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2008

Interfacing continuum and discrete methods: convective diffusion of microparticles and chemical species in microsystems

J. Berthier



Contents

- Diffusion of species, nanoparticles, macromolecules in microfluidics
 - Model
 - Applications to diffusion in cellular networks
- Convective transport
 - Model: interfacing COMSOL (carrier fluid flow field) and Monte-Carlo (particles)
 - Applications :
 - transport in a straight channel
 - recirculation regions for particle trapping
 - flow past micropillars
 - flow through a micro/nano aperture
- Conclusions and perspectives

Modeling diffusion at the microscale: the Monte-carlo approach

- Starting point: Langevin's equation
(Newton's equation + stochastic term)
- Mimic of the random walk ($l \ll L$)
- Geometry description
- Boundary conditions

2D

$$X_{i+1} = X_i + \sqrt{4D\Delta t} \cos(\alpha)$$

$$Y_{i+1} = Y_i + \sqrt{4D\Delta t} \sin(\alpha)$$

$$\alpha = \text{random}(0, 2\pi)$$

3D

$$X_{i+1} = X_i + \sqrt{4D\Delta t} \cos(\alpha) \sin(\beta)$$

$$Y_{i+1} = Y_i + \sqrt{4D\Delta t} \sin(\alpha) \sin(\beta)$$

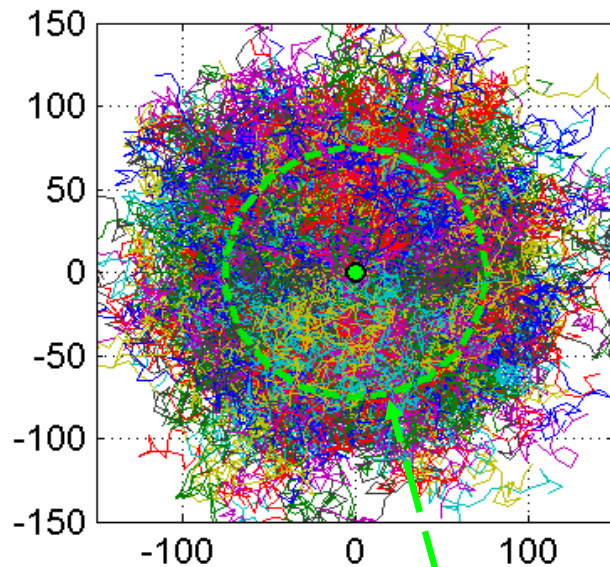
$$Z_{i+1} = Z_i + \sqrt{4D\Delta t} \cos(\beta)$$

$$\alpha = \text{random}(0, 2\pi)$$

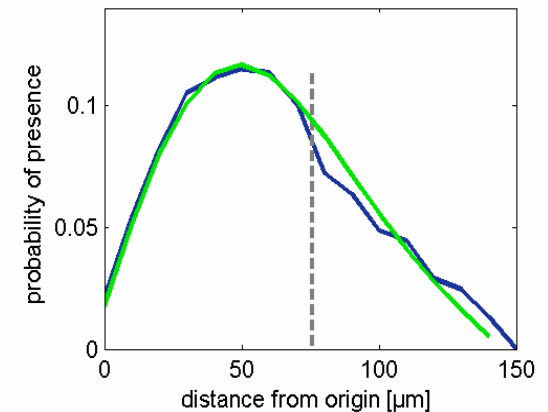
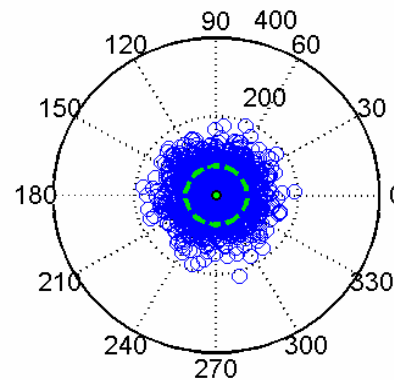
$$\beta = a \cos(1 - 2 \text{random}(0, 1))$$

(a)

(b)



$$l = \sqrt{4Ddt}$$

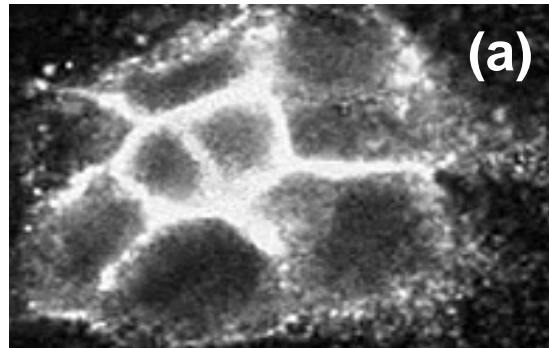


Comparison between COMSOL
and Monte-Carlo model

In-vivo application: modeling diffusion in extra-cellular spaces

Application to cellular uptake

Fluorescent image of a cellular network



Schematic of the network

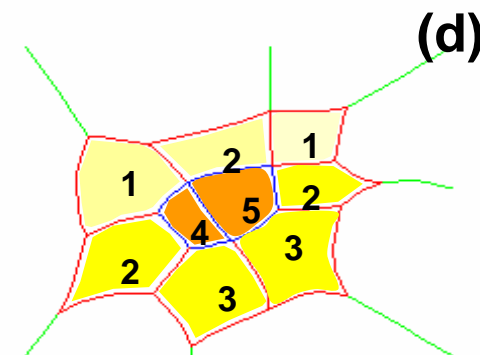
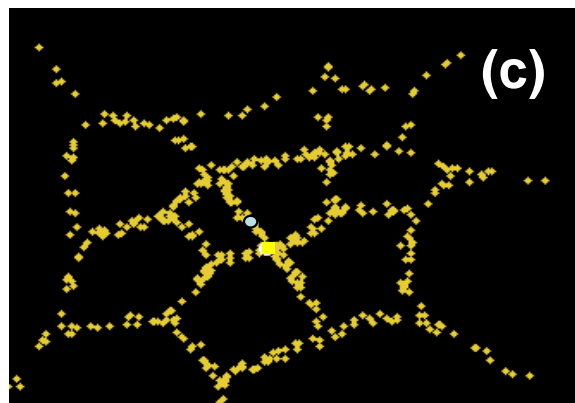
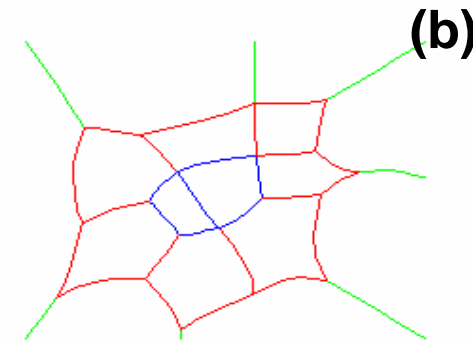


Image of the calculated network after 10 mn

Cellular uptake rate after 10 mn

Modeling diffusion at the microscale: the Monte-carlo approach

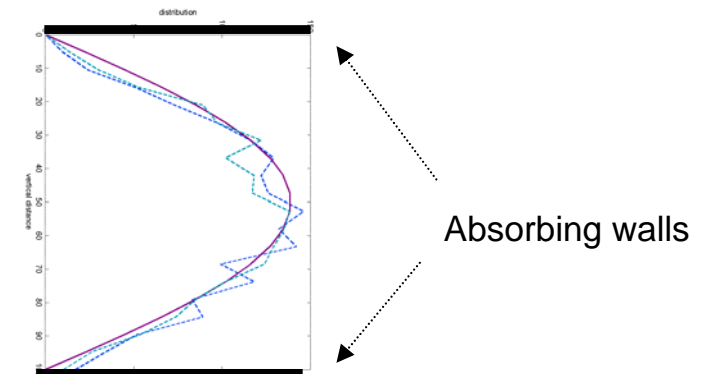
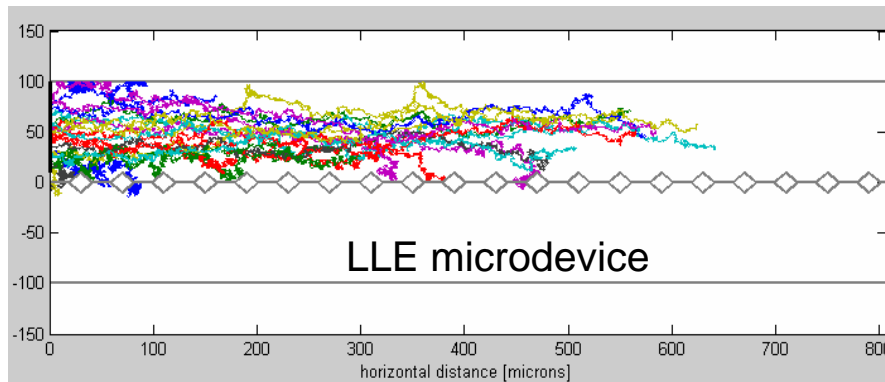
- Carrier fluid is a continuum ($Kn \sim 1 \rightarrow l \sim 10$ nm for a liquid)
- Transported species may be in a discrete quantity
- Starting point: Langevin's equation
- Geometry description + boundary conditions

2D

$$X_{i+1} = X_i + V_{x,i} \Delta t + \sqrt{4D \Delta t} \cos(\alpha)$$
$$Y_{i+1} = Y_i + V_{y,i} \Delta t + \sqrt{4D \Delta t} \sin(\alpha)$$
$$\alpha = \text{random} (0, 2\pi)$$

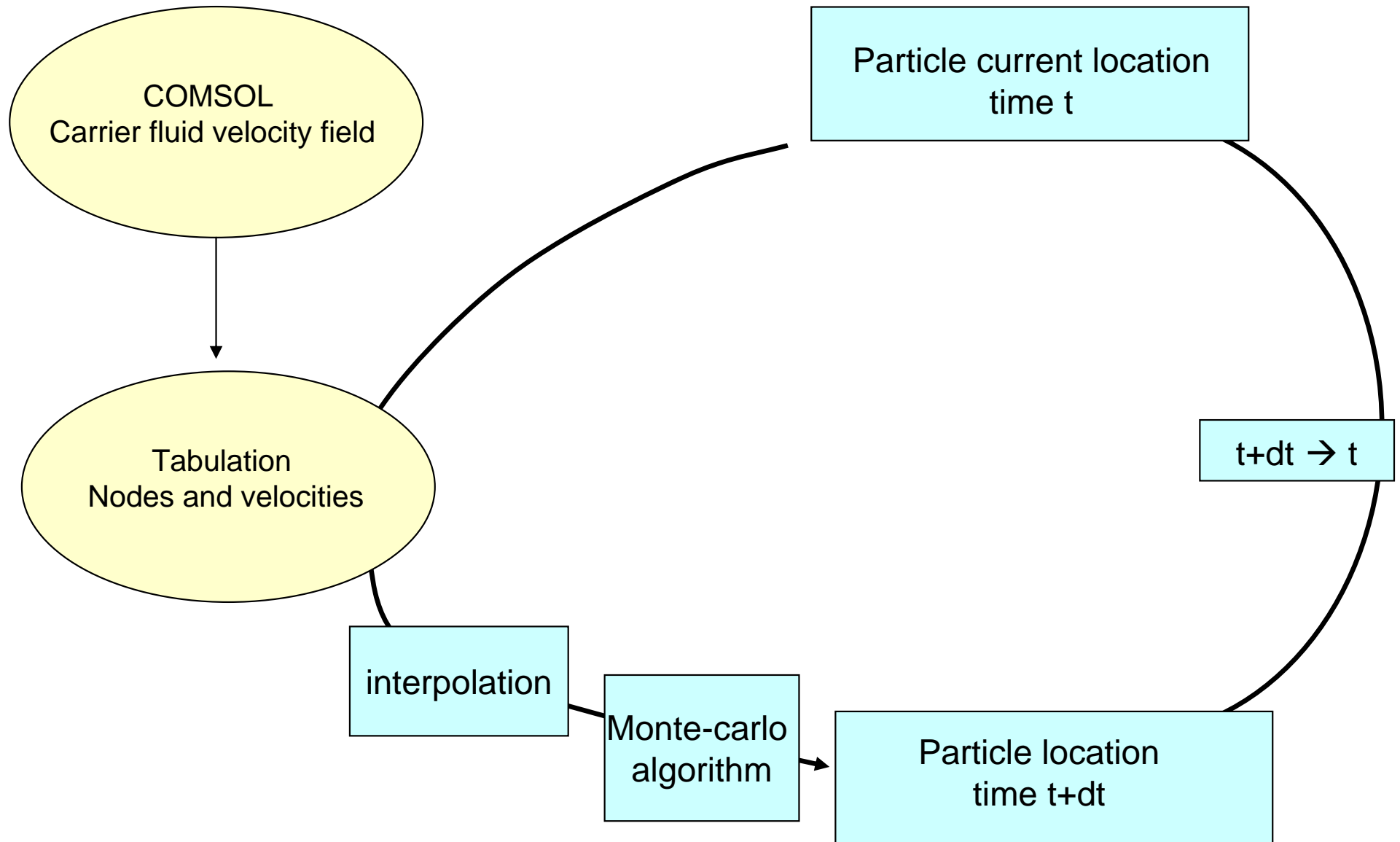
3D

$$X_{i+1} = X_i + V_{x,i} \Delta t + \sqrt{4D \Delta t} \cos(\alpha) \sin(\beta)$$
$$Y_{i+1} = Y_i + V_{y,i} \Delta t + \sqrt{4D \Delta t} \sin(\alpha) \sin(\beta)$$
$$Z_{i+1} = Z_i + V_{z,i} \Delta t + \sqrt{4D \Delta t} \cos(\beta)$$
$$\alpha = \text{random} (0, 2\pi)$$
$$\beta = a \cos(1 - 2 \text{random} (0, 1))$$

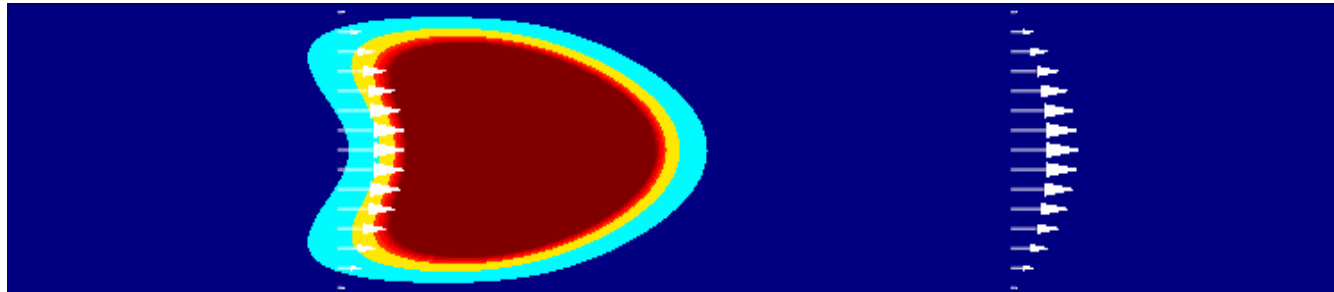


J. Berthier et al., "The physics of a coflow micro-extractor: interface stability and optimal extraction length," *Sensors and Actuators A*, in print.

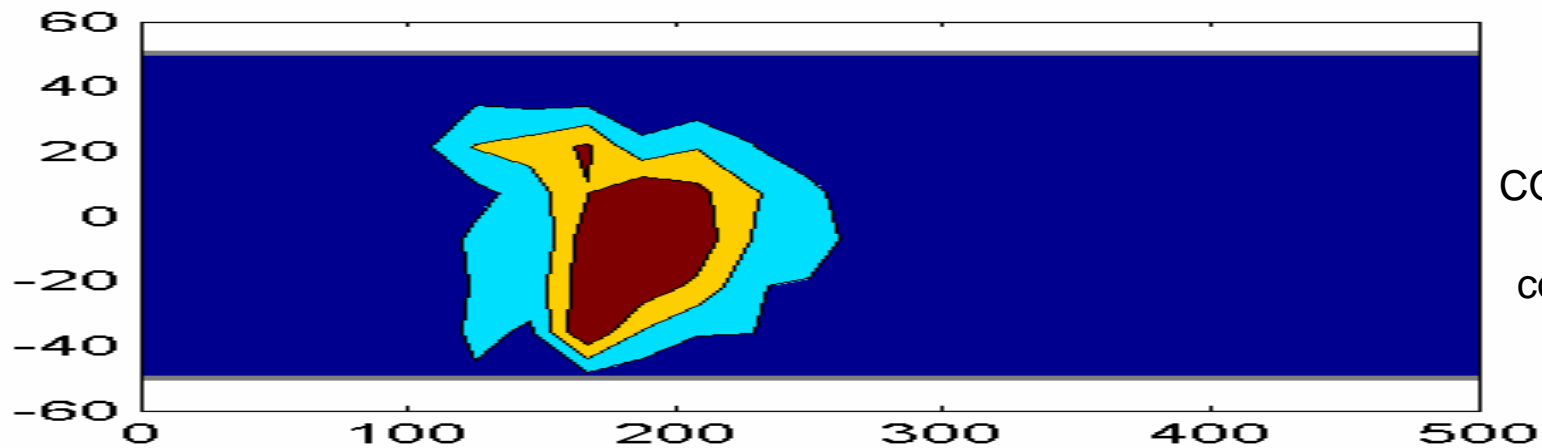
Schematic of the numerical approach



Assessment of the method in the geometry of a straight channel



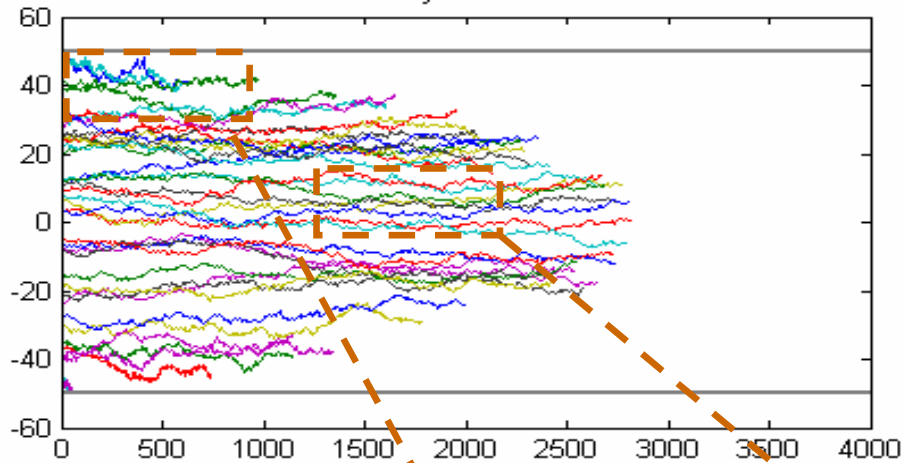
COMSOL (velocity field
+ convective transport)



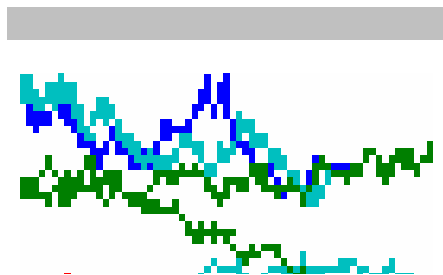
COMSOL velocity field
+ Monte-Carlo
convective transport
(200 particles)

Convective diffusion in a straight channel - viscoelastic liquid

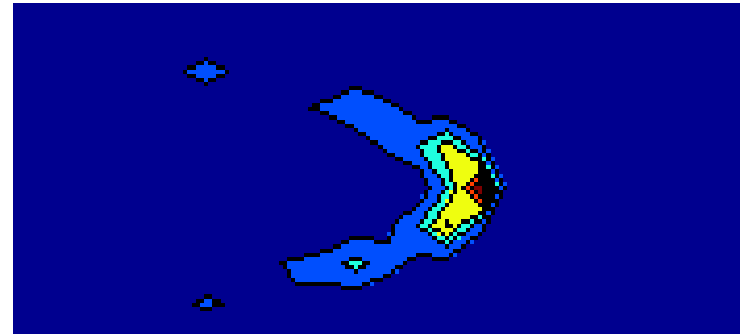
Ballistic random walk of particulates



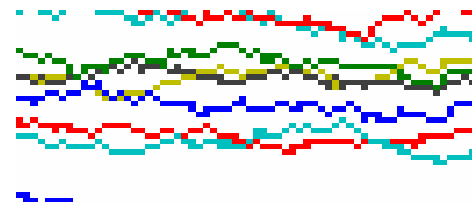
Random walk near the wall
(large shear rate)



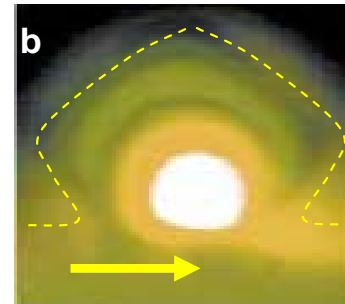
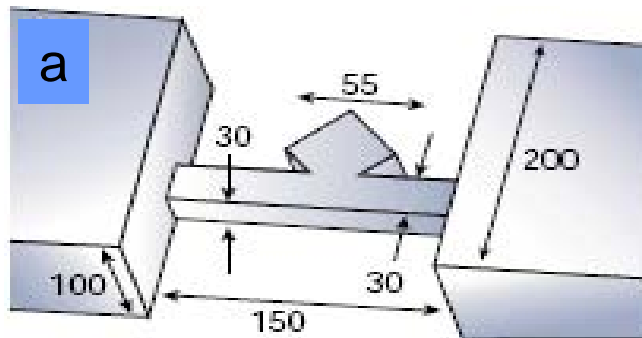
Concentration after 7 mm transport



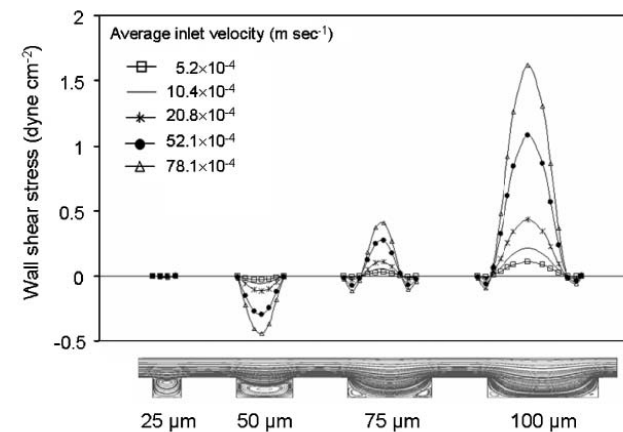
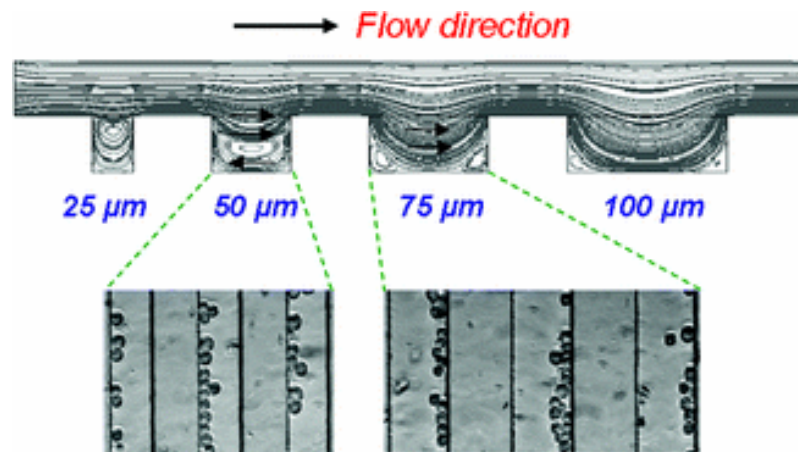
Random walk in the channel middle
(small shear rate)



Application: Convective trapping of particles in recirculation regions



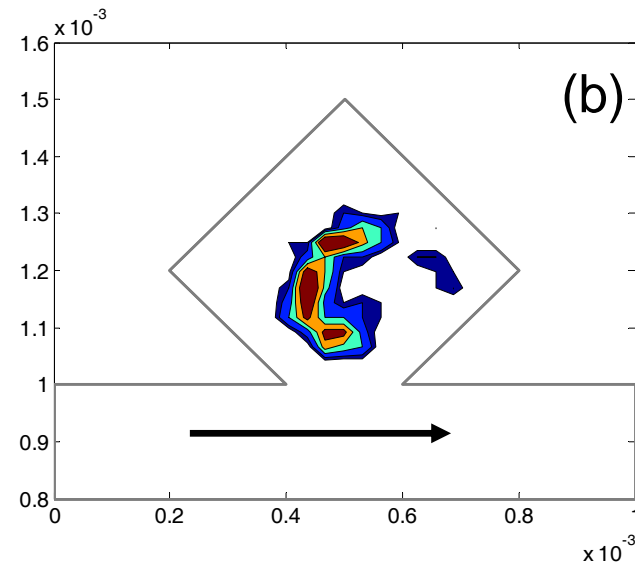
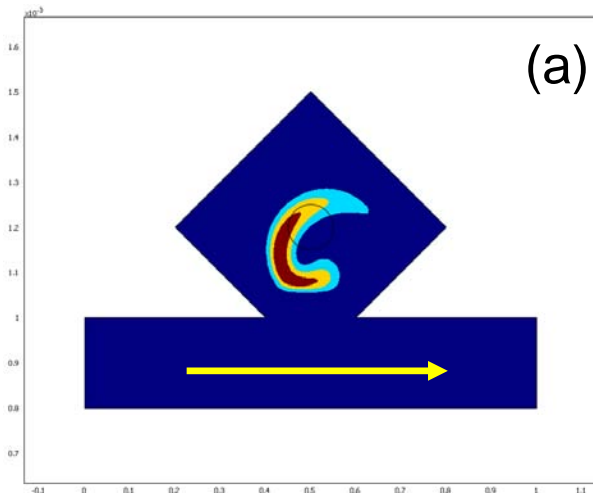
J.P. Shelby, D. S.W. Lim, J. S. Kuo, D. T. Chiu, "High radial acceleration in microvortices," *Nature*, Vol. 425, p.38, 2003



A. Manbachi et al. "Microcirculation within grooved substrates regulates cell positioning and cell docking inside microfluidics channels," *Lab chip*, **8**, 747–754, 2008

Application: Convective trapping of particles in recirculation regions

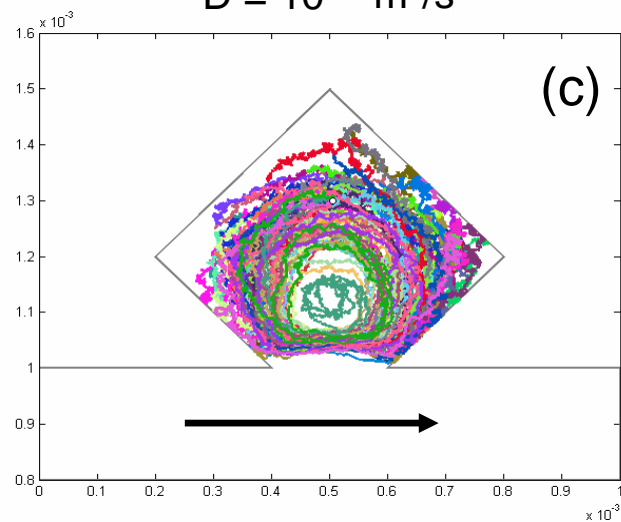
COMSOL



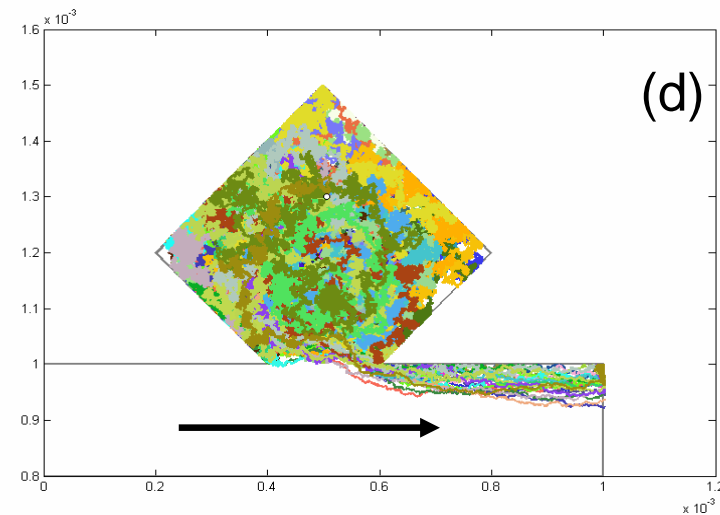
COMSOL / Monte-Carlo

Small diffusion coefficient
 $D = 10^{-10} \text{ m}^2/\text{s}$

Larger diffusion coefficient
 $D = 10^{-9} \text{ m}^2/\text{s}$



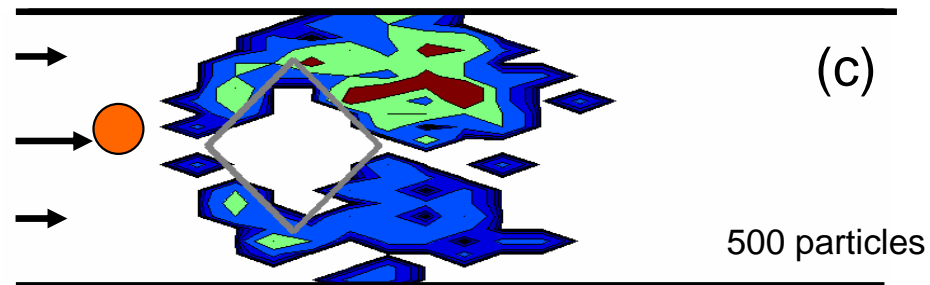
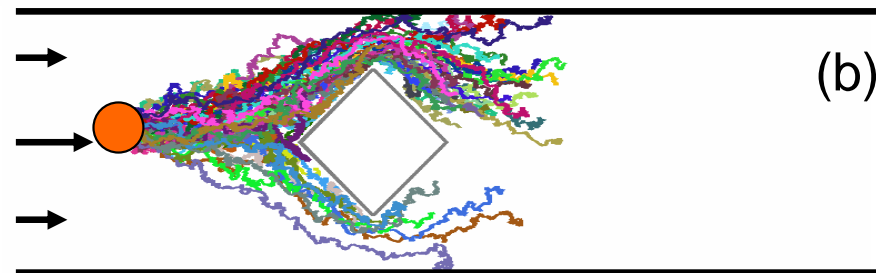
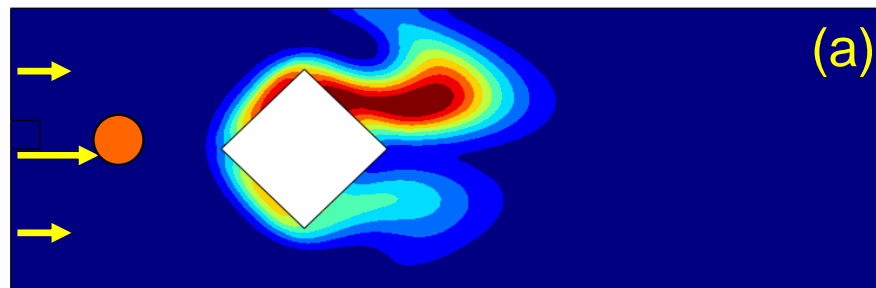
Particles do not escape



Particles slowly escape

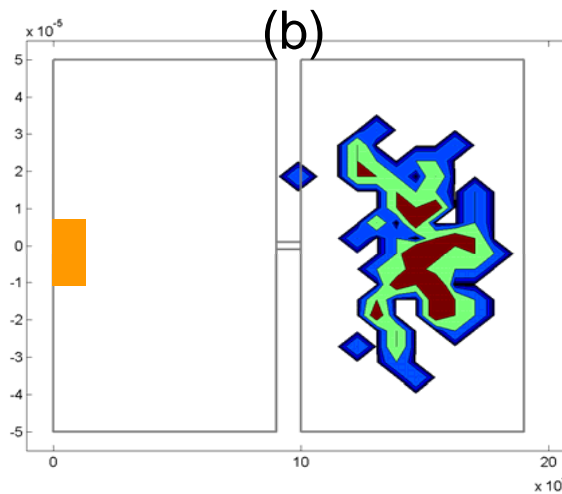
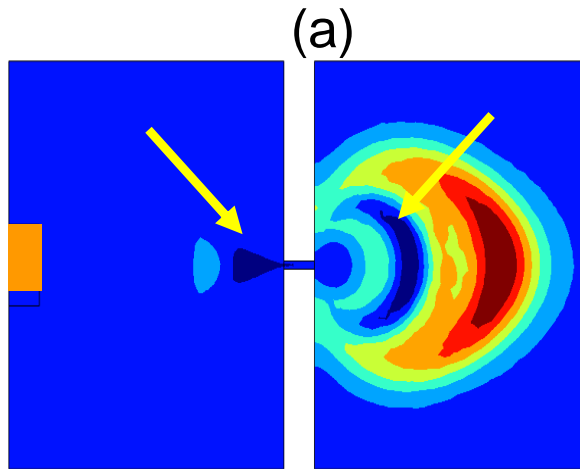
Transport of species past a micropillar

- Carrier fluid flow: Poiseuille profile at inlet
- Bolus of concentration released at $t=0$ from a circular volume above the middle axis

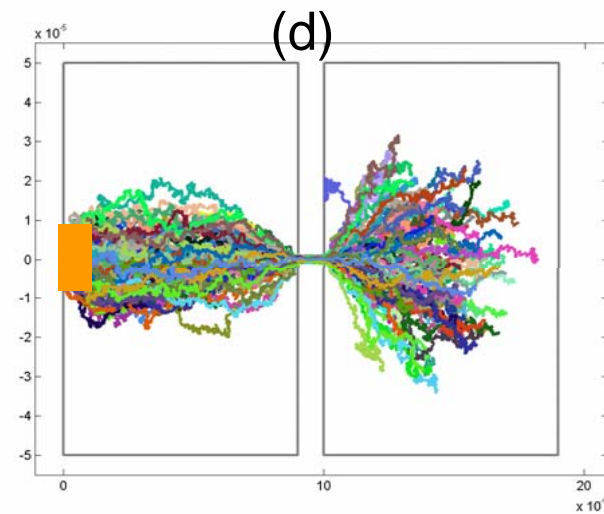
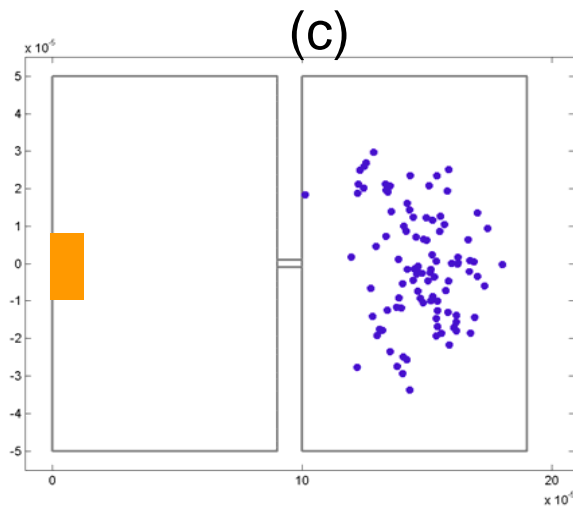


Convective diffusion of species / nanoparticles through a micro-hole

- Two chambers separated by a 10 μm aperture
- Bolus of concentration released at $t=0$ from a rectangular volume



200 particles



3D approach: feasible ?

- Limitation in the number of particles
- Algorithm not complicated
- Geometry and boundaries much more difficult to describe

$$X_{i+1} = X_i + \sqrt{4Ddt} \cos(\alpha) \sin(\beta)$$

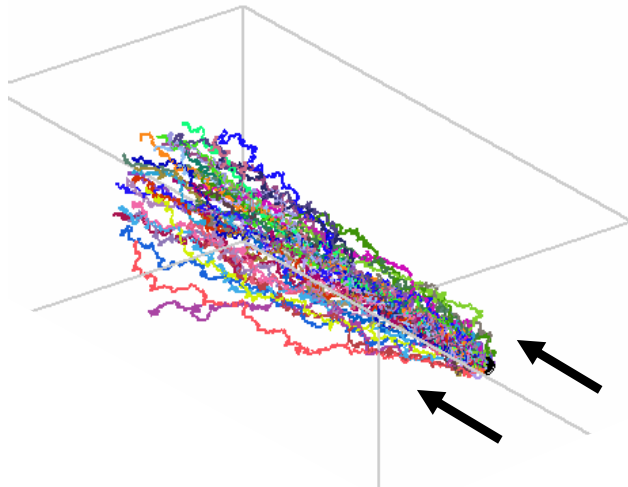
$$Y_{i+1} = Y_i + \sqrt{4Ddt} \sin(\alpha) \sin(\beta)$$

$$Z_{i+1} = Z_i + \sqrt{4Ddt} \cos(\beta)$$

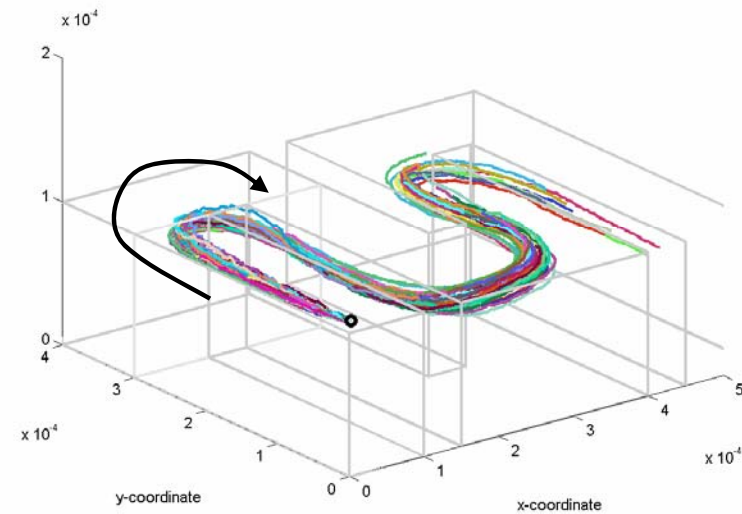
$$\alpha = \text{random}(0, 2\pi)$$

$$\beta = a \cos(1 - 2 \text{random}(0, 1))$$

Convective diffusion from a source point

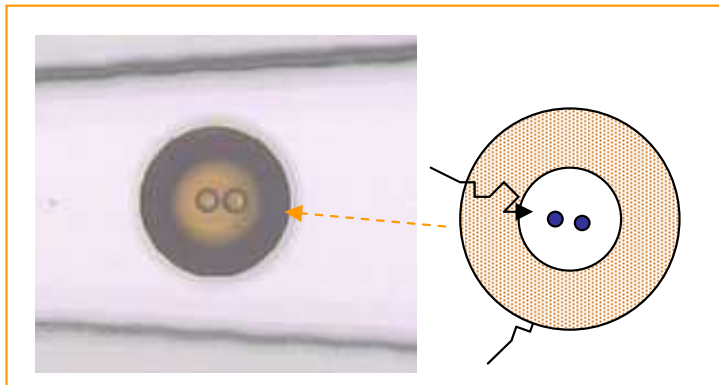


Convective diffusion in a curved micro-channel

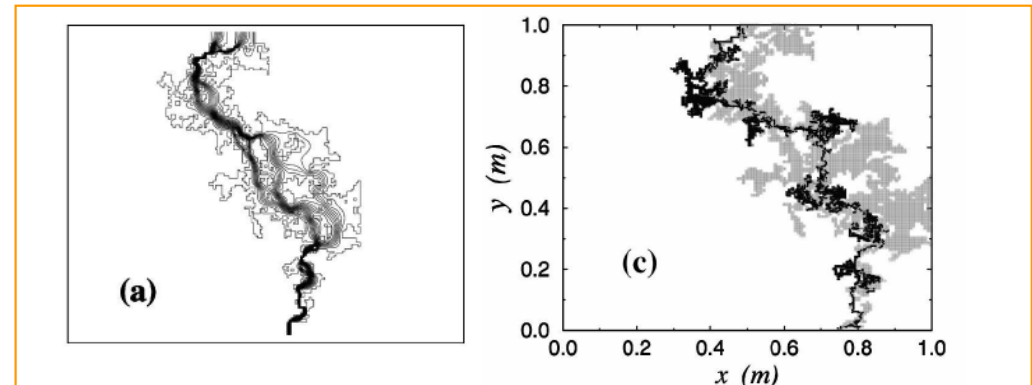


Conclusion / perspectives

- Continuum method → probability of presence
- COMSOL/ Monte-Carlo interfacing
 - Mimic the transport (to some extent) – visualization tool
 - Avoid numerical problems
 - Easy statistics
 - Necessity for nanopores, nanoporous materials, encapsulation



Le Vot et al. "Non-Newtonian fluids in Flow Focusing Devices: encapsulation with alginates," *Proceedings of the 1st European Conference on Microfluidics - Microfluidics 2008 - Bologna, December 10-12, 2008*



H.A. Makse et al., "Tracer dispersion in a percolation network with spatial correlations," *Physical Review E*, Vol 61, n°1, p. 583, 2000



Thanks