Modeling the Acoustic Scattering from Axially Symmetric Fluid, Elastic, and Poroelastic Objects due to Nonsymmetric Forcing Using COMSOL Multiphysics

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Abstract: A frequency-domain finite element (FE) technique for computing the acoustic scattering from axially symmetric fluid-loaded structures subject to a nonsymmetric forcing field based on Ref. 1 is extended to poroelastic media and implemented in COMSOL Multiphysics. This method allows for the scattering body to consist of any number of acoustic, elastic, and poroelastic domains. The elastic and poroelastic domains are implemented using "Weak Form PDE" interfaces. Verification cases that illustrate scattering due to fields where the Fourier coefficients of the incident field are and are not known analytically are considered.

Keywords: Underwater acoustics, Biot theory, poroelasticity, acoustic scattering, axisymmetric.

1. Introduction

Thanks to advances in computing technology, the finite element method (FEM) is now a feasible choice for the calculation of the scattering response of geometrically complex structures due to their interaction with an acoustic field. One such usage of the FEM in underwater acoustics is prediction of the acoustic signature of objects on or in the seabed for classification purposes, as discussed in Ref. 2. Full-scale simulations like those described in Ref. 2 can be computationally expensive, especially when the domain being simulated is large compared to an acoustic wavelength. This cost can be somewhat mitigated when the scattering body is axially symmetric through the technique described by Zampolli and coworkers in Ref. 1, where Fourier decomposition allows for the calculation of the full three-dimensional solution using a series of less computationally intensive two-dimensional simulations. It is the goal of this paper to describe how this technique can be extended to poroelastic media and implemented in COMSOL Multiphysics.

2. Problem Description

The general class of problem being addressed in this work is shown in Figure 1.

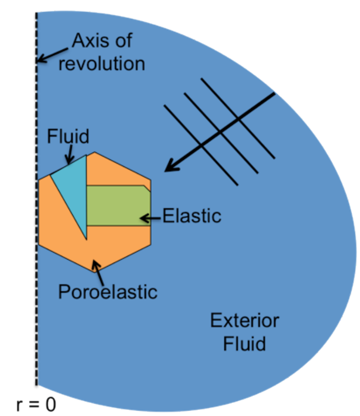

Figure 1. Depiction of axisymmetric scattering problem.

An axially symmetric yet otherwise arbitrarily shaped object consisting of fluid, elastic, and poroelastic domains is surrounded by an exterior fluid domain of infinite extent and insonified by a nonsymmetric acoustic field, such as an obliquely incident plane wave. The governing equations used to model the fluid, elastic, and poroelastic domains are standard and are as given in Ref. 3. In general, the incident field (or any field) can be decomposed into a Fourier series and can be expressed as

$$p_{inc}(r,\theta,z) = \sum_{m=-\infty}^{\infty} A_m(r,z) e^{-im\theta}, \qquad (1)$$

where

$$A_m(r,z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} p_{inc}(r,\psi,z) e^{im\psi} d\psi \,. \qquad (2)$$

This decomposition is the basis of the axisymmetric formulation described in the next section.

3. Axisymmetric Formulation

The Fourier decomposition of the incident field given by Eqs. (1) and (2) points the way toward a method in which the fully three-dimensional solution can be assembled from a series of two-dimensional simulations. Since the model geometry is axially symmetric and each $A_m(r, z)$ is independent of θ, it can be shown that the fully three-dimensional solution can be constructed by solving for the scattering response due to each $A_m(r, z)$ and using the following equation:

$$b(r, \theta, z) = \sum_{m=-\infty}^{\infty} \underline{b}_m(r, z) e^{-im\theta}, \qquad (3)$$

where $b(r, \theta, z)$ is a field quantity (dependent variable) such as pressure or the displacement in a given direction (i.e., u, v, w) and $\underline{b}_m(r, z)$ is the component of the scattered field quantity corresponding to the forcing $A_m(r, z)$.

In short, the steps of the Fourier decomposition technique are as follows. First, the Fourier coefficients, A_m, of the incident field are calculated using Eq. (2). Next, these Fourier coefficients, which are by definition axisymmetric, are used as the incident fields in a series of axisymmetric calculations to obtain $\underline{b}_m$, where each $\underline{b}_m$ is the scattered field solution corresponding to a given A_m. Finally, the corresponding fully three-dimensional scattered field solution b is constructed using Eq. (3).

Application of the proposed technique to fluid and elastic domains is well described in Ref. 1 and the reader is directed to that reference for more detail. Application to poroelastic media is carried out in a similar way by substituting Eq. (3) for the dependent variables in the poroelastic weak formulation given in Refs. 3 and 4.

4. COMSOL Implementation

This section deals with how models using the Fourier decomposition method described above can be constructed using COMSOL Multiphysics. First, fluid domains are discussed. Next, elastic and poroelastic domains and the added complexities they bring are covered. Then, implementation of the incident fields described by Eqs. (1) and (2) are considered. Finally, meshing considerations are briefly mentioned.

4.1 Fluid Domains

The axisymmetric formulation described in Section 3 is easily implemented for fluid domains (both interior and exterior) using the "Pressure Acoustics, Frequency Domain" interface of the Acoustics Module. When the "Pressure Acoustics, Frequency Domain" interface is added to the model, the user is able to easily input a desired circumferential wavenumber, which corresponds to the m in Eqs. (1), (2), and (3). When the circumferential wavenumber is nonzero, the equations of motions implemented by the Pressure Acoustics interface are equivalent to those given for fluid domains in Ref. 1.

For exterior fluid domains, the Sommerfeld radiation condition should be enforced through use of either radiating conditions, infinite elements, or perfectly matched layers. It is the opinion of the current authors that perfectly matched layers are the most robust of these techniques due to their superior performance in absorbing obliquely incident waves; it is therefore recommended that perfectly matched layers with thickness equal to one acoustic wavelength and rational coordinate stretching be used in most cases.

4.2 Elastic and Poroelastic Domains

Unlike the case with fluid domains, implementing the axisymmetric formulation for elastic and poroelastic domains is nontrivial. While the axisymmetric form of the equations of motion for an acoustic fluid implemented in COMSOL allow for a nonzero circumferential wavenumber, the elastic and poroelastic equations of motion included in the "Elastic Waves" and "Poroelastic Waves" interfaces always assume $m = 0$. Therefore, the governing equations must be completely entered by the user. While there are many ways to go about entering these equations in COMSOL, the authors decided to use several "Weak Form PDE" interfaces.

In general, three "Weak Form PDE" interfaces are necessary for problems including both elastic and poroelastic domains (one where the three elastic displacement components are defined, one where the three poroelastic displacement components are defined, and one where the poroelastic pressure is defined). The volume integral portions of the weak forms given by Ref. 1 for elastic domains and Refs. 3 and 4 for poroelastic domains are entered using the "Weak Form PDE" nodes. In order to couple the fluid, elastic, and poroelastic domains, the coupling conditions given by Ref. 3 are implemented using either "Weak Contribution" nodes or "Dirichlet Boundary Condition" nodes, depending on whether the coupling condition in question is natural or essential.

4.3 Incident Field

As in most acoustic scattering calculations, the incident field is implemented using a "Background Pressure Field" node in the "Pressure Acoustics, Frequency Domain" interface. If the integral in Eq. (2) can be evaluated analytically (as in the case of an obliquely incident plane wave), the analytical expression for A_m can be entered as a user defined background field. If the integral in Eq. (2) does not admit an analytical solution, the integral itself can be numerically evaluated in COMSOL using the *integrate* command. For instance, for a generic incident field defined as an analytic function under the definitions tab called *pinc* with arguments r, θ, z, Eq. (2) can be implemented by setting the user defined background field to *1/(2*pi)*integrate(pinc(r,psi,z)*exp(1i*m*psi), psi,-pi,pi)*.

4.4 Mesh

When attempting to resolve wave physics using quadratic shape functions, as is the default in COMSOL, a good rule of thumb is to have elements be of relatively uniform spacing and sized no larger than $\lambda/6$, where λ is the wavelength corresponding to the slowest wave supported by the medium. The current authors have found that the performance of perfectly matched layers is improved when a structured or mapped mesh is used for PML domains and the elements are sized no larger than $\lambda/10$.

5. Verification Cases

A relatively simple problem will be used to showcase the effectiveness of the method presented in this work. The model consists of an aluminum shell (elastic) filled with sand (poroelastic) surrounded by water (fluid), as shown in Figure 2. The outer radius of the shell is 0.1 m. The shell thickness is 25%. The water is surrounded by PMLs equal to one acoustic wavelength in thickness. The sound speed and density of the water are set to 1530 m/s and 1023 kg/m^3, respectively. The compressional speed, shear speed, and density of the aluminum are set to 6300 m/s, 3000 m/s, and 2700 kg/m^3, respectively. The poroelastic properties used for sand are given in the Appendix. The frequency is fixed at 10 kHz.

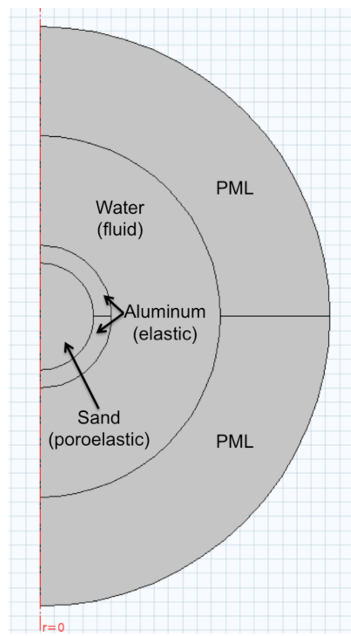

Figure 2. Verification model geometry.

Two incident fields are considered to demonstrate the implementation of plane wave and general field incidence. For each case, the target strength is calculated by assigning a "Far-Field Calculation" node set to "Full Integral" on the PML-water interface. The total far-field pressure at 100 m range is obtained through use of Eq. (3), with the Fourier series truncated at some acceptable $m = M$. The target strength is then calculated using the following expression:

$$TS(\theta) = 20 \log_{10} \frac{r|p_{far}(r,\theta)|}{|p_{inc}|}, \quad (4)$$

where p_{far} is evaluated in the $z = 0$ plane, $r = 100$ m, and $|p_{inc}|$ is assumed equal to 1 Pa.

For both incident fields considered, the accuracy of the proposed technique will be verified by comparing the target strength for a normally incident (axisymmetric) field calculated using the built-in physics to the results for an obliquely incident field calculated following Section 4. Since the scatterer is spherical, the target strength should be independent of incident angle.

5.1 Plane Wave Incidence

For the first model, a plane wave incident field is used to demonstrate the case where an analytical expression for A_m is known. For a plane wave, the analytical Fourier coefficients are

$$A_m = i^m \exp(ikz\sin\phi)\, J_m(kr\cos\phi), \qquad (5)$$

where ϕ is the incident angle ($\phi = 90°$ denotes a wave traveling in the negative z-direction) and $J_m(x)$ represents the Bessel function of order m.

The target strengths for incident angles of 90° (calculated with the built-in physics) and 0° (calculated with the user implemented physics) are shown in Figure 3. For the off-normal case, 17 terms were calculated in accordance with the convergence criteria given in Ref. 1. It is clear from the figure that the target strengths are in almost exact agreement, verifying the accuracy of the proposed method.

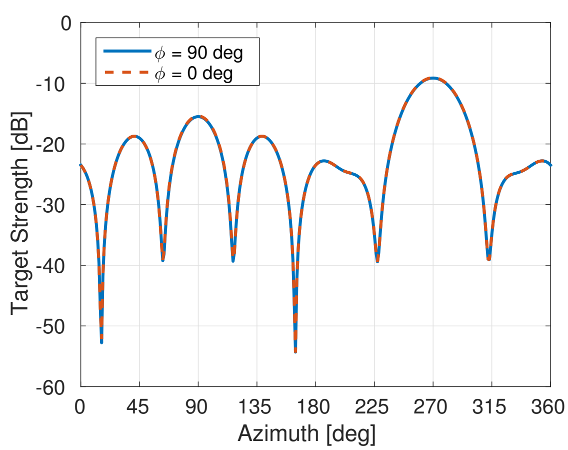

Figure 3. Target strength due to an incident plane wave.

5.2 General Field Incidence

The second model is meant to demonstrate the case where an analytical expression for A_m is not known. For this case, a Gaussian tapered plane wave as given in Ref. 5 is used as the incident field. The waist of the beam is set equal to the outer diameter of the shell and the Fourier coefficients are found numerically, as discussed in Section 4.3.

The target strengths for incident angles of 75° and 60° are compared with that calculated for normal incidence, as shown in Figure 4. 10 terms are used to construct the curves corresponding to the oblique cases. Again, excellent agreement is seen between the normal and oblique cases for the range of azimuthal angles considered.

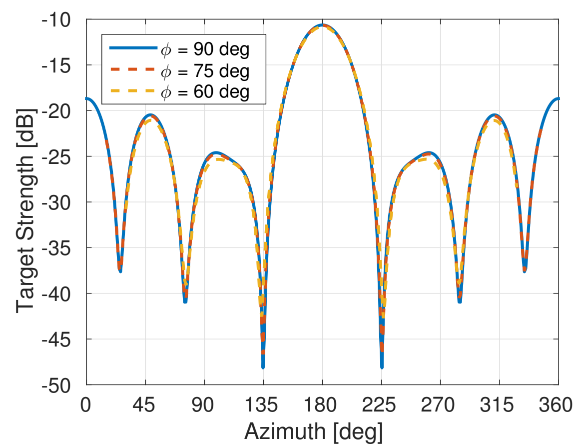

Figure 4. Target strength due to an incident Gaussian-tapered beam.

6. Conclusions

In this work, a method used to construct a full three-dimensional acoustic scattering solution for the case of axially symmetric geometry and nonsymmetric loading is presented. The scattering body can consist of any number of fluid, elastic, and poroelastic domains. Weak Form PDE interfaces are used to implement the required physics for the elastic and poroelastic portions of the model. Verification models are shown to illustrate the cases of plane wave and general field incidence. These verification examples clearly demonstrate the accuracy and utility of the proposed decomposition technique.

7. References

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8. Acknowledgements

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9. Appendix

Table 1: Poroelastic Properties

Parameter	**Value**	**Unit**
Fluid density	1023	kg/m^3
Fluid bulk modulus	2.395	GPa
Fluid viscosity	0.00105	kg/(m*s)
Sediment grain density	2690	kg/m^3
Bulk modulus of grains	32	GPa
Frame bulk modulus	43.6+2.08i	MPa
Shear bulk modulus	29.2+1.80i	MPa
Permeability	2.5×10^{-11}	m^2
Tortuosity	1.35	--
Porosity	0.385	--
Pore size parameter	26.5	μm