

Weak Form & LiveLink™ for MATLAB® Based Modified Uzawa Method for Solving Steady Navier-Stokes Equation

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Abstract

We all know COMSOL Multiphysics® software can solve nonlinear PDE easily by built-in Newton method, and one typical example is Navier-Stokes equation. But sometimes we can't figure out a good initial guess for Newton method, or maybe we have some other good algorithms. In these cases, The Laminar Flow interface may not be so suitable.

So I want to demonstrate a simple example about how to design my own algorithm for steady N-S equation by weak form PDE (Figure 1) and LiveLink™ for MATLAB® (Figure 2).

First, I add geometry, each step (weak form equation) of my algorithm, mesh to COMSOL. Then I assign loop variables by adding a simple coefficient equation ($u_k = u$). Last, load the .mph model to MATLAB®, and let MATLAB® execute the loop, draw the answer and analyze the data.

After the computation, We get the same solution as other FEM packages (FreeFEM++ & FeniCs) at each iterative step (Figure 3), and we can see the N-S equation's solution and errors between each step (Figure 4).

With other physics models or equations, the mathematics branch and LiveLink™ for MATLAB® can also do the numerical experiment easily.

Reference

Puyin Chen, Jianguo Huang, Huashan Sheng, Some Uzawa methods for steady incompressible Navier–Stokes equations discretized by mixed element methods, *Journal of Computational and Applied Mathematics* 273 (2015) 313–325.

Figures used in the abstract

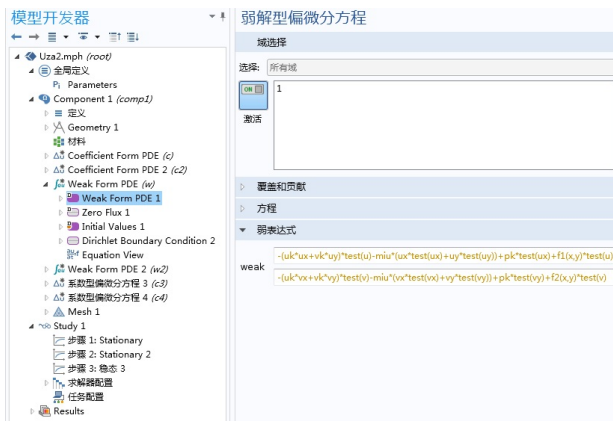


Figure 1: Weak Form PDE of Steady N-S Equation.

```
File Edit Text Go Cell Tools Debug Desktop Window Help
- 1.0 + ÷ 1.1 x
1 - model = mphload('Uza2.mph');
2 - ERROR = 1;
3 - COUNT = 1;
4 - ERRUVEC = [];
5 - ERRUPRE = [];
6 - while ERROR > 1e-6 && COUNT < 1000
7 -     model.sol('sol1').run;
8 -     Q1 = mphmax(model,'abs(u-uk_bk)', 'surface');
9 -     Q2 = mphmax(model,'abs(v-vk_bk)', 'surface');
10 -    Q3 = mphmax(model,'abs(p-pk_bk)', 'surface');
11 -    B1 = mphmax(model,'abs(uk_bk)', 'surface');
12 -    B2 = mphmax(model,'abs(vk_bk)', 'surface');
13 -    B3 = mphmax(model,'abs(pk_bk)', 'surface');
14 -    ErrU = max(Q1/B1,Q2/B2);
15 -    ErrP = Q3/B3;
16 -    fprintf('Iter. Step: %d \t \n ',COUNT);
17 -    fprintf('Velc. Err. Btw Iwo Step: %.2e \t \n ',ErrU);
18 -    fprintf('Pres. Err. Btw Iwo Step: %.2e \t \n ',ErrP);
19 -    ERROR = max(ErrP,ErrU);
20 -    COUNT = COUNT+1;
21 -    ERRUVEC = [ERRUVEC;ErrU]; %#ok<AGROW>
22 -    ERRUPRE = [ERRUPRE;ErrU]; %#ok<AGROW>
23 - end
24 - subplot(2,2,1)
25 - mphplot(model,'pg1');
```

Figure 2: MATLAB Code for Looping.

```

Command Window
News MATLAB Watch this Video, see Examples, or read Getting Started
File Edit View Search Terminal Help
Itr. Step: 194
Velc. Err. Btw Two Step: 1.36e-06
Pres. Err. Btw Two Step: 1.62e-06
Itr. Step: 195
Velc. Err. Btw Two Step: 1.28e-06
Pres. Err. Btw Two Step: 1.52e-06
Itr. Step: 196
Velc. Err. Btw Two Step: 1.21e-06
Pres. Err. Btw Two Step: 1.43e-06
Itr. Step: 197
Velc. Err. Btw Two Step: 1.14e-06
Pres. Err. Btw Two Step: 1.35e-06
Itr. Step: 198
Velc. Err. Btw Two Step: 1.07e-06
Pres. Err. Btw Two Step: 1.27e-06
Itr. Step: 199
Velc. Err. Btw Two Step: 1.01e-06
Pres. Err. Btw Two Step: 1.20e-06
Itr. Step: 200
Velc. Err. Btw Two Step: 9.46e-07
Pres. Err. Btw Two Step: 1.13e-06
Itr. Step: 201
Velc. Err. Btw Two Step: 8.91e-07
Pres. Err. Btw Two Step: 1.06e-06
Itr. Step: 202
Velc. Err. Btw Two Step: 8.38e-07
Pres. Err. Btw Two Step: 9.97e-07
-----
Pressure L2 Norm: 5.04e-05
Velocity L2 Norm: 1.11e-07
Velocity H1 Norm: 3.29e-05
LiveLinkForMATLAB
Two Step Error of (U,V) :1.18351e-06
Two Step Error of P :1.34948e-06
.....
Out itn Number:198
Two Step Error of (U,V) :1.11395e-06
Two Step Error of P :1.27017e-06
.....
Out itn Number:199
Two Step Error of (U,V) :1.04847e-06
Two Step Error of P :1.19551e-06
.....
Out itn Number:200
Two Step Error of (U,V) :9.86844e-07
Two Step Error of P :1.12525e-06
.....
Out itn Number:201
Two Step Error of (U,V) :9.28839e-07
Two Step Error of P :1.05911e-06
.....
Out itn Number:202
Two Step Error of (U,V) :8.74243e-07
Two Step Error of P :9.96858e-07
End Itr Errors:9.96858e-07
End Itn Number:202
Mesh size h: 1/32
alpha: 0.2
kappa: 1
FreeFEM++
/>>>

```

Figure 3: Solving Process Compared with FreeFEM++.

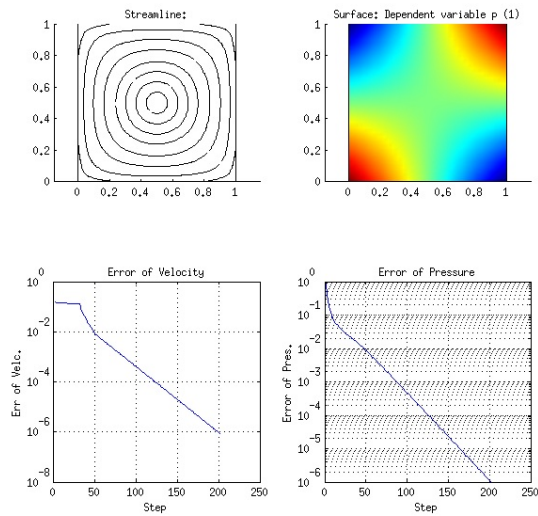


Figure 4: Solution and Iterative Errors of the Numerical Example.