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Lava Tubes at Shallow Depth

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Abstract: Many theoretical studies concerning lava tubes focus on the thermal disturbances generated on the earth surface. Recently *Dragoni and Tallarico* (2008) suggested the solution of a lava tube located at a deep depth *h* in the soil, where the ratio between *h* to the major axis of the ellipse *a* is higher than 10. But lava tubes more frequently are located at shallower depths .

In this work we focus our attention to the stationary thermal field generated by an elliptical lava tube at constant temperature located at shallow depths. Moreover we take into account the effect of the heat lost from the earth surface by air convention. The problem is approached by the COMSOL Multiphysics

Keywords: lava tube, thermal disturbance.

1. Introduction

Lava tubes form when a flow of fluid lava cools on the upper surface in contact with the cold air and by thermal irradiation, sufficiently to form, over a prolonged period of time, a solidified crust to insulate the underlying liquid layers against cooling. Often the top of the lava undergoes to a flash frozen in contact with the atmosphere.

Beneath this excellent insulator, the molten rock continues to flow covering many kilometers from the vent without cooling appreciably . Such a conduit generates by crusting over an existing lava channel on the flanks of volcanoes. The shapes and the length of the tubes depend on many factors such as the type and the viscosity of the lava, the eruption rate, the slope of the ground, the effusion rate and so on (e.g. basalt lava flow confined within a channel can reach velocities >30 km/hour). In their simplest form, lava tubes are long tunnels of uniform circular/elliptical cross-section shape, the axis oriented down to the slope of an erupting vent from where lava pours or oozes. But generally lava tubes have complicated cross-section shapes [1].

In many cases lava tubes are evidenced by clear surface features (tumuli, skylights, collapses, secondary vents and so on) but this is not the rule.

The aim of this paper is focused on the thermal anomaly generated on the earth surface by a hot lava tube in a conductive soil. Such kind of problem was handled recently by *Dragoni and Tallarico* [2]. Their analytical solution is weakly affected by the assumed constraints so that it holds only for deep lava tubes. Observations of *Berthelote et al.* [3] show on the contrary that many lava tubes are located at shallow depths.

We use a numerical approach by the COMSOL Multiphysics to evaluate the thermal anomaly generated on the earth surface by lava tubes of different cross-section size, located at any depth.

2. Analytical model

Dragoni and Tallarico [2] consider a cylindrical tube embedded both in an unbounded and in a half-space solid medium. The tube is horizontal, has an elliptical cross-section in the vertical plane *xz*, and their semiaxes *a* and *b* are directed along the *z* and *x*-axis, respectively. The tube is assumed infinitely long in the *y*-direction, so that the problem is two-dimensional.

The lava filling the tube is assumed to exist since a sufficiently long time so that it has reached a steady-state thermal condition.

The thermal field T(*x*,*z*) outside the tube is a solution of the Laplace equation:

$$\nabla^2 T = 0 \,.$$

Dragoni and Tallarico [2] assume different boundary conditions depending on the host medium is unbounded or bounded (half-space). In the former case they assume that the temperatures both at the wall of the tube θ_w and at a very large ellipse confocal with the section of the tube θ_o are uniform and constant. This problem satisfies the Dirichlet's conditions, so the solution can be expressed in terms of Fourier series for arbitrary temperature θ_o and θ_b at the boundaries [4]. *Boley and Wiener* [5] and *Nehari* [6] suggested a conformal mapping technique to solve the problem. The temperature field around the tube, is trivially described by isothermal lines with elliptical shape confocal with the one on the wall decreasing in eccentricity to the increasing distance.

More interesting is the latter more realistic case. As boundary conditions they assume uniform and constant temperatures at the wall of tube θ_w and on the earth surface θ_S. The solution is searched by the image source method. As the source (the cross-section of the tube) is finite in size the method provides a satisfactory solution on condition that the size of the source (e.g. the major semi-axis of the ellipse) is much smaller than its depth h. This condition restrict the pertinence of the results to a very limited cases, i.. when the ratio between the depth h and the major semi-axis is higher than 10. *Berthelote et al.* [3] by field observations show that this ratio ranges from 1 to 11, with mode 3.

In this paper we take up the approach of *Dragoni and Tallarico* [2] by the COMSOL Multiphysics to enable a better characterization of the buried lava tubes (mainly their geometrical properties) detected by radiometric observations. In addition, we study the evolution of the transient temperature field of the first stage of crusting, up to the steady-state.

3. Numerical model

Our working hypothesis are the following both in the case of a bounded and an unbounded host medium: i) the cross-section is an ellipse of semi-axes *a* and *b* and eccentricity $e = \sqrt{(a^2 - b^2)/a^2}$; ii) the tube is filled up by a liquid lava at constant temperature throughout its length; iii) the host medium is homogenous and isotropic so that its thermal diffusivity and conductivity can be assumed constant, iv) the temperature of the host medium before the filling up of the tube by the hot lava is assume to be constant.

At first instance we studied a tube located at the centre of a wide unbounded medium with *a*=1 m and *e*=0.5. The temperature at the boundaries of our working domain *D* is maintained constant at any time after filling. To avoid that the this hypothesis can affect the results, *d* (a characteristic length of *D*) is chosen so that the ratio $a/d<<1$ (Fig. 1).

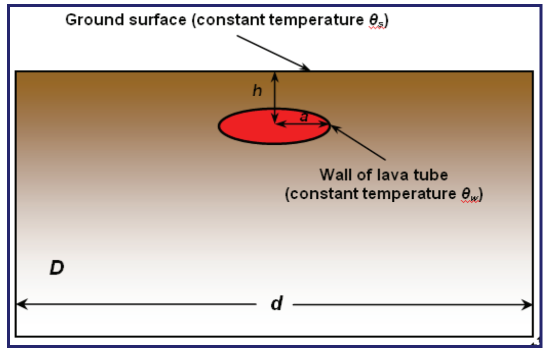

Figure 1. Elliptical lava tube.

The results are the ones we expected: the isotherms T(*x*,*z*)=costant are elliptical lines confocal with the ellipse of the wall, which become progressively more and more round as the distance increases from the centre of the ellipse (the eccentricity of the isotherms $e\rightarrow 0$ as observed by *Dragoni and Tallarico* [2]).

In the case the host medium is semi-infinite and bounded by the earth surface (both at the constant temperature θ_S) we locate the centre of a lava tube at the shallow depth *h* (e.g. 2 m) with its major (minor) axis in parallel with the earth surface, the thermal field around the lava tube is shown in Fig. 2. As it can be seen the structure of the thermal field is described by isotherms normalized with respect to the constant temperature θ_w of the wall of the tube ($\theta_w >> \theta_S$), while the arrows perpendicular to the isotherms stand for the heat flow *F* (in intensity and direction) inside the medium ($F = -K\nabla T$). For its calculations we assumed θ_w=1000°C and thermal conductivity *K*= 4 W m^{-1} K^{-1} for the entire host medium.

The typical curves of the surface heat flow are bell-shaped with the maximum F_{max} positioned in correspondence to the centre of the tube.

If we the major axis of the ellipse is inclined with respect to the surface of an angle φ>0 the curve assumes an asymmetrical shape with its maximum F_{max} displaces toward the point of the tube nearest the surface. The asymmetry of the curve is maximum to φ=π/4 and the value of F_{max} is highest to φ=π/2.

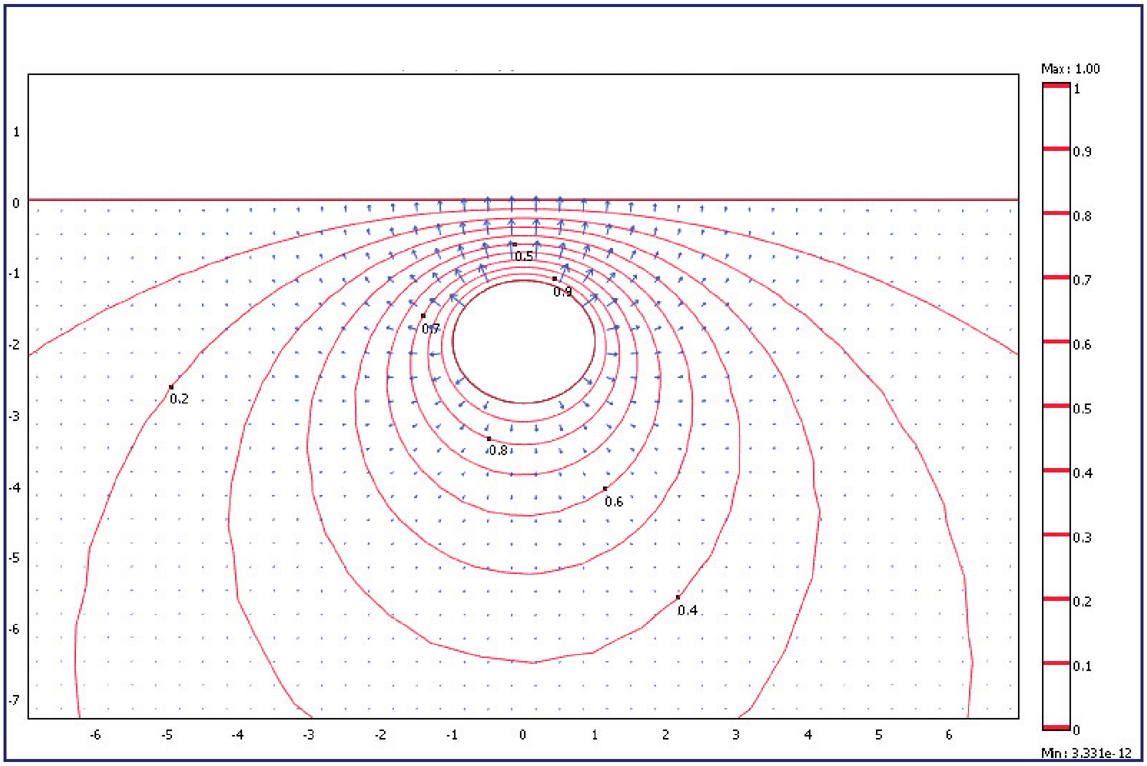

Figure 2. Thermal field around a lava tube (*a*= 1 m, *e*= 0.5, *h*= 2 m). Isotherms are normalized with respect to the constant temperature θ_w of the wall of the tube. The blue arrows stand for the heat flow.

The dependence of the F_{max} on the angle φ for a tube with *a*= 1 m, *e*= 0.5 and *h*= 2 m is shown in Fig. 3.

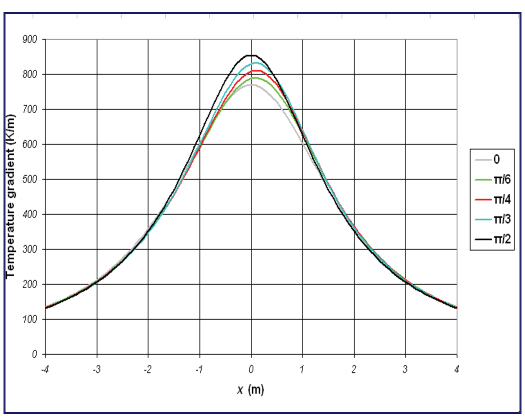

Figure 3. Dependence of the F_{max} on the angle φ (*a*= 1 m, *e*= 0.5, *h*= 2 m).

The dependence of the F_{max} on the angle φ is shown in Fig. 4 for a tube (*a*=2 m and *e*=0.5) buried at three different depths (3, 5, and 8 m) inclined of an angle φ respect to the surface. The curves are normalized respect to the highest value of F_{max} which refers to an elliptical tube whose major axis is perpendicular to the surface (φ=π/2). If φ decreases, F_{max} lowers as well. The lowering is more evident for the shallower tube (3 m) but can be neglected for deeper ones (*h*> 8 m), because of F_{max} are near equal for any φ. More generally, by increasing the depth of the tube the asymmetry of the curve vanishes progressively and the isotherms become more and more circular.

The effect on the value of F_{max} of the depth *h* is shown in Fig.5 by assuming a tube *a*=1 m and *e*=0.5. The thermal gradient at the earth surface is maximum if the tube is very near the surface and diminishes progressively as the depth of the tube increases. Nevertheless at 12 m depth, in the steady-state conditions, the maximum surface gradient F_{max}/K is about 50 K m^{-1}, so the heat flow anomaly at the earth surface is so far detectable by the thermal radiometers.

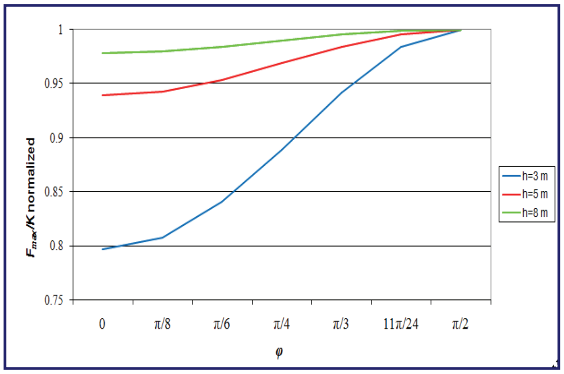

Figure 4. Dependence of the F_{max} on the angle φ

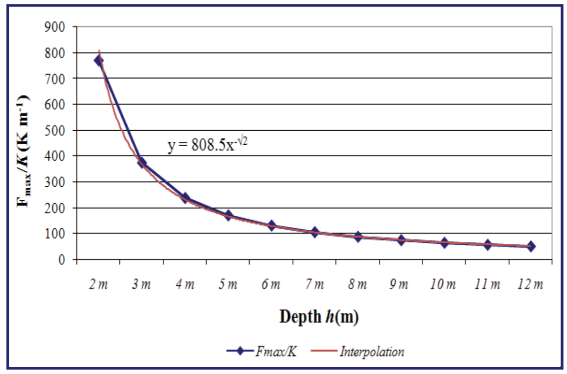

Figure 5. Dependence of the F_{max} on the the depth h

The dependence of the maximum gradient F_{max}/K on the size of the tube a is shown in Fig. 6. At the fixed depths 5, 8, and 12m we located a tube with increasing dimension a=1, 2, 3, 4, 5 m and given eccentricity e. As it can be seen, the range of variation is less and less significant with the increasing depth. As a consequence, downward from a given depth the heat flow anomaly at the earth surface is poor sensitive to the size (a and e) of the tube.

In the real cases, the temperature at the earth surface, i.e. at the boundary of the half-space, is not constant for a wide belt along the axis of the tube. At the surface, normally to the axis, the temperature attenuates progressively towards the surroundings. This means that the Dirichlet conditions cannot be satisfied. This is a severe obstacle to adopt the analytical image-source method to reproduce the thermal field in the surroundings of the conduit.

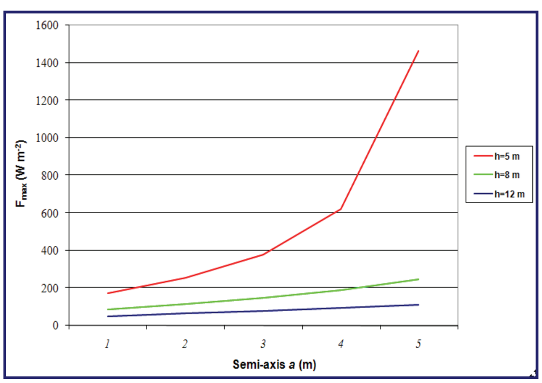

Figure 6. Dependence of the F_{max} on the on the semi-axis a.

3.1 Surface convective cooling

The air convection play an important role in cooling this area. The equilibrium temperature at the earth surface depends on the heat transfer coefficient h_s, which in turn depends of the wind speed near the surface. By laboratory experiments *Berthelothe et al.* [3] showed that this coefficient changes in a wide range, from 0.1 to 80 W $m^{-2}K^{-1}$. *Keszthelyi and Delinger* [7], by experiments above cooling pahoehoe a different higher range of variation, from 35 to 150 $Wm^{-2}K^{-1}$. Recently *Keszthelyi et al.* [8] suggested a better estimation of 45-50 $Wm^{-2}K^{-1}$ for pahoehoe temperature surface at 400-550 °C cooled by a wind speed of about 10 ms^{-1}.

The cooling of the earth surface due to the air convection is shown in Fig. 7. The boundary condition at the earth surface is the heat convective flow (Neumann condition) for different h_s ranging from 1 to 80 $Wm^{-2}K^{-1}$ as suggested by *Berthelothe et al.* (2008), whereas the temperature of the other boundaries of D and that of the air is constantly maintained to 10°C (see Fig. 1). The relationship between the cooling of the surface and h_s is direct: higher h_s higher the cooling and the area subject to cooling.

4. Discussion and conclusions

These theoretical results are wide-ranging and provide a convincing aid to understand the problem concerning the characterization of the lava tube where the analytical methods fail.

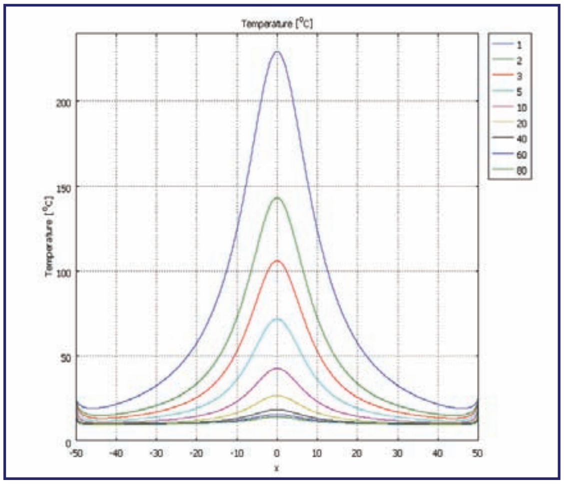

Figure 7. Cooling of the earth surface due to the air convection. The heat convective flow for different h_s ranging from 1 to 80 $Wm^{-2}K^{-1}$.

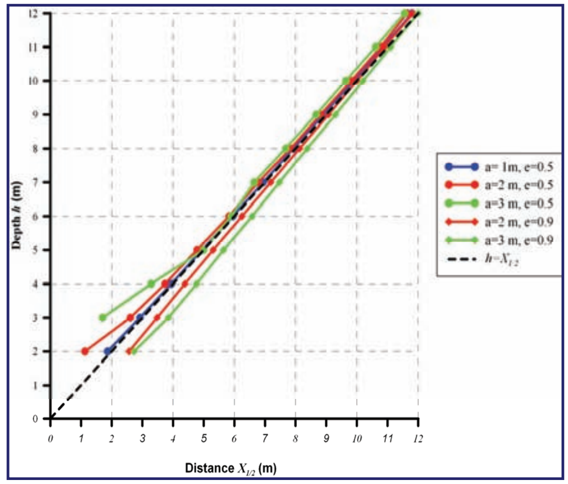

Figure 8. Relation between $X_{1/2}$ and h.

Some parameters on the tube can be estimate from observations on the earth surface by thermal devices.

The maximum and the lateral dimension of the bell-shaped curves (Fig. 3) are clearly related to the depth of the conduit. Let X_1 and X_2 the points on the curve where the thermal gradient is the half of the maximum (Y= ½(F_{max}/K)), the depth is:

$$h = \frac{1}{2}\left(|X_1| + |X_2|\right) = X_{1/2}$$

as it is shown in Fig. 8.

The steady-state thermal field around a lava tube is reached after a time τ depending on the depth h by the following relationship:

$$\tau \approx \frac{h^2}{k}$$

where k is the thermal diffusivity ($k=10^{-6}$ m^2 s^{-1}). This implies that deep conduit reach the steady-state after some years. Presumably during this time the lava inside the tube cools. Both analytical and numerical approach fail when the tube is located at depths greater than 5 m. In these cases the transient method appears to be more realistic in studying the evolution of the thermal field after that lava filled the conduit.

5. References

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