|\overrightarrow{B_o}\right| = k_N \left|\overrightarrow{B_\infty}\right|$$

Coil with a work piece:

$$k_N^{\ *} = k_N\left(1-\left(\frac{D_w-\delta_w}{D_c+\delta_c}\right)^2\right)+\left(\frac{D_w-\delta_w}{D_c+\delta_c}\right)^2$$

Short coils have non-uniform fields both axially and radially. The field is strongly influenced by the work piece/coil geometry and the electromagnetic penetration depth in the work piece.

Modelling of magnetic fields and induction heating with COMSOL®

- Should the coil be voltage or current driven?

- How big does the magnetic domain need to be to simulate an infinite external volume? i.e. when is the coil flux density estimated with 100% accuracy for a given applied magneto-motive force (NI)?

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Modelling of magnetic fields and induction heating with COMSOL®

- Which domain, “single-turn” or “multi-turn” can be used and under what circumstances?

- What mesh is required to obtain accurate results at different frequencies? How do we relate this to the physics?

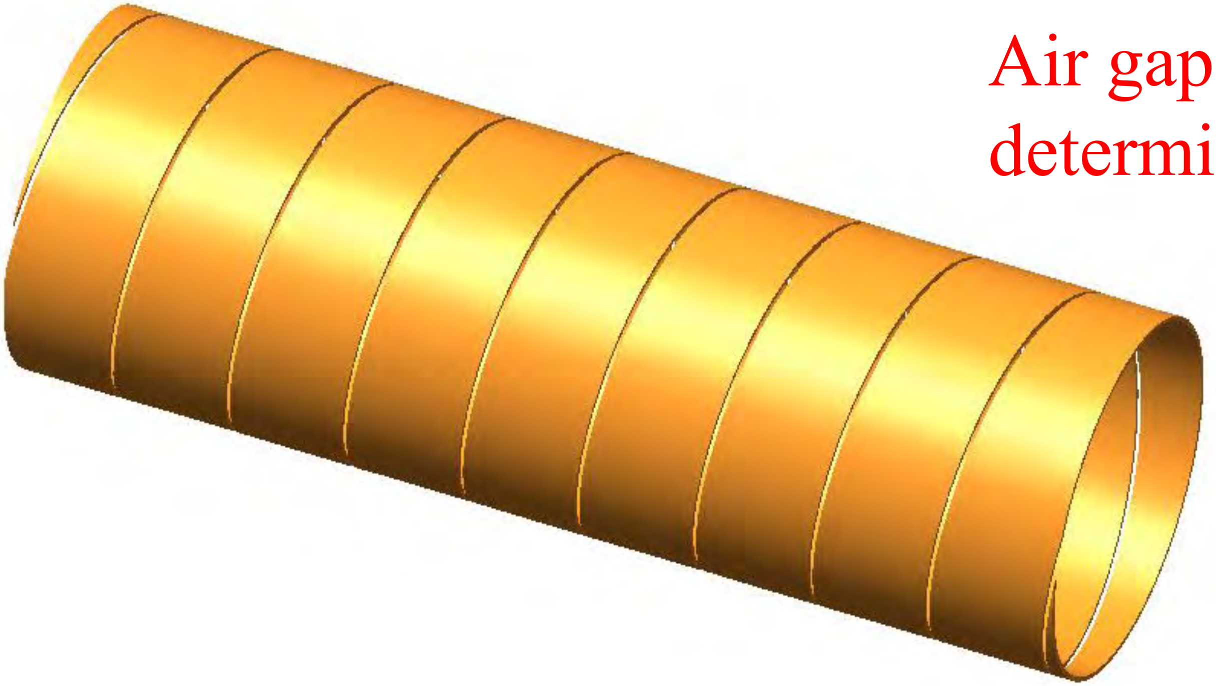
- How accurately can a 2D axial symmetric model estimate magnetic fields and heating rates for cylindrical work pieces in experimental helical coils?

How to find the required magnetic domain size?

$$L_o = \frac{8\mu_0 N^2 r^3}{3\ell^2}\left[\frac{2k^2-1}{k^3}E(k) + \frac{1-k^2}{k^3}K(k) - 1\right] \quad k = \sqrt{\frac{4r^2}{4r^2+\ell^2}}$$

$$L_o = \frac{k_N A_c N_c \left|\overrightarrow{B_\infty}\right|}{I_c}$$

k_N = Nagaoka short coil correction factor. Can be solved to double precision accuracy.



Air gap flux density^2 determines the heating rate!

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Comparison of COMSOL® and analytical inductance of a current sheet

Ratio of Magnetic Domain Dimensions to Coil Dimensions	COMSOL Calculated Inductance (µH)	COMSOL - Analytical Solution Difference (%)
2.00	22.7563	-13.82
4.00	25.9502	-1.72
6.00	26.2783	-0.48
10.00	26.3870	-0.07
14.00	26.4057	0.00
20.00	26.4129	0.03

Error in inductance is the same as for the flux density and is then squared when calculating heating rate!

Theoretical answer = 26.4051 µH.
Ratio of 14 gives ideal results.

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Induction heating instrumentation

Electrical conductivity accuracy of +/- 0.5%

Standards +/- 0.01% IACS

Magnetic field measurements

Axial/Transverse

From 0.1µT-30T

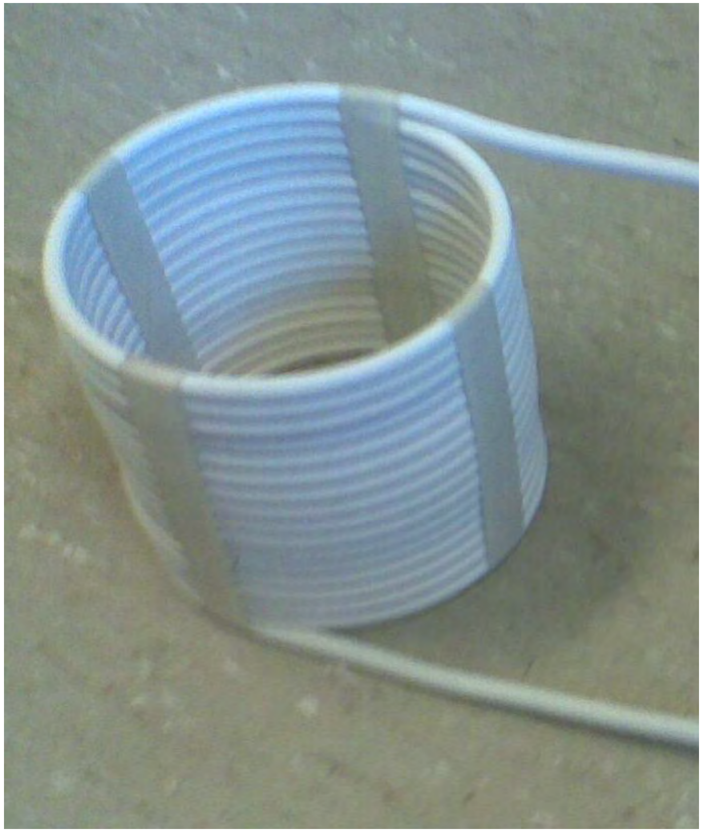
+/- 1.0% AC

Standards from 500-2000 Gauss

Electrical analysis:

1. V, I, P (+/-100 W), p.f.
2. Inductance
3. Harmonics
4. Current +/- 1% (usable up to 100 kHz)

Coil and work piece



6 mm by 1 mm wall, DHP copper
80% IACS electrical conductivity
Insulated with glass fibre sleeves

A356 aluminium alloy
48% IACS conductivity

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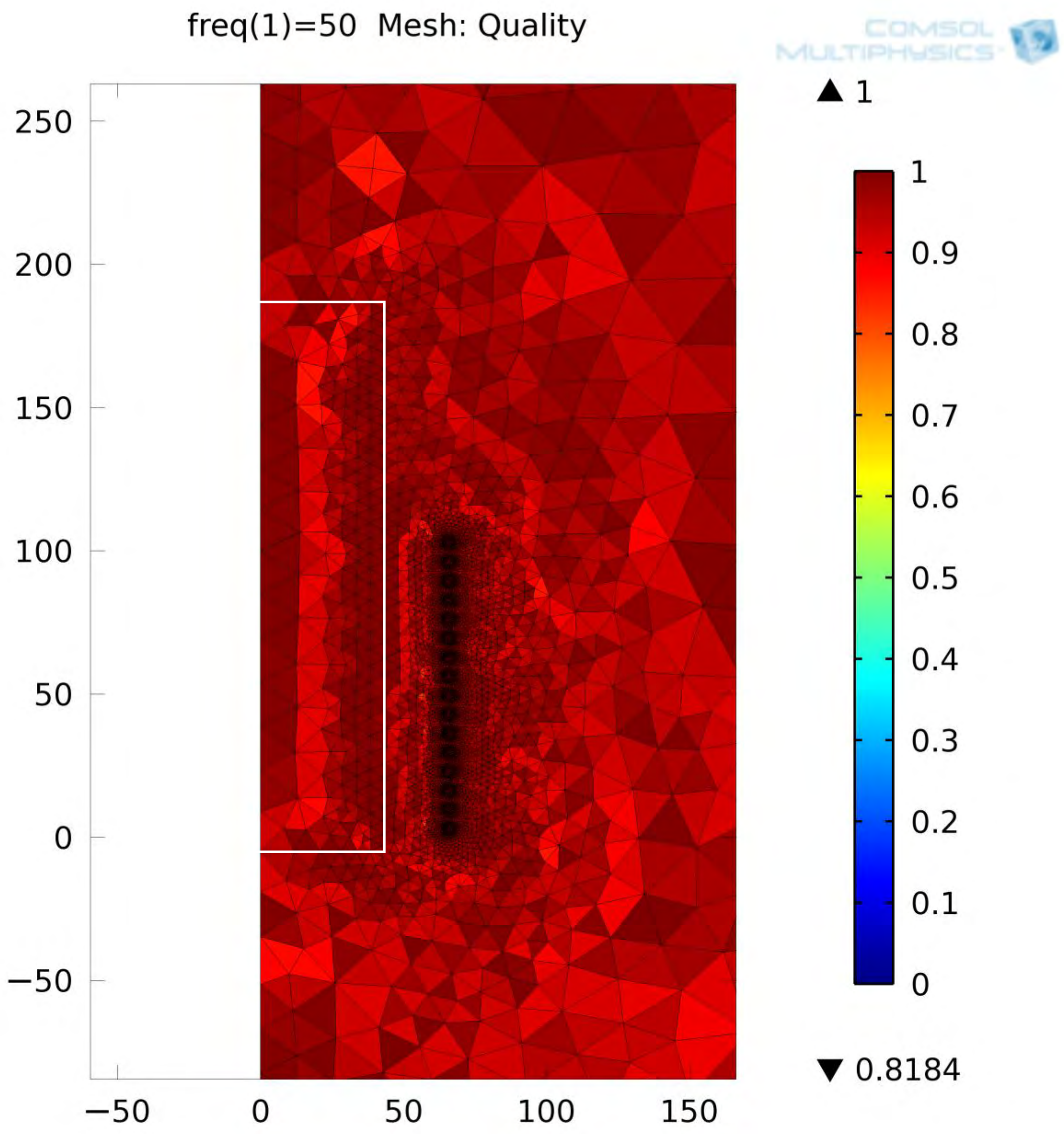
Mesh 1

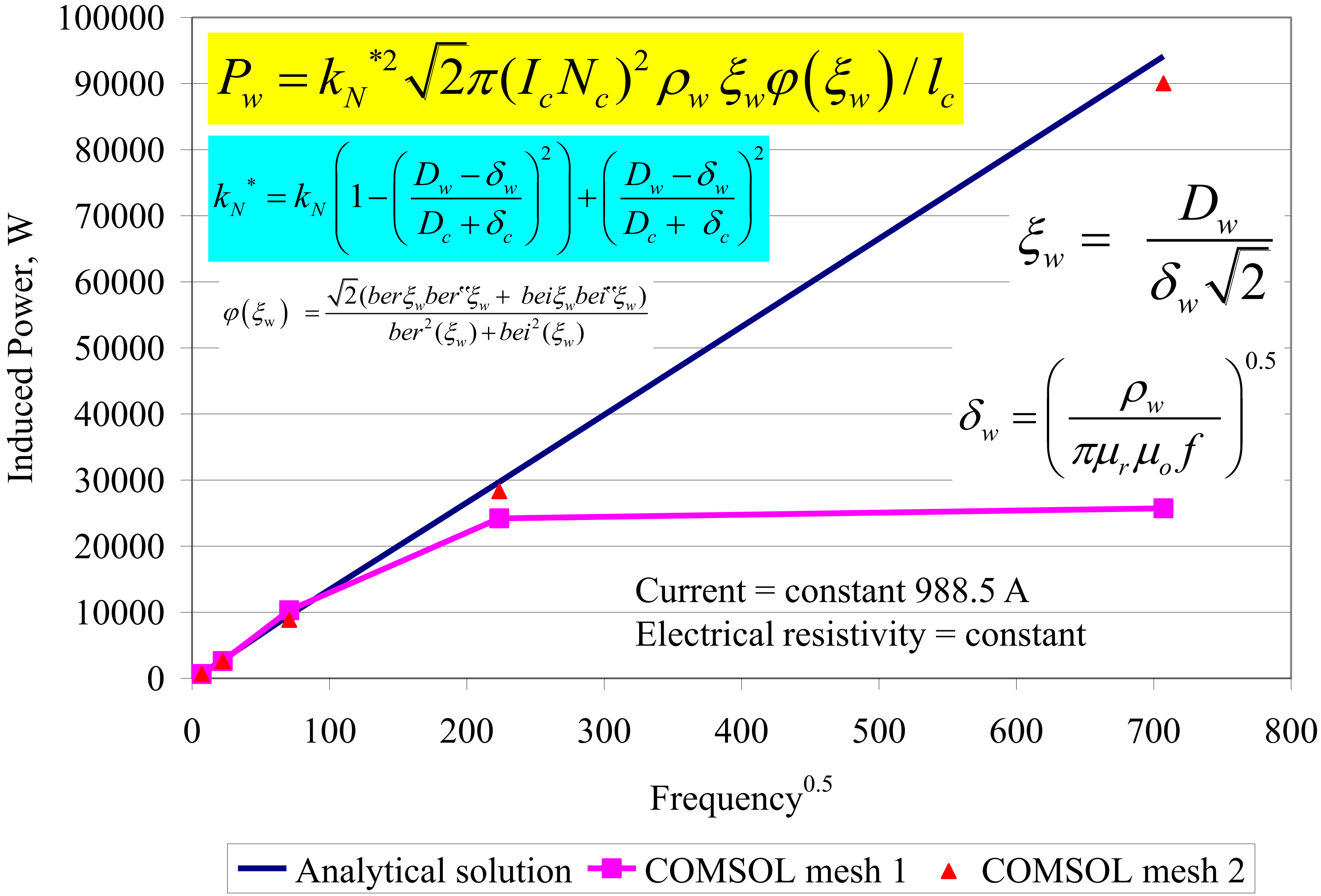

freq(1)=50 Mesh: Quality
COMSOL MULTIPHYSICS
250
200
150
100
50
0
−50
−50
0
50
100
150
▲ 1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
▼ 0.8184

Induction heating using mesh 1

Frequency (Hz)	Experimental Power (W)	Analytical Power (W)	Mesh 1 Power (W)	Mesh 1-Analytical Difference (%)	δ (mm)
50	696	691	650	-6.0	14.50
500	N/A	2768	2604	-5.9	4.59
5000	N/A	9549	10280	7.7	1.45
50000	N/A	29697	24211	-18.5	0.46
500000	N/A	94123	25728	-72.7	0.14
	Mesh 1spacing at work piece interface =				5.10

At „High Frequency"the power induced should change by $\sqrt{f}$. Also the first electromagnetic penetration depth will contain 63% of the total current and 86% of the power, with an **exponential gradient squared**.

Variation of heating rate with frequency$^{0.5}$



$$P_w = k_N^{*2}\sqrt{2}\pi(I_c N_c)^2 \rho_w \xi_w \varphi(\xi_w)/l_c$$

$$k_N^{*} = k_N\left(1-\left(\frac{D_w-\delta_w}{D_c+\delta_c}\right)^2\right)+\left(\frac{D_w-\delta_w}{D_c+\delta_c}\right)^2$$

$$\varphi(\xi_w) = \frac{\sqrt{2}(ber\,\xi_w\,ber'\xi_w + bei\,\xi_w\,bei'\xi_w)}{ber^2(\xi_w)+bei^2(\xi_w)}$$

$$\xi_w = \frac{D_w}{\delta_w\sqrt{2}}$$

$$\delta_w = \left(\frac{\rho_w}{\pi\mu_r\mu_o f}\right)^{0.5}$$

Current = constant 988.5 A

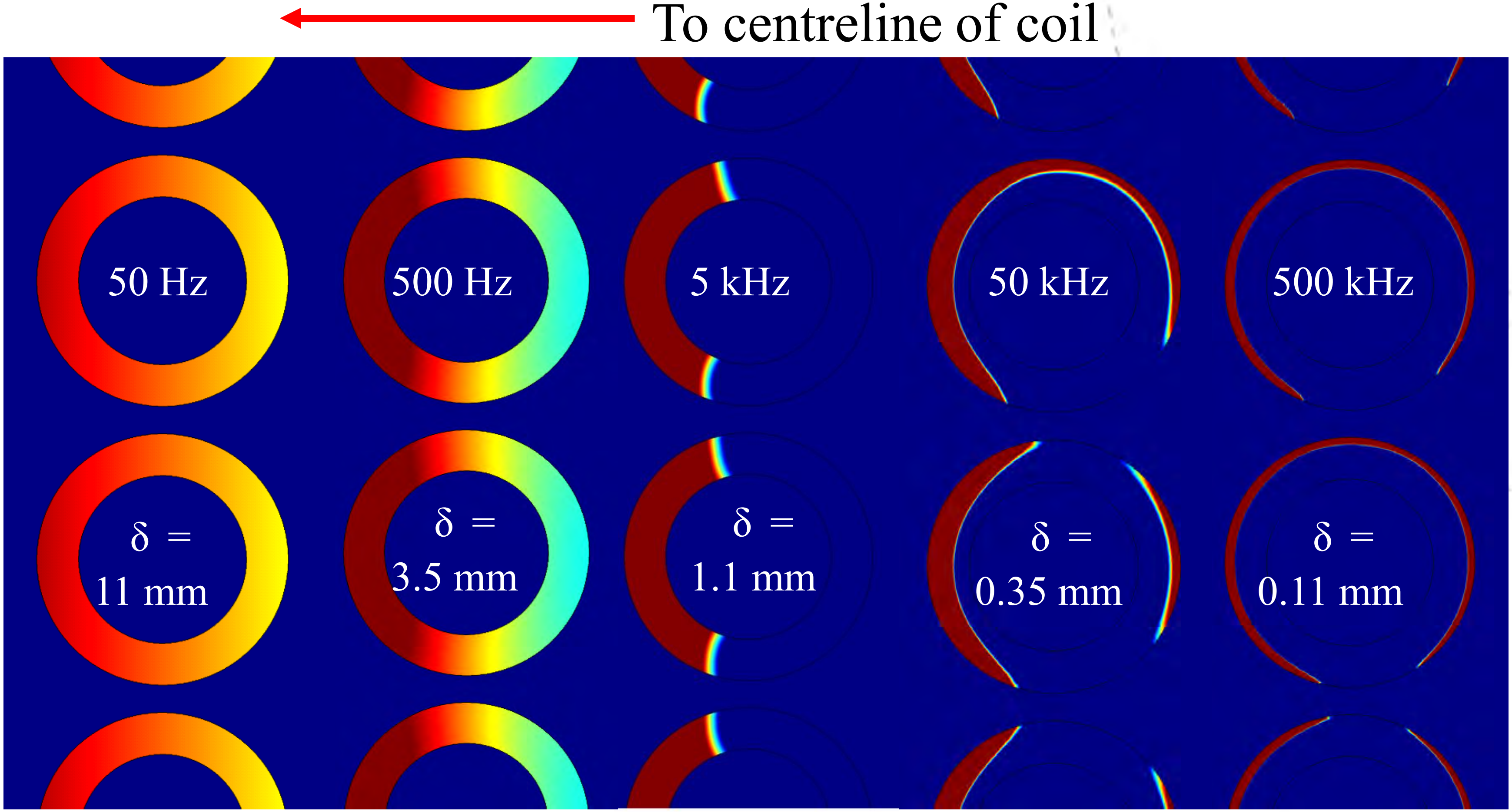
Electrical resistivity = constant

Induction heating using boundary meshes

Frequency (Hz)	Experimental Power (W)	Analytical Power (W)	Mesh 2 Power (W)	Mesh 2-Analytical Difference (%)	δ (mm)
50	696	691	650	-6.0	14.5
500	N/A	2768	2597	-6.2	4.59
5000	N/A	9549	8834	-7.5	1.45
50000	N/A	29697	28305	-4.7	0.46
500000	N/A	94123	90029	-4.3	0.14
	Mesh 2 spacing at work piece interface =				0.02

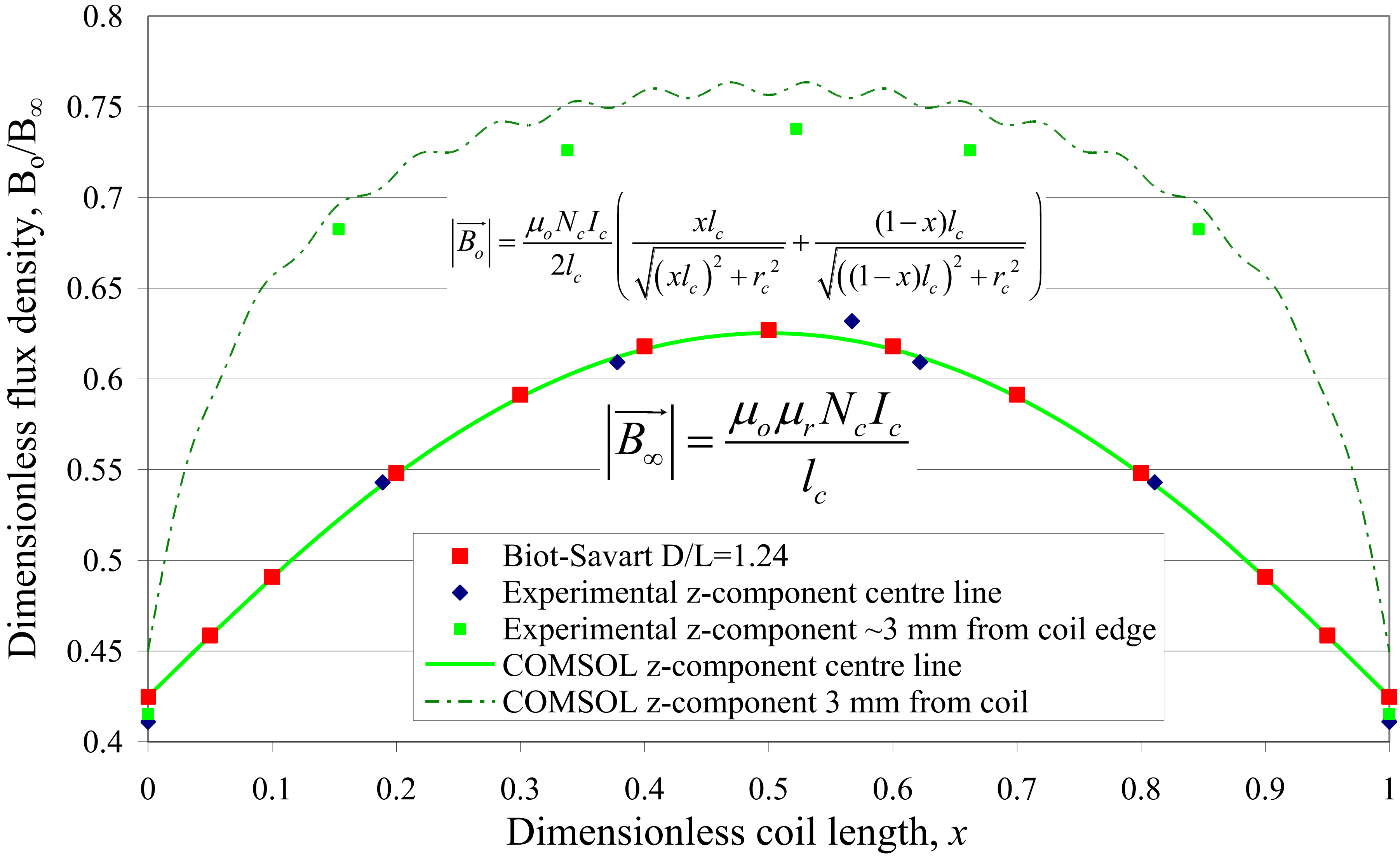
Boundary meshes allow accurate calculation to extremely high frequency. Mesh spacing should be < δ.

„Single-turn“ vs. „Multi-turn“ domain



If $\delta <$ tubing diameter, must use "Single-turn" domain and ideally a current driven coil,
voltage driven „Multi-turn" results will be wrong.

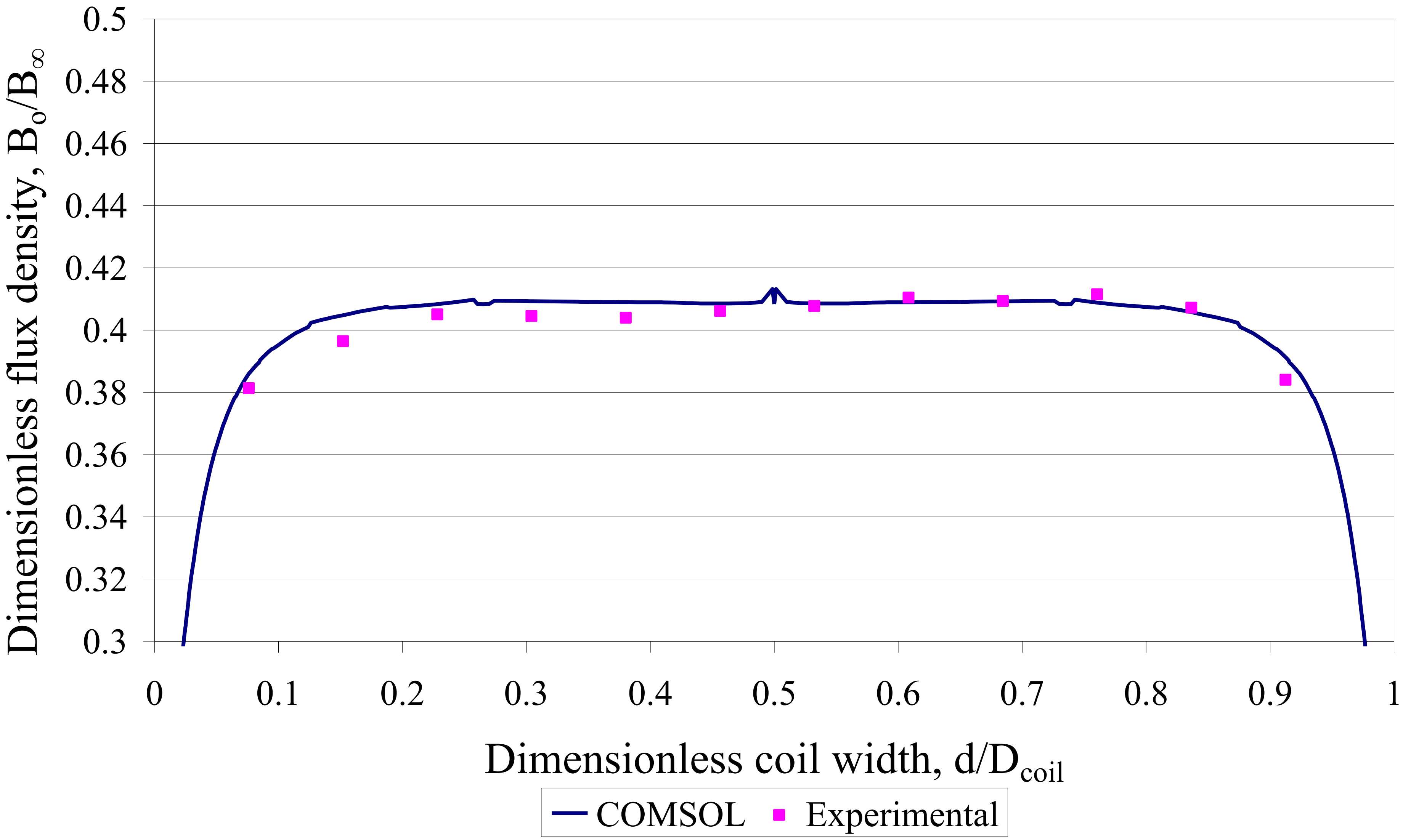
Dimensionless flux density vs. position



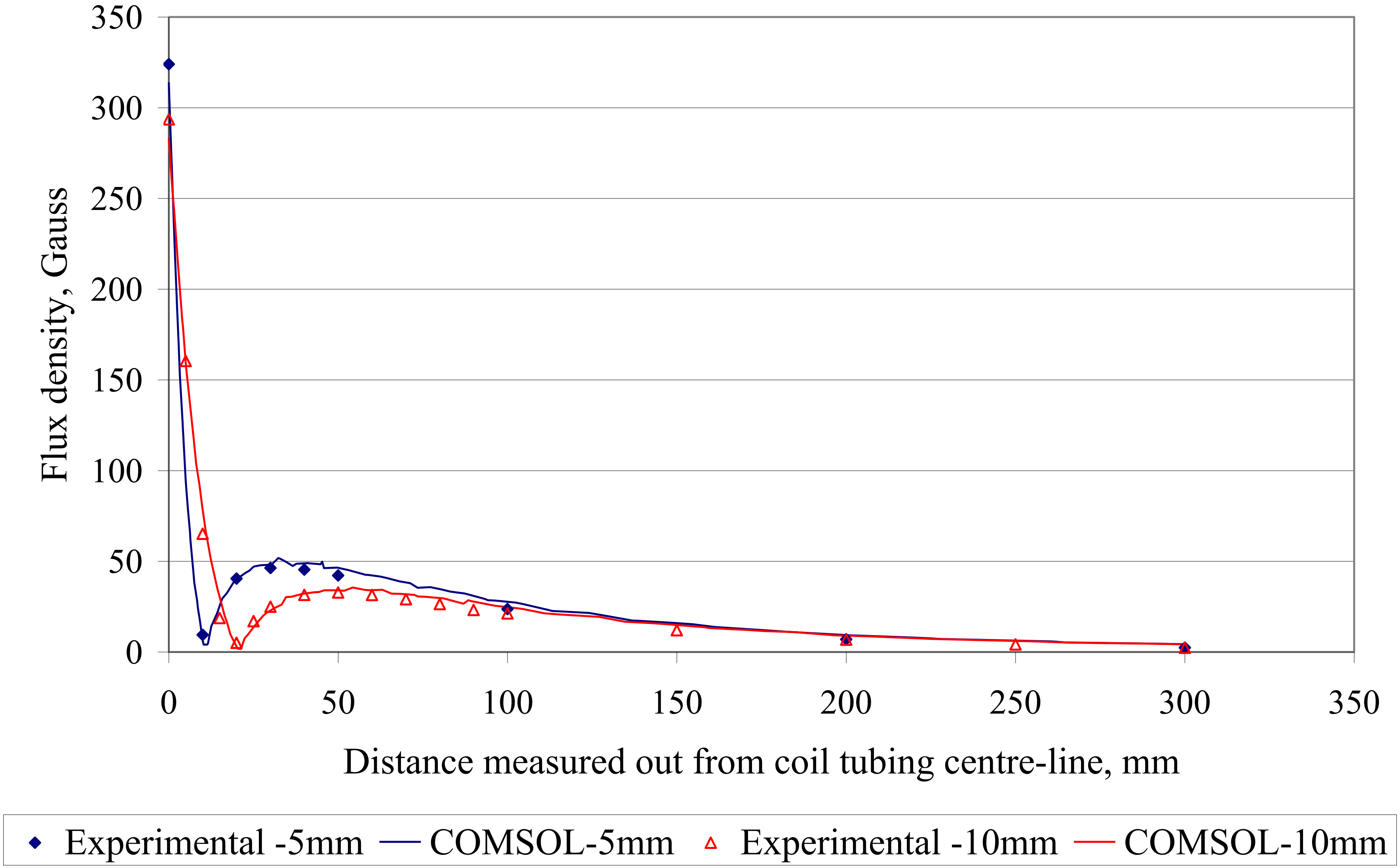
$$\left|\overrightarrow{B_o}\right| = \frac{\mu_o N_c I_c}{2 l_c}\left(\frac{x l_c}{\sqrt{(x l_c)^2 + r_c^2}} + \frac{(1-x) l_c}{\sqrt{((1-x) l_c)^2 + r_c^2}}\right)$$

$$\left|\overrightarrow{B_\infty}\right| = \frac{\mu_o \mu_r N_c I_c}{l_c}$$

Transverse probe magnetic field measurements 2 mm over top of short coil



Transverse probe magnetic field measurements from side of short coil



Conclusions

- Current driven coils are recommended, they give the correct magneto-motive-force (NI).
- 14 times the coil size is sufficient to simulate an infinite external volume, for 2D axial symmetric models.
- „Single-turn“ domain is recommended, it gives correct results at all frequencies.

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Conclusions

- „Multi-turn“ domain can be used if the electromagnetic penetration depth is greater than the coil tubing diameter.

- Due to the extremely steep current gradients at the surface of the work piece at high frequencies (small δ), boundary meshes should be used to give a mesh spacing $< \delta$.

- Magnetic field estimates with error < 1-2% and heating estimates with errors< 6% can be obtained. (Note: New calorific measurements have verified errors to be < 2%!)

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Thank you for your attention !

Mark William Kennedy
Department of Materials Science and Engineering
Phone: +47 73595164
Mobile: +47 92219891
E-mail: mark.kennedy@material.ntnu.no

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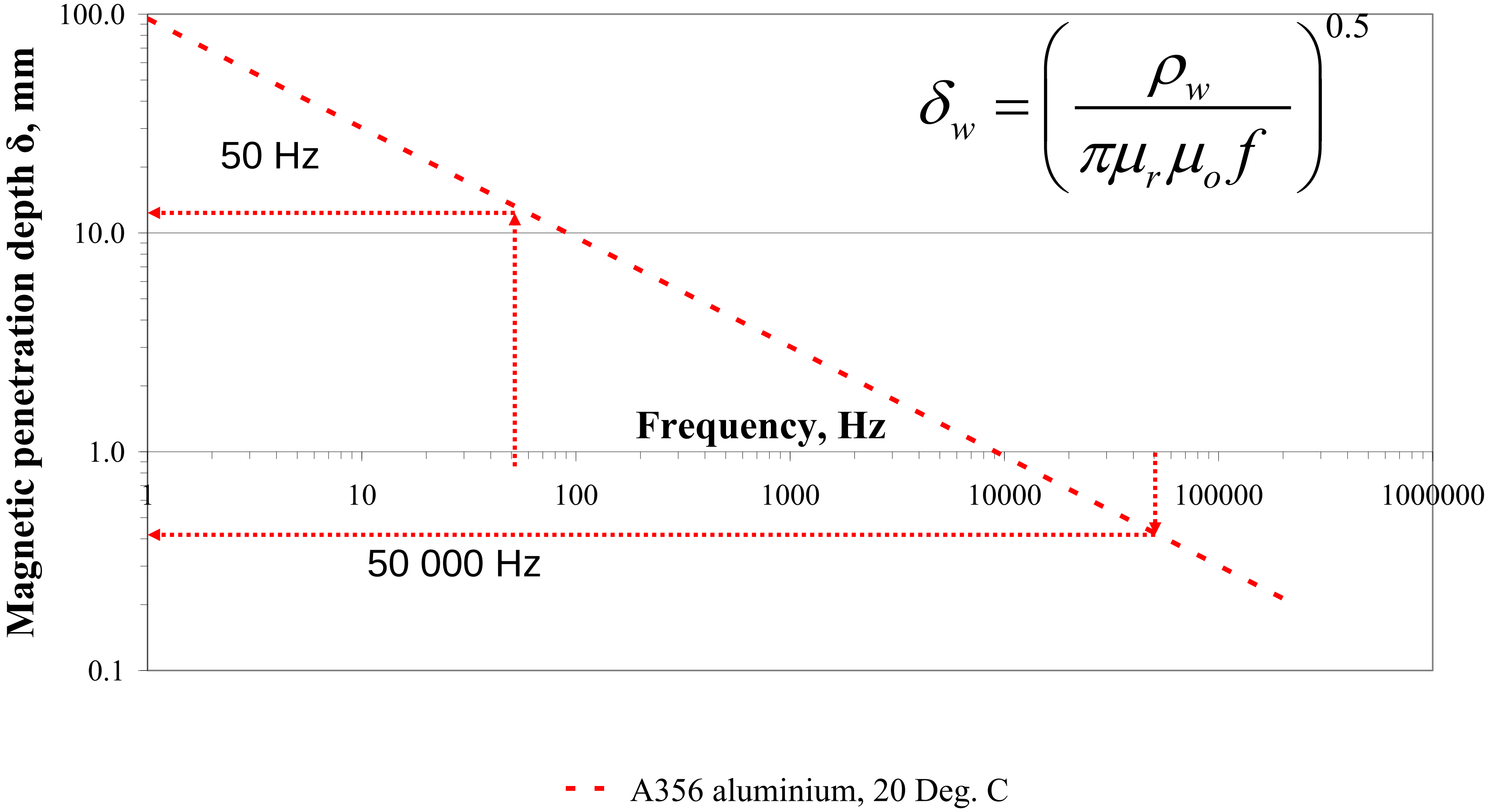
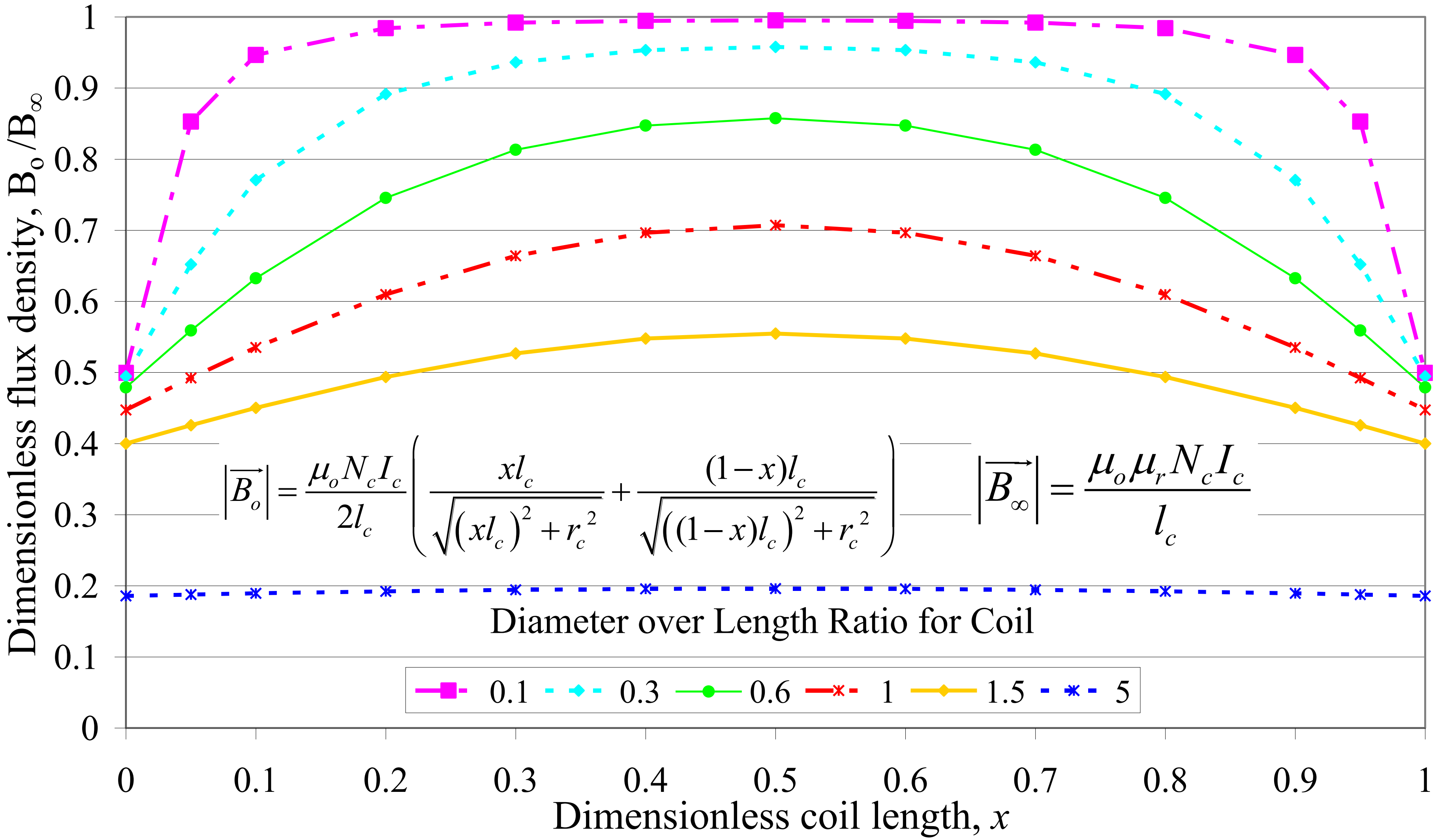
Variation of magnetic penetration depth with frequency



$$\delta_w = \left(\frac{\rho_w}{\pi \mu_r \mu_o f} \right)^{0.5}$$

Centreline magnetic fields of short coils



$$\left|\overrightarrow{B_o}\right| = \frac{\mu_o N_c I_c}{2l_c}\left(\frac{xl_c}{\sqrt{(xl_c)^2 + r_c^{\,2}}} + \frac{(1-x)l_c}{\sqrt{((1-x)l_c)^2 + r_c^{\,2}}}\right)$$

$$\left|\overrightarrow{B_\infty}\right| = \frac{\mu_o \mu_r N_c I_c}{l_c}$$

Equations for 1D analytical and 2D axial symmetric model

$$P_w = k_N^{*2}\sqrt{2}\pi(I_c N_c)^2 \rho_w \xi_w \varphi(\xi_w)/l_c$$

$$k_N^{*} = k_N\left(1-\left(\frac{D_w-\delta_w}{D_c+\delta_c}\right)^2\right)+\left(\frac{D_w-\delta_w}{D_c+\delta_c}\right)^2$$

$$\varphi(\xi_w) = \frac{\sqrt{2}(ber\xi_w ber'\xi_w + bei\xi_w bei'\xi_w)}{ber^2(\xi_w)+bei^2(\xi_w)}$$

$$\xi_w = \frac{D_w}{\delta_w\sqrt{2}} \qquad \delta_w = \left(\frac{\rho_w}{\pi\mu_r\mu_o f}\right)^{0.5}$$

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{B} = \mu_o\mu_r\vec{H}$$

$$\nabla\times\vec{H} = \vec{J}$$

$$\vec{J} = \sigma\vec{E} + \vec{J^e}$$

$$\vec{E} = \nabla V - \frac{\partial\vec{A}}{\partial t}$$

$$j\omega\sigma\vec{A_\phi} + \nabla\times\vec{H} = \vec{J^e}$$

Factors for analytical solutions