Hybrid Resistive-Capacitive and Ion Drift Model for Solid Gas Dielectrics

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Abstract: Many electrical insulation systems use a combination of gaseous and solid dielectrics. This article describes a model, implemented in COMSOL Multiphysics, for simulating the electric field distribution in such systems under DC stress.

The solid dielectrics can be modeled with a resistive model, while for the gas an ion drift model is more appropriate. The two models are implemented using the "electric currents" and the "transport of diluted species" interfaces and connected using charge sources. An additional simplification is made by assuming the ion flow is quasi-static. This speeds up the model significantly and improves numerical stability.

The results from simulation are compared to measurements performed at ETH Zürich [1]. Two geometries with different gas volumes are simulated and compared to experimental results. The results show that the model describes the experimental results well with reasonably short simulation times and good numerical stability.

Keywords: Surface charging, HVDC, gaseous insulation, GIS.

1. Introduction

In high voltage engineering, various types of concepts are employed for electrically insulating between high voltage and ground and between different phases. This article considers electrical insulation systems that are composed of a combination of solid and gaseous media under DC stress. This is a common combination in dielectric systems and typical examples include gas insulated switchgear (GIS), which employ a combination of solid epoxy insulation and SF6 gas.

A breakdown of the insulation is initiated when part of it becomes electrically overstressed. It is therefore important to understand the electrical field distribution as a function of time. In DC applications, this requires a model for charge transport in addition to Poisson's equation to model the electric field distribution as a function of time. While a resistive model typically describes conduction in solid insulation adequately, such a description is typically not applicable to gases, since they are characterized by a more complex current-voltage characteristic [2]. For the electric fields typically used in applications, that are large but much below the corona inception level, conduction in gases is best described using an ion drift model. This article describes how a hybrid model can be formulated and efficiently implemented in COMSOL Multiphysics. The numerical results from the model are compared to measurements performed at ETH Zürich [1]. Results obtained using this model has been published in [3]. Here, we focus on explaining the numerical background of the model, and demonstrate its ability to describe the physics of the problem in different geometries.

The article is structured as follows. In section 2, we first provide a general theoretical summary of the concepts needed to describe charge transport in hybrid solid-gas insulation systems under DC stress. The COMSOL Multiphysics implementation of the model is discussed in section 3. In section 4, we show how the model accurately reproduces the measurements published in [1]. A discussion regarding the applications of the model is presented in section 5, followed by conclusions in section 6.

2. Theory

Understanding the electric field distribution is vital to modelling a dielectric system. For ideal insulation performance no part of the system should be overstressed. For an AC system the electric field distribution can be calculated simply using the Laplace equation, while for a DC system the situation is more complicated since one has to take into account the full time-dependence of the field distribution associated with the transport of charge through the system.

Starting from first principles, the electric field of a general DC system can be modeled using the following equations:

$$-\nabla\cdot(\varepsilon\nabla\phi) = \rho \quad (1)$$

$$\boldsymbol{E} = -\nabla\phi \quad (2)$$

$$\partial\rho/\partial t = -\nabla\cdot\boldsymbol{J} \quad (3)$$

Here ϕ is the electric potential, $\boldsymbol{E}$ the electric field, ρ is the charge density and $\boldsymbol{J}$ the current. To effectively model the system, one needs a way of determining the current from the electric field and other system properties that may influence the flow of charge, such as the local availability of ions. The current depends on the electric potential, making the equations coupled.

2.1 Solids

The simplest model for the current is Ohm's law which can be formulated as

$$\boldsymbol{J} = \sigma\boldsymbol{E} \quad (4)$$

where σ is the electric conductivity which depends on the medium.

This model generally works quite well for solid dielectrics for a range of electric fields. The equation can be combined with Poisson's equation and the continuity equation described above to model the transient behavior of a system. This model is generally referred to as a resistive-capacitive (RC) model.

2.2 Gas

The conductions in gaseous dielectrics is fundamentally different compared to solid dielectrics. In gases, the dominating conduction mechanism is ion transport. Below the critical field, the current is mainly limited by the ion production rate. This leads to a voltage current characteristic which remains Ohmic only for very low fields and quickly saturates [2, 4] due to the limited amount of available ions. Similar properties apply for liquid dielectrics [5], but the focus of this article is on gases.

The critical field is the level of stress for which electrons are accelerated to high enough energies to cause electron-impact ionization at a rate that is faster than the attachment rate of electrons to neutral molecules. These effects are the cause of properties such as corona and streamers which are also important aspects of dielectric systems but beyond the scope of this article (see for instance [6, 7] on how such aspects can be included). For air at atmospheric pressure the critical field is typically around 2.5kV/mm [2] and for SF6 approximately 8.9kV/mm/bar [2]. Below the critical field, electrons quickly attach to neutral particles and the current arises from the drift of positive and negative ions. In this regime, the maximal current that can be carried through the gas is a function of the ion production rate.

The production mechanism for ions in a gas below the critical field is background radiation, typically from sources such as cosmic rays and, in air, radon decay. The expected background ion pair production rate in air is typically $10 s^{-1} cm^{-3}$. In SF6 at 0.4MPa, values between 20 and 60 ion pairs per second and cm^3 have been reported [8], depending on the location of the laboratory where the measurement has been done. The production rate varies approximately linearly with pressure.

The system can be modelled with the ion drift equations.

$$\partial p/\partial t + \nabla\cdot(\mu_p\boldsymbol{E}p - D_p\nabla p) = R \quad (5)$$

$$\partial n/\partial t - \nabla\cdot(\mu_n\boldsymbol{E}n + D_n\nabla n) = R \quad (6)$$

$$\rho = q(p-n) \quad (7)$$

Here p and n are the positive and negative ion densities, q the elementary charge, μ is the mobility, D diffusion constant and R the source term from background radiation.

This is the conservative form of the equation. For dielectric gasses the term relating to the divergence of the electric can be neglected in the gas due to typically very low charge carries densities.

$$\nabla\cdot(\mu_p\boldsymbol{E}p) \approx \mu_p\boldsymbol{E}\nabla p \quad (8)$$

This model also depends on the choice of boundary conditions. In this article we assume a boundary conditions with no charge injection. This means that any charge that reaches the surface sticks there and no charge is injected into the gas. This can be modelled with the following equations.

$$p = 0 \qquad \text{if } \boldsymbol{N}\cdot\boldsymbol{E} < 0 \quad (9)$$

$$\boldsymbol{N}\cdot\nabla p = 0 \qquad \text{if } \boldsymbol{N}\cdot\boldsymbol{E} \geq 0 \quad (10)$$

Here $\boldsymbol{N}$ is the surface normal vector and same equations with opposite inequalities applies for the negative ion polarity.

3. Numerical Implementation

The equations for each of the two media can be implement directly in COMSOL Multiphysics. The resistive-capacitive model can

be implemented using the "electric currents" interface. The electric currents interface contains Poisson's equation and needs to be applied to the whole geometry. The gas volume is modeled with zero or near zero, i.e. orders of magnitude lower than the solid insulation, conductivity inside the electric currents interface. The boundary conditions used are electric potentials.

The gas dynamics can be implemented using the "transport of diluted species" interface. This interface is only activated for the appropriate areas of the geometry. The boundary conditions are set to the "open boundary" condition in COMSOL which is an implementation of equations (9) and (10).

A source term is needed for connecting the two interfaces in a consistent way and this can be implemented in several ways. The main area of interest is the boundary between gas and solids since this is where the main charge build up happens.

One way of coupling the interfaces is to simply integrate the charge on surfaces of the dielectrics. This can be done in COMSOL by using a boundary current source in the electric currents interface with the flow across the boundary as input, which is available among the variables in the "transport of diluted species" interface. This is the approach used in this article. The approach neglects the net charge in the gas, which is negligible compared to the surface charge as is discussed in detail later in this section.

One other way of facilitating the coupling between the interfaces is adding an external current in the electric currents interface based on the flow of ions, which can also be implemented in COMSOL. The motivation for not selecting this method is based on numerical considerations near the boundary. The flow of ions is only present in the gas domain and therefore discontinuous at the boundary between solid and gas. Using the ions as an external current means transforming this discontinuous ion flow into a current defined on the whole geometry and then differentiating to determine the time derivative of the surface charge. We have realized that the boundary conditions in the "transport of diluted species" interface most likely provides a numerically better value, by comparing a 1D version of the model with the analytical solution.

There is one further approximation that can be made. The velocity of ions in the gas is typically of the order of 100 m/s, meaning that the time of flight for an ion is at most a fraction of a second for all geometries of interest. When all electrical fields are kept below the critical value, the characteristic timescale over which the electric field varies is of the order hours up to months, depending on the geometry and the conductivity of the solid insulator. This means that the flow pattern of ions is essentially stationary at any given time, except directly after a change in the voltage.

Based on this argument, one can simplify the model significantly by considering the ion flow as stationary and neglecting the time derivative in equations (5) and (6). This can be implemented in COMSOL by changing the equation form to stationary and has been found to significantly decrease simulation time and increase numerical stability. The main error introduced with this approximation comes when the voltage is changed and the flow is in non-equilibrium. However, the total amount of charge accumulated during this period is small compared to the charge accumulated throughout the experiment. The physics described with this final model is similar to the one described in [9], but the numerical implementation is quite different.

One more aspect of the implementation needs to be considered, namely how to initialize the solver. The desired initial condition is to start from a state with zero charge. Simply starting the time dependent solver from a state with voltage would results in a resistive solution as initial condition. One option is to ramp the up voltage during the first few seconds, but this method results in a lot of extra computational time. A better way to proceed is to initialize the electric currents module by solving the frequency dependent problem which gives the desired zero charge solution assuming

$$\varepsilon\omega >> \sigma \qquad (11)$$

where ω is the angular frequency and ε the permittivity. After a solution for the electric field has been found the stationary ion flow can be calculated and after that the time dependent solver called.

4. Results

We apply the model introduced in the previous section to the calculation of the surface potential in a configuration especially designed

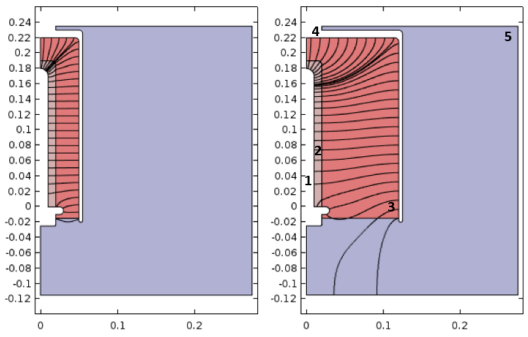

Figure 1. The two geometries considered in the present article. Both geometries are axisymmetric. Left panel: geometry with 5cm cup. Right panel: geometry with 12cm cup. The different components of the geometry are numbered on the right panel: grounded electrode (1), epoxy layer (2), SF6 gas (3,5), and high voltage electrode (4). On both panels, the coloring denotes the simulation domains, while the lines are streamlines of the capacitive electric field. The ion-drift equations are solved only in the red-colored domain.

to highlight the importance of charge transport through the gas volume. We consider the two geometries investigated experimentally in [1], and sketched in figure 1.

The experimental setup is fully 2d-axisymmetric. It consists of a central grounded electrode, covered with a solid insulation layer made of epoxy. A cylindrical high voltage electrode whose radius can be adjusted (5cm or 12cm) surrounds the central electrode. Both electrodes are themselves located in a large pressure vessel filled with 0.5MPa of SF6 gas.

In the experiment [1], the voltage on the outside electrode is quickly ramped to a value of 1kV, and remains at this level for 12 hours. After this time, the voltage is ramped down, and the surface potential at the surface of the insulator is measured. This surface potential stems from the charge that has accumulated at the gas-insulator interface over the whole time of the experiment.

The model described in section 3 is applied to both geometries. An additional electrostatic interface is added in order to compute the electric potential based on the accumulated surface charge. An extra volume is added to the geometry in order to calculate the electrostatic potential from the surface charge accumulated in the simulation. The material parameters used in the simulation are specified in table 1.

Table 1: Simulation parameters.

Description	Symbol	Value
Ion mobility	μ	$1.2\cdot10^{-5}m^2/Vs$
Ion diffusion	D	$3.1\cdot10^{-7}m^2/s$
Epoxy conductivity	σ	$10^{-17}S/m$
Background ionization rate	R	$3.8\cdot10^{7}IP/m^3s$

The ion mobility is taken from [10] and diffusion parameters estimated using the Einstein relation.

$$D = \mu kT/q \qquad (12)$$

Here k is the Boltzmann constant and T the temperature. Changing the mobility or diffusion constant has only small impact on the results.

The parameters that have the highest impact on the results are the background ionization rate and the epoxy conductivity. Both have been fitted to the data in [1]. In [1], a background ionization rate of 29 IP/cm^{3}s was measured. The simulations show that this value is not sufficient to generate enough charge to describe the measured surface potential and a somewhat higher value of 38 IP/cm^{3}s was therefore used. This value remains in line with standard values for the given pressure [7].

A comparison of the measured and simulated potential at the insulator surface after the application of 1kV for 12 hours is shown in figure 2. In general there is a good agreement between the results and the experiments. The simulation reproduces the main features seen in the experiment. The peak in the 12cm cup data is

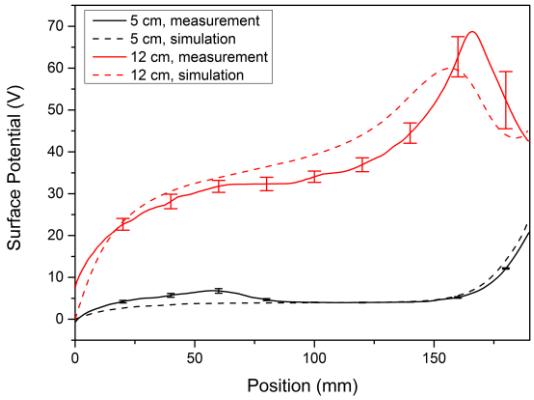

Figure 2. Comparison of the measured (solid line) and simulated (dashed line) surface potential after the application of a DC 1kV stress on the high voltage electrode for a period of 12 hours, for the two geometries plotted in Figure 1 (5cm radius: black curve, 12 cm radius: red curve). The measurement is done along the insulating rod (z-direction), with position zero corresponding to the bottom of the rod.

not quite as sharp as in the experiment, and its location is slightly different. The results with the 5cm cup are on the other hand very well reproduced. Overall, and given the experimental uncertainty (error bars on figure 2), the agreement between the model and experiment can be considered very satisfactory.

The peak seen with the 12cm electrode radius in both simulation and experiment can be qualitatively understood by looking at the field lines in figure 1. A higher density of field lines crossing the epoxy surface means higher ion flow due to a larger capture volume. This means that the highest charge accumulations happens near the corner of the epoxy rod in the 5cm geometry and about 2cm away from the corner in the 12cm geometry, corresponding to the peak in figure 2.

We also performed simulations to validate the quasi-static approximation described in section 3. The same simulation was run with time dependent terms activated. The initial ion density was set to $2 \cdot 10^{10} m^{-3}$. The simulation time increased from around one minute to more than five minutes and the simulation results were changed by less than 0.1% compared to the quasi-static simulation. Almost all the extra simulation time is spent simulating the initial sweeping out of the equilibrium charge density via the electric field. This phenomena occurs within a fraction of the first second, and has very little impact on the long term behavior of the insulation system. Explicitly simulating this initial phase of the problem is time consuming and can often turn out to be unstable for more complicated geometries.

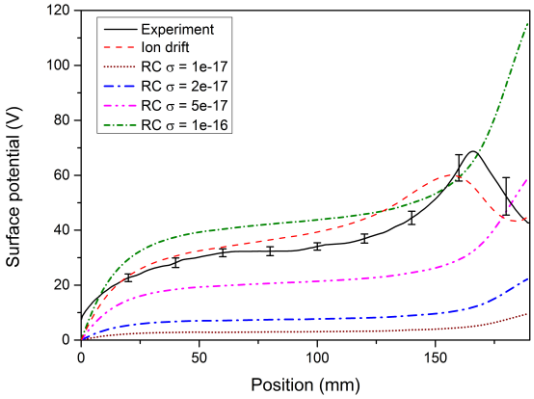

Figure 3. Surface potential as a function of position after applying 1kV on the high voltage electrode for 12 hours, in the 12 cm radius configuration. Comparison of the experimental results (solid line) to those obtained using either the ion-drift model for the charge transport in the gas (dashed line) and using an Ohmic, RC description for the gas, with different values for the gas conductivity.

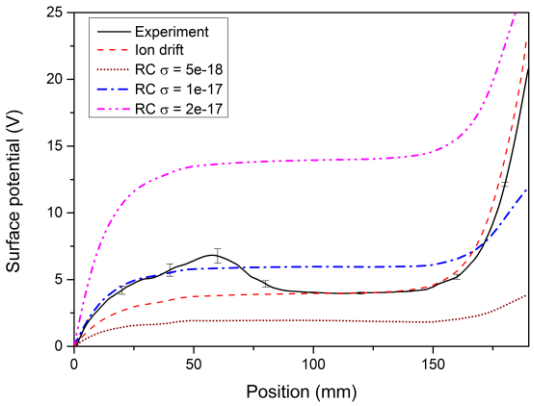

Figure 4. Surface potential as a function of position after applying 1kV on the high voltage electrode for 12 hours, in the 5 cm radius configuration. Comparison of the experimental results (solid line) to those obtained using either the ion-drift model for the charge transport in the gas (dashed line) and using an Ohmic, RC description for the gas, with different values for the gas conductivity.

The hybrid model was also compared to the results from a pure RC model. Simulations were run for a number of different SF6 conductivities; we show in figure 3 and 4 the results obtained with the values that best reproduce the experimental curves.

The general fit to experiment is much worse for the RC model. The main structure of the experimental results with a peak about 2 cm from the end of the for the 12 cm cup geometry is not reproduced at all. It is also worth noting that the best match is achieved only when varying the conductivity by an order of magnitude between the two geometries. All this indicates that the RC model is not applicable to the gas.

5. Discussion

This article presents a numerical implementation of a hybrid ion drift and RC model. The ion flow is assumed to be quasi-stationary, making the simulations faster and more robust without any significant impact on the results compared to the full ion drift equations. The resulting simulations describe the two test geometries well. Having a fast and robust model makes it applicable to more

complicated insulations systems than those simulated here, making it possible, for instance, to use such simulations with 3D geometries.

A pure RC model was also applied to the geometry, but it failed to adequately describe the features present and relies on fitting the gas conductivity separately for each geometry. The comparison with pure RC illustrates the importance of including the effects from ions for gaseous dielectrics.

All the theory presented in this paper is also applicable to other insulation gasses such as air by changing the material parameters. The challenge with air insulated systems is that they are often very large and may involve open gas volumes. It is therefore not straightforward to implement the right boundary conditions for such cases. This is certainly worth investigating.

The model is also potentially applicable to systems involving a combination of liquid and solid dielectrics such as in [5]. Here some care needs to be taken verifying the quasi-static approximation. The ion densities in liquids are much greater compared to gases and the dependence on the initial state may therefore be more important.

6. Conclusions

In this paper a hybrid ion drift and resistive-capacitive model for modelling combinations of gas and solid dielectric under DC stress. The model is implemented in COMSOL Multiphysics and a quasi-static ion flow approximation is introduced making the simulations much faster and more robust. The simulation results are compared to results from experiments performed at ETH Zürich [1] and the simulations capture all the main features of the experimental results.

7. References

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