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Time-Harmonic Modeling of Squirrel-Cage Induction Motors: A Circuit-Field Coupled Approach

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Overview

- Introduction
- Magnetic Field Equations (Low-Frequency).
- Induction Machine Basics.
- Circuit-field coupled problem.
- Example: Induction machine operating at steady state.
- Conclusions.

- Equivalent circuit to represent induction machines.
- Accurate representation of geometry, material properties and excitations is required.
- Finite-element modeling of induction machines is an alternative.
- Squirrel-cage asynchronous machines are coupled with external circuits.

- A current fed approach is forbidden.
- Circuit and field equations must be simultaneously solved.
  - Strong coupling.
  - weak coupling.
- Other difficulty is the small air-gap that requires special attention while obtaining a proper meshing of this domain.

AC/DC module of COMSOL Multiphysics

- Quasi-3D model: Representation of overhang effects and coupling with external circuits.
- Solid and filamentary conductors are properly accounted for.
- The modeling of rotor and stator with meshes of different density is possible (air-gap).
- Rotor motion is achieved through modification of rotor conductivities and resistances.

Magnetic Field Equations

Low Frequency

Maxwell's equations can be combined to give:

$$\sigma \frac{\partial A}{\partial t} + \nabla \times (\frac{1}{\mu_0 \mu_r} \nabla \times A) - \sigma v \times (\nabla \times A) = \frac{\sigma \Delta V}{d} + J^e$$

- Displacement current and free charges have been disregarded.
- A is the magnetic vector potential.
- J^e is an external current density imposed in conductor regions.
    - Unknown since its value depends on A and the external circuit elements.
- Conductors have length d with a potential difference V which is unknown.

- Classical phasor concept of circuit theory.
- $\frac{\partial A}{\partial t}$ is simply substituted by $j\omega$.

$$j\omega\sigma A + \nabla \times (\frac{1}{\mu_0\mu_r}\nabla \times A) - \sigma v \times (\nabla \times A) = \frac{\sigma\Delta V}{d} + J^e$$

- It is possible to have a static geometry in the frequency domain.

- Three-phase induction machines produce a magnetic field that rotates at synchronous speed.
- The rotor slips from the synchronous speed.
- The magnetic field of the rotor currents rotates at the same speed of the stator field.
- The finite-element model must consider these facts.
- Slot effects are disregarded. Proper positioning of the rotor reduces their impact.

- Two main sub-domains: stator and rotor.
- Stator: stator core, phase conductors and part of the air gap.
- Rotor: rotor core, rotor conductors and remaining air gap.
- Stator winding conductors are considered filamentary (negligible eddy-current effects).

$$\nabla \times (\frac{1}{\mu_0}\nabla \times A) = J^e$$
$$\nabla \times (\frac{1}{\mu_0}\nabla \times A) = 0$$
$$\nabla \times (\frac{1}{\mu_0 \mu_r}\nabla \times A) = 0$$

Induction machine

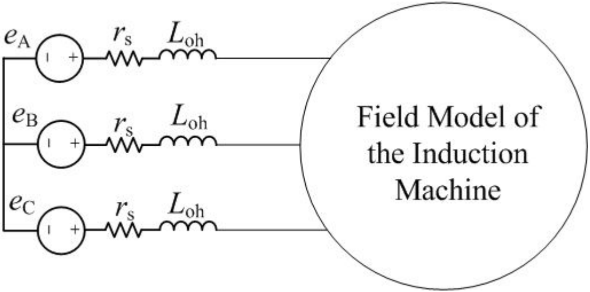
Rotor Equations

The conductors of squirrel cage type rotors must consider the eddy current effect.

$$j\omega\sigma(s)A + \nabla \times (\frac{1}{\mu_0}\nabla \times A) = \frac{\sigma(s)\Delta V}{d}$$
$$\nabla \times (\frac{1}{\mu_0}\nabla \times A) = 0$$
$$\nabla \times (\frac{1}{\mu_0\mu_r}\nabla \times A) = 0$$

$\sigma(s)$ is proportional to the actual rotor bar conductivity and refers the rotor quantities to the stator frequency.

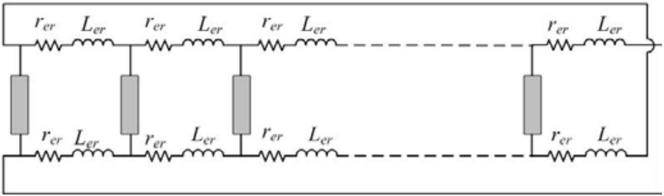
The stator is connected to a three-phase voltage source through the end-winding impedance.



End-winding impedance is calculated using classical formulae.

External circuit conditions
Rotor

For a squirrel cage rotor, the conductors are solid and short-circuited through conducting end rings of finite impedance.



- End-ring resistances must be divided by s.
- Rotor circuit accounts for anti-periodic boundary conditions.
- Only one pole pitch is required to represent the whole machine behavior.

Current and voltage relationships

Coupling of circuit and field variables requires expressions for the voltages in all machine conductors.

- Filamentary

$$e = ir + Ndj\omega A \qquad J^e = \frac{Ni}{S}$$

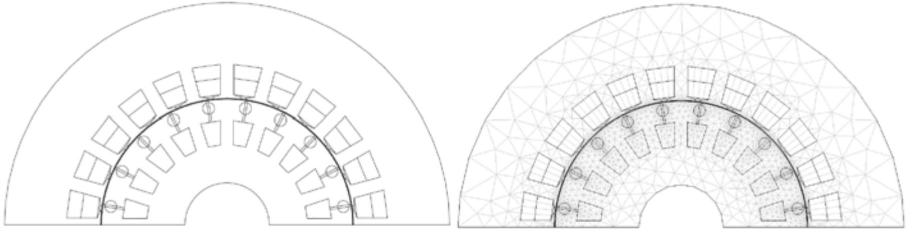
- Solid

$$i_{sol} = \int \left(\frac{s\sigma \Delta V}{L} - js\omega\sigma A \right) d\Omega \qquad \Delta V = r_{sol} \left(i_{sol} + js\omega\sigma \int A d\Omega \right)$$

- Anti-periodic boundary conditions means that:

$$A(r,\theta) = A(r,\theta)e^{j180^0}$$

The induction machine considered in this work is a double squirrel cage induction motor. It is a two-pole, 7.5 kW, 380 V, 50 Hz, three-phase star connected motor.



Lagrange-quadratic elements were use to perform the numerical simulations.

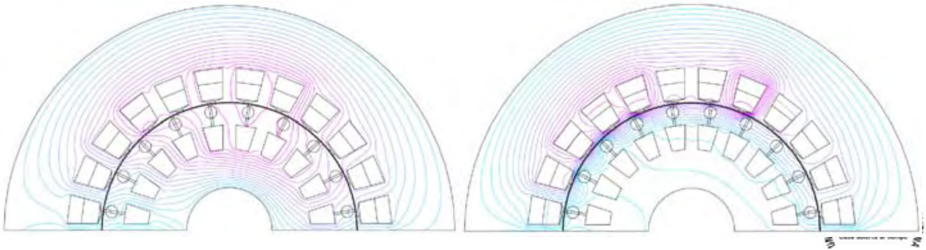
- Two independent meshes were constructed and stitched together using the create pairs capability of COMSOL.
- Perpendicular induction currents, vector potential application mode of the AC/DC module of COMSOL.
- Voltages can be computed using integration coupling variables at appropriate sub-domains.

- Resultant values of the integration coupling variables are inputted as a potential difference in the sub-domain settings of the rotor bars.
- Filamentary current densities are calculated as global scalar expressions that are later used as a sub-domain setting of stator conductor.
- Hence, the COMSOL interface with SPICE circuit lists becomes available and the problem is fully set up.

Numerical Results

Comsol

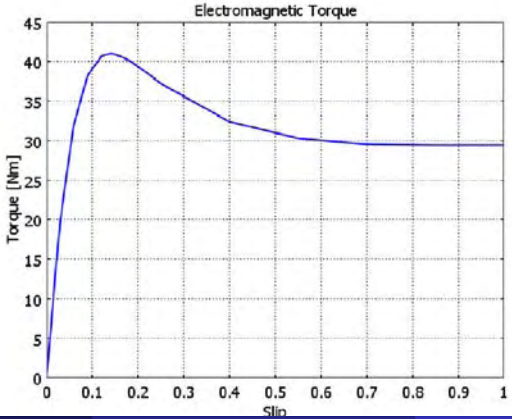
- The magnetic field can penetrate well into the rotor core because the slip frequency is small for rated operation.
- For locked-rotor operating, the effective rotor conductivity equals the actual conductivity of the rotor bars and the magnetic field is only reaching the outer parts of the rotor body (stronger eddy currents).



Numerical Results

Comsol

- A parametric analysis can be easily performed by allowing the slip to vary from 0 to 1 (motor operation).
- The electromagnetic torque for each slip is computed using the Maxwell stress tensor.



- Time-harmonic simulations can be used to estimate initial conditions of transient problems: Computational savings.
- Finally, it is important to emphasize that neglecting the overhang effects, such as the end-ring impedances, leads to large errors.

Calculated currents at rated operating conditions: A value of 8.74 A is obtained when the end-ring impedances are considered while 12.67 A is found when they are disregarded.

Conclusions

- Ferromagnetic materials are considered linear (a rather complicated procedure to find an equivalent complex permeability can be used).
- It has been found that interconnection of external systems to the field model is important to avoid large errors.
- Correct identification of solid and filamentary conductors must be carried out to avoid large problems and to properly model eddy current effects.
- The capacity of COMSOL for stitching two meshes of different densities provides a convenient way of forming quality meshes at the air gap.

Thank You!