

## Identification of Noise Sources by Means of Inverse Finite Element Method

COMSOL Conference

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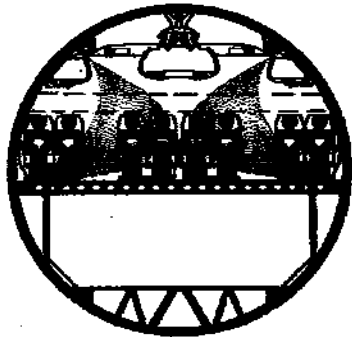
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# Identification of Acoustic Hot Spots in an Aircraft



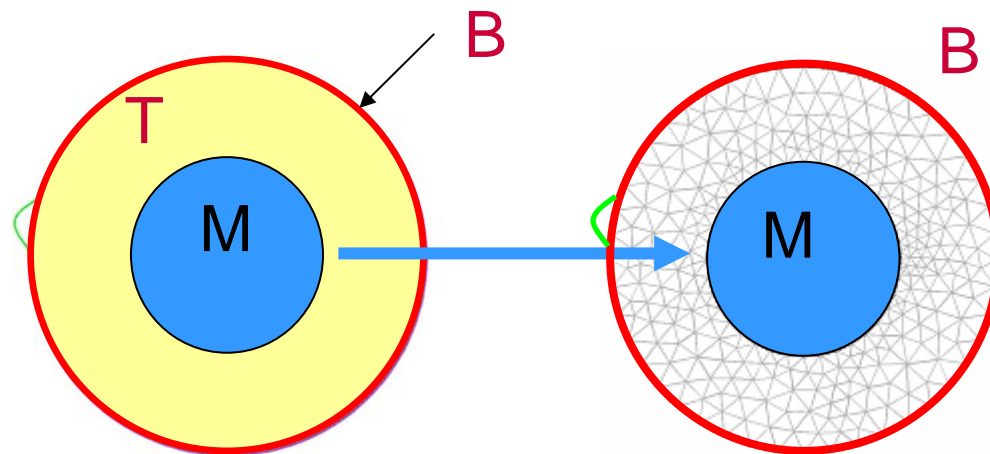
## Problem:

Standing waves and reflections

**IFEM approach:** Inverse Finite Element Method

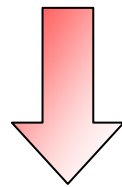
## Overview:

1. Motivation
2. Solution approach
3. 3D simulation
4. Experimental validation
5. Conclusion & outlook



# Governing Equations

Wave Equation  $\Delta p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$   $p = p(x, y, z, t)$



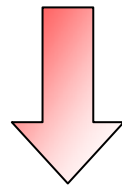
Fourier Transform

$$\hat{p}(\omega) = \int_{-\infty}^{+\infty} p(t) e^{-j\omega t} dt$$

Helmholtz  
Equation

$$\Delta \hat{p} + \frac{\omega^2}{c^2} \hat{p} = 0$$

$$p = p(x, y, z, \omega)$$

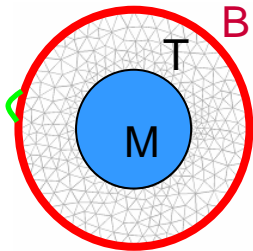


Weak Formulation

Finite Element  
Formulation

$$\mathbf{K} \mathbf{p} = \mathbf{v}$$

# Inverse Finite Element Method (IFEM)



$$\mathbf{Kp} = \mathbf{v}$$

$$\begin{bmatrix} \mathbf{K}_1 & \mathbf{K}_2 & \mathbf{K}_3 \\ \mathbf{K}_4 & \mathbf{K}_5 & \mathbf{K}_6 \\ \mathbf{K}_7 & \mathbf{K}_8 & \mathbf{K}_9 \end{bmatrix} \begin{bmatrix} \mathbf{p}_{MK} \\ \mathbf{p}_{TU} \\ \mathbf{p}_{BU} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{MK} \\ \mathbf{v}_{TK} \\ \mathbf{v}_{BU} \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{K}_2 & \mathbf{K}_3 & \mathbf{0} \\ \mathbf{K}_5 & \mathbf{K}_6 & \mathbf{0} \\ \mathbf{K}_8 & \mathbf{K}_9 & -\mathbf{E} \end{bmatrix} \begin{bmatrix} \mathbf{p}_{TU} \\ \mathbf{p}_{BU} \\ \mathbf{v}_{BU} \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_1 \mathbf{p}_{MK} \\ -\mathbf{K}_4 \mathbf{p}_{MK} \\ -\mathbf{K}_7 \mathbf{p}_{MK} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{K}_2 & \mathbf{K}_3 \\ \mathbf{K}_5 & \mathbf{K}_6 \end{bmatrix} \begin{bmatrix} \mathbf{p}_{TU} \\ \mathbf{p}_{BU} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{MK} - \mathbf{K}_1 \mathbf{p}_{MK} \\ \mathbf{v}_{TK} - \mathbf{K}_4 \mathbf{p}_{MK} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{K}_8 & \mathbf{K}_9 & -\mathbf{E} \end{bmatrix} \begin{bmatrix} \mathbf{p}_{TU} \\ \mathbf{p}_{BU} \\ \mathbf{v}_{BU} \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_7 \mathbf{p}_{MK} \end{bmatrix}$$

ill-conditioned

over-determined for  $n_M > n_B$

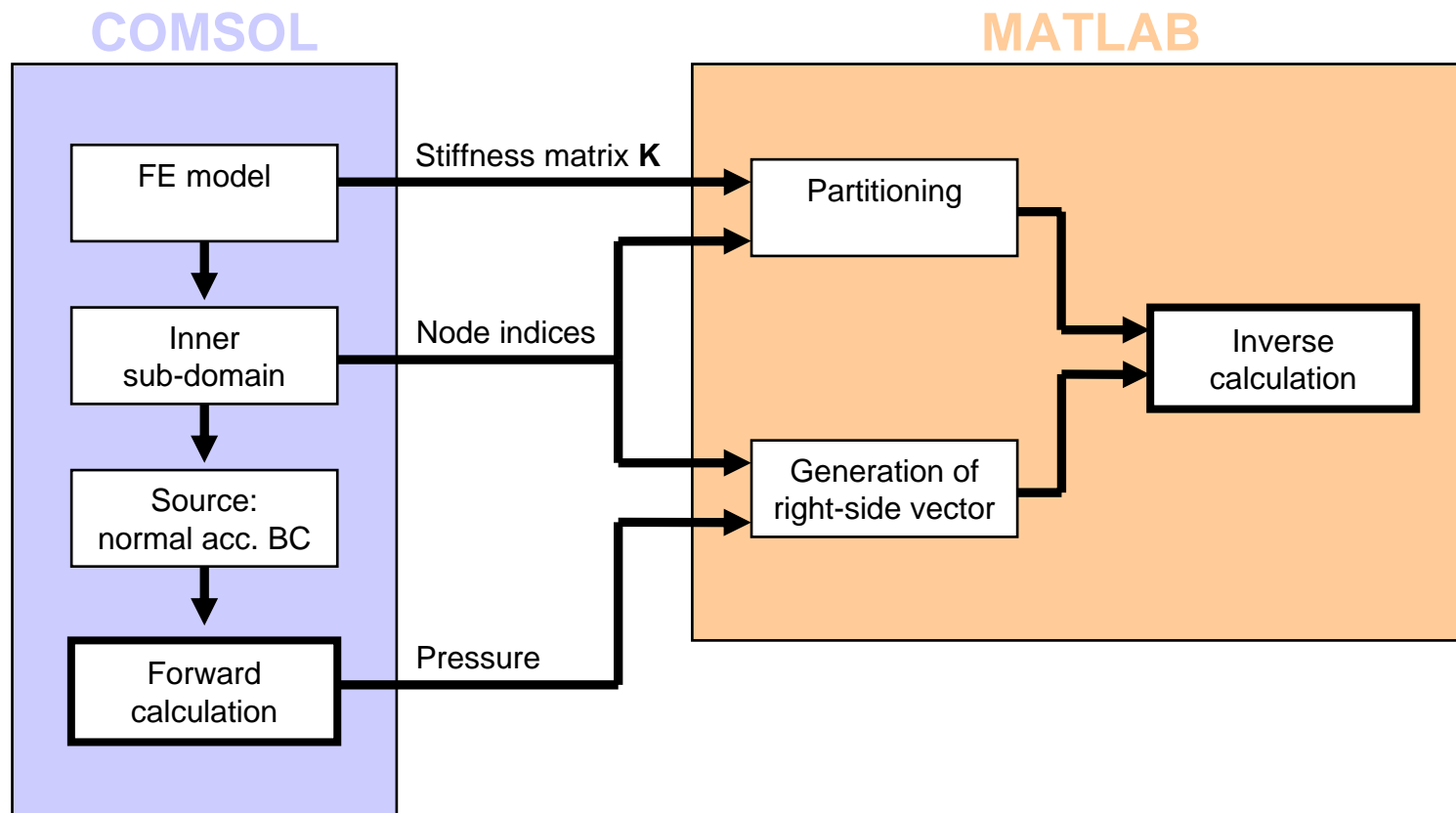
=> Regularization:

- Tikhonov
- Conjugated Gradients (CG)
- Truncated Singular Value Decomposition (TSVD)

Sachau, D.; Drenckhan, J.: Sound source localization in cabins by inverse finite element analysis, DAGA'06, Braunschweig

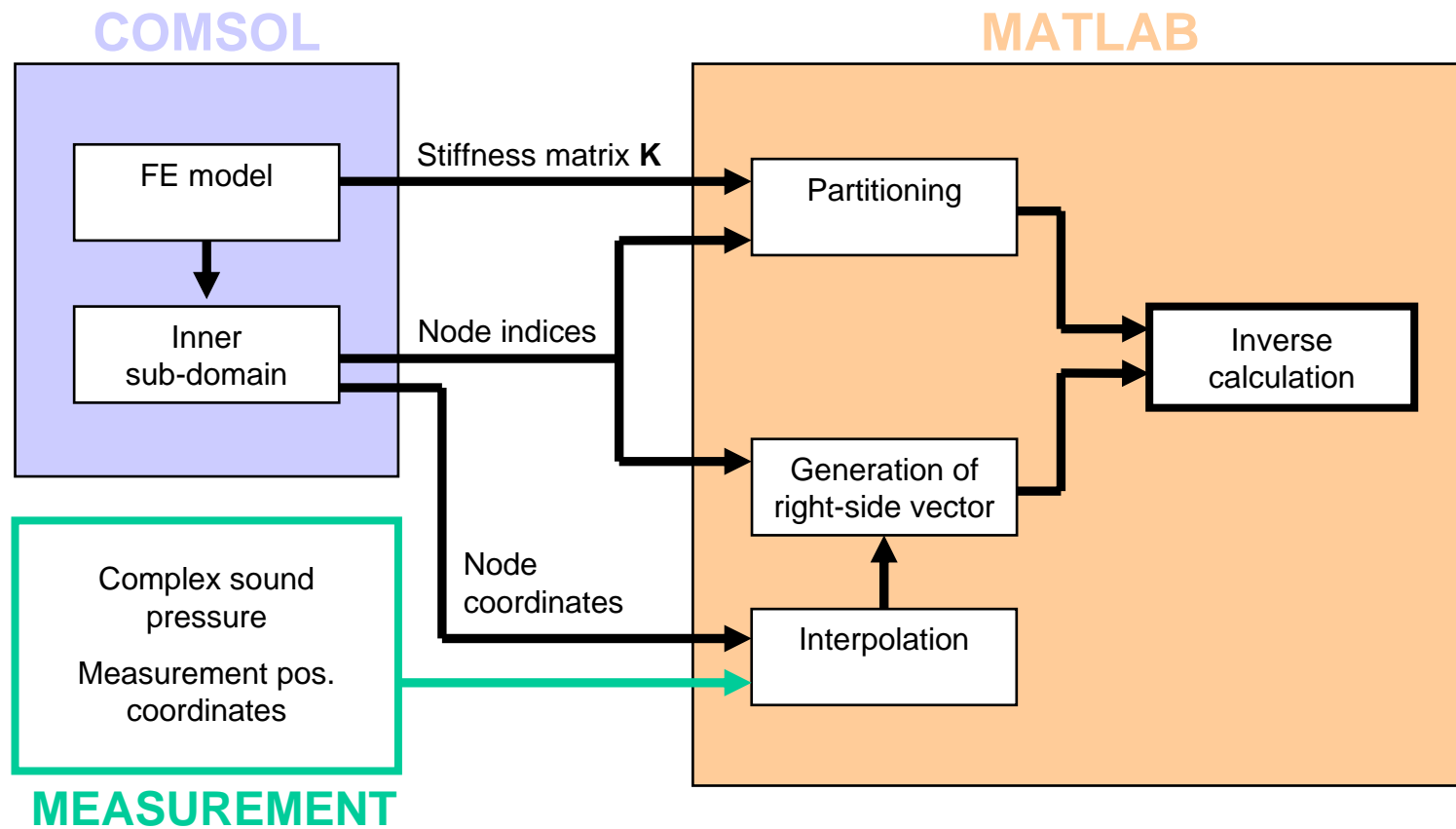
# COMSOL / MATLAB Interaction

## A) Simulated Data:

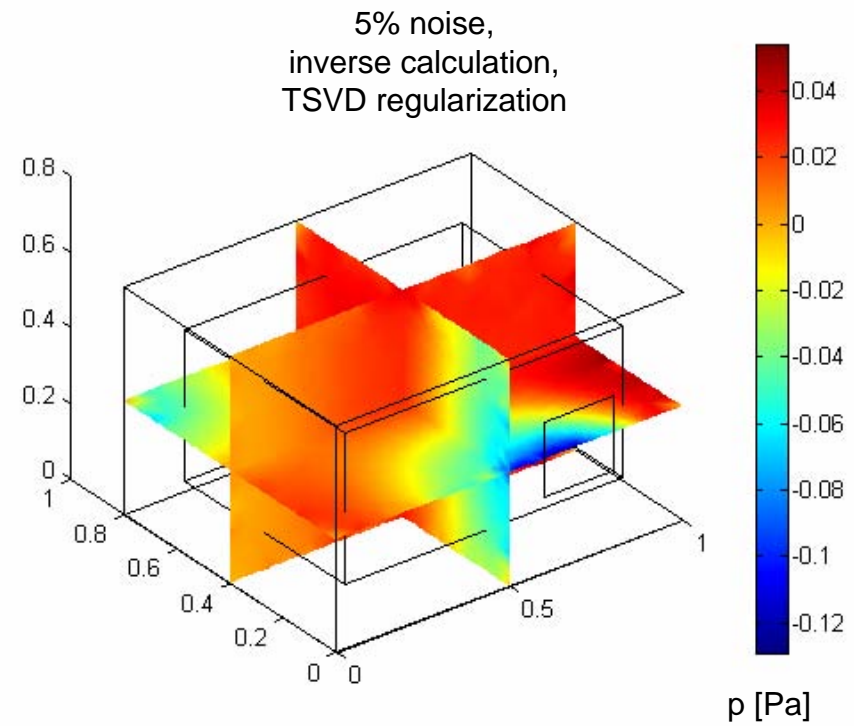
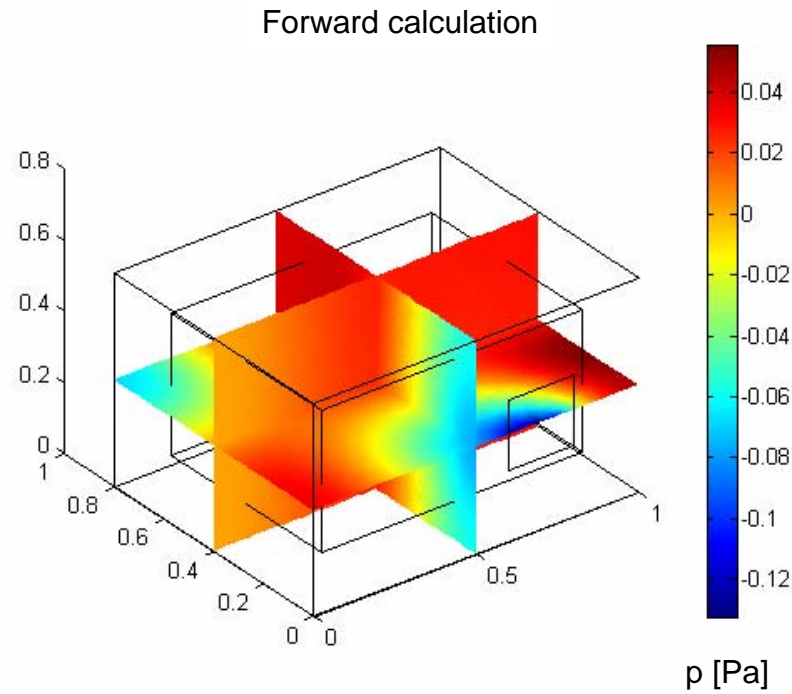


# COMSOL / MATLAB Interaction

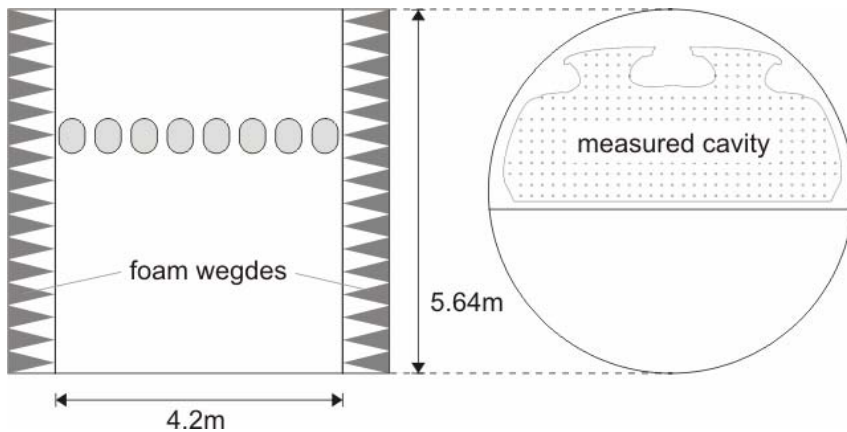
## B) Measured Data:



# 3D Simulation with Artificial Measurement Noise



# Mapping of a Long Range Airliner Cross Section



## Wideband noise excitation

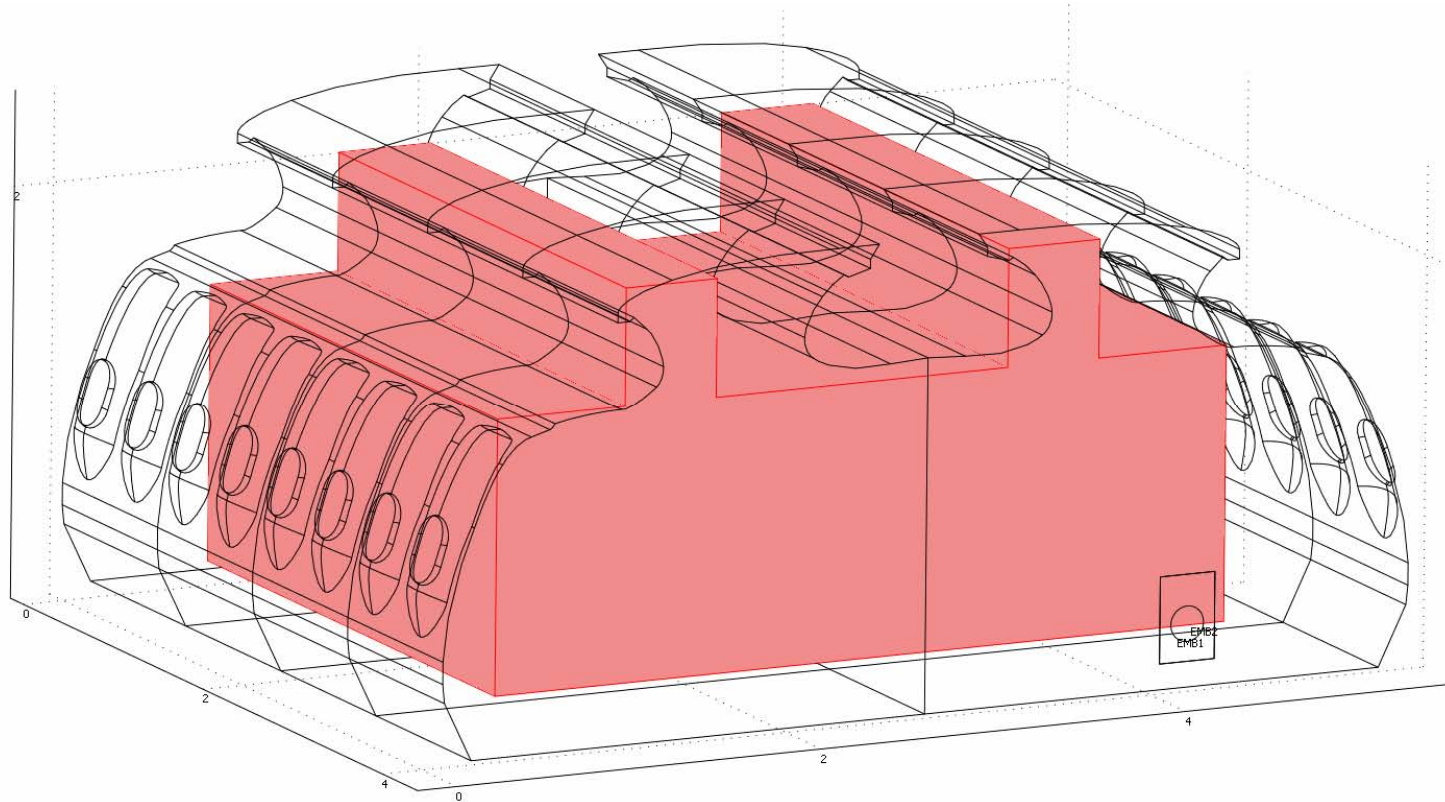
- inner loudspeaker
- outer loudspeakers

Microphone distance: 0.17m

► 7172 positions



# FE Model of the Cavity



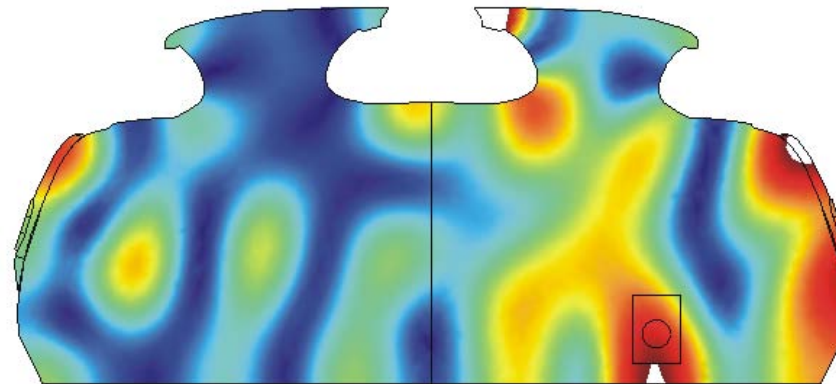
## 1. Coarse Mesh

Total nodes:	27,000
Inner sub-domain:	9,900
Boundary:	8,700

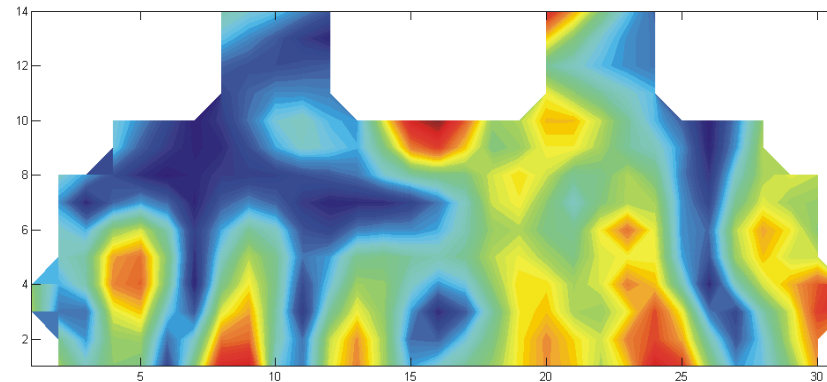
## 2. Fine Mesh

Total nodes:	68,000
Inner sub-domain:	31,000
Boundary:	12,500

# Exemplary Comparison of Model and Mapping ( $f = 293\text{Hz}$ )

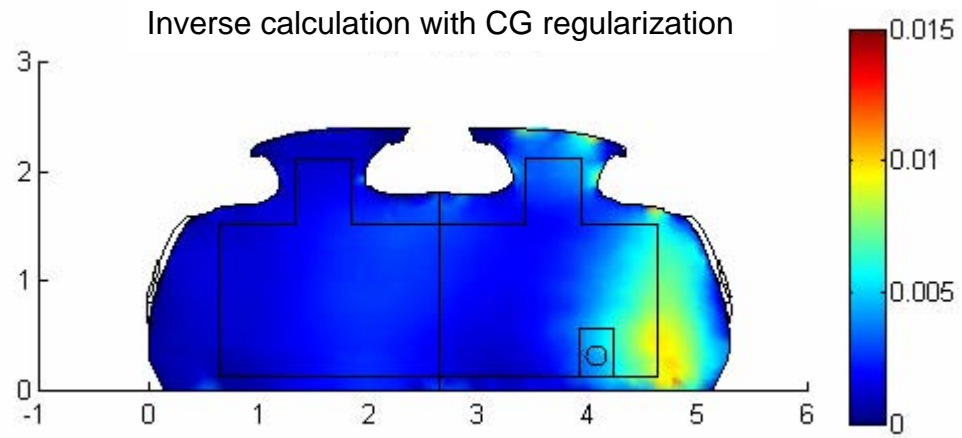
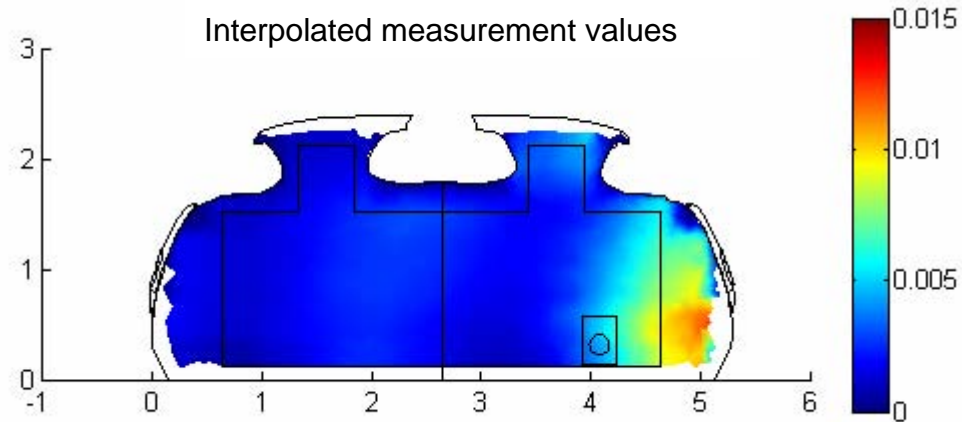


Simulated sound pressure from source with normal acceleration BC



Measured sound pressure from excitation with internal loudspeaker

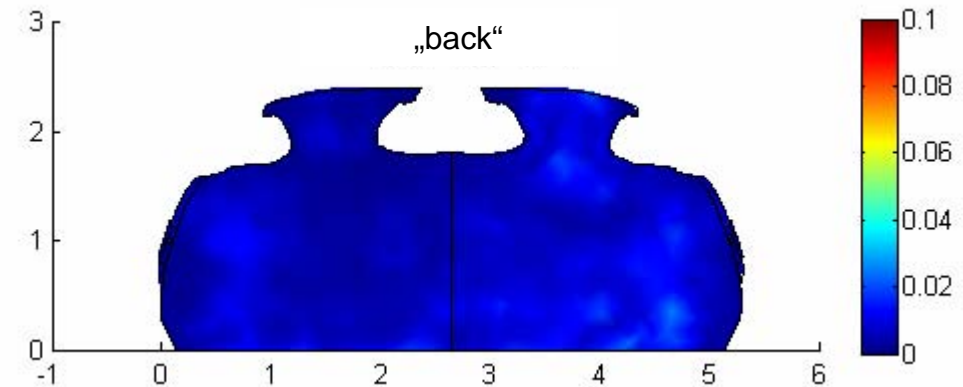
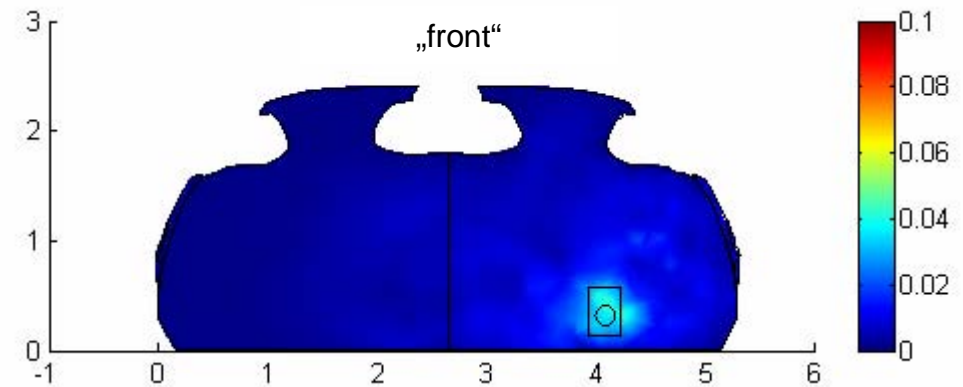
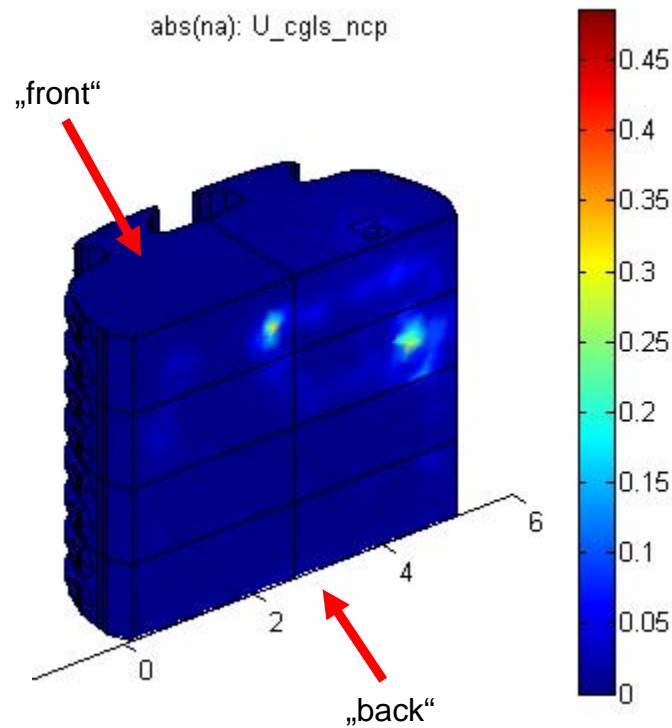
# Inverse Calculation: Sound Pressure (Magnitude) (Inner Loudspeaker, $f = 90\text{Hz}$ )



## Inverse Calculation: Normal Acceleration (Magnitude) (Inner Loudspeaker)

**f = 90Hz, coarse mesh**

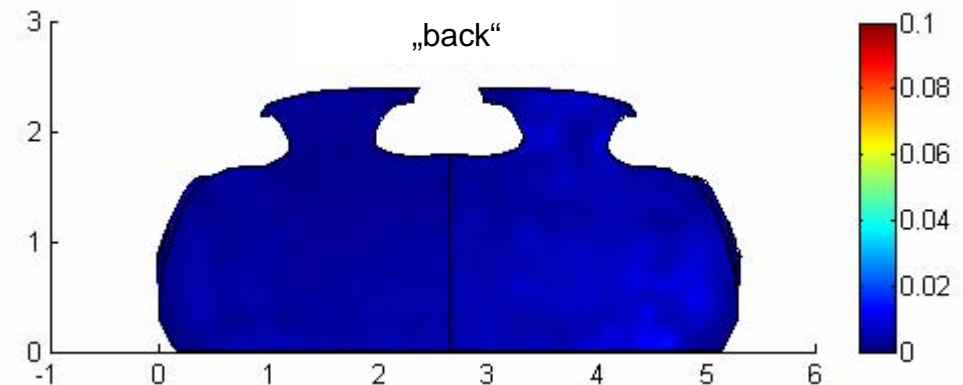
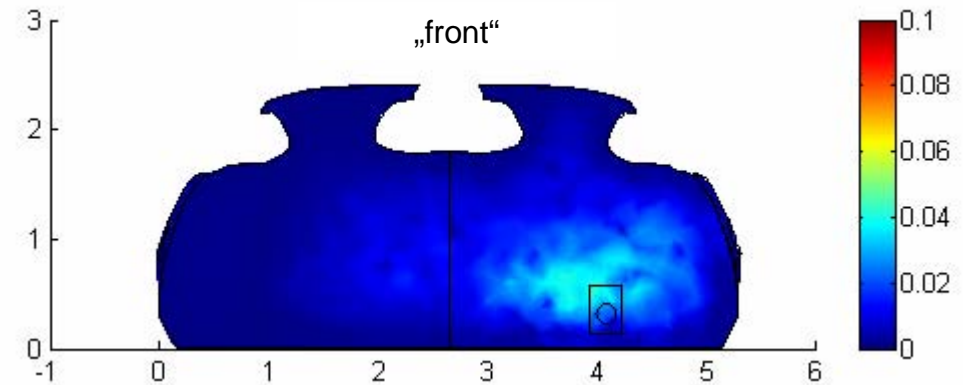
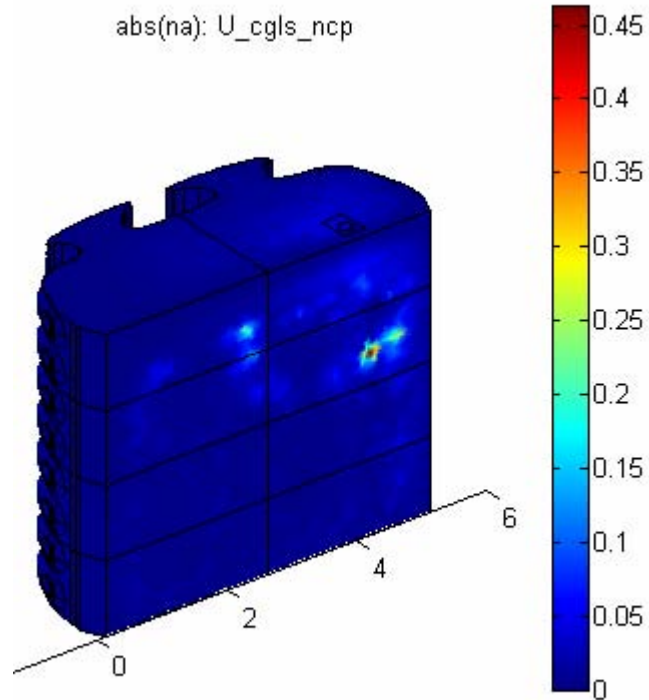
max. element size:  $\lambda/20$



# Inverse Calculation: Normal Acceleration (Magnitude) (Inner Loudspeaker)

**f = 90Hz, fine mesh**

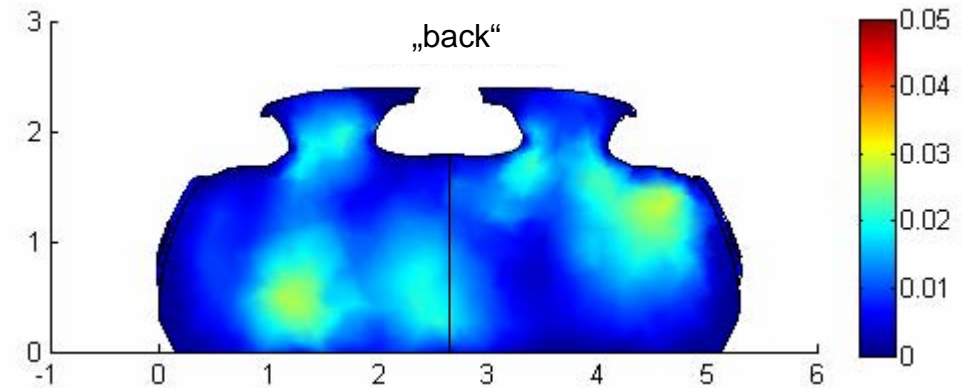
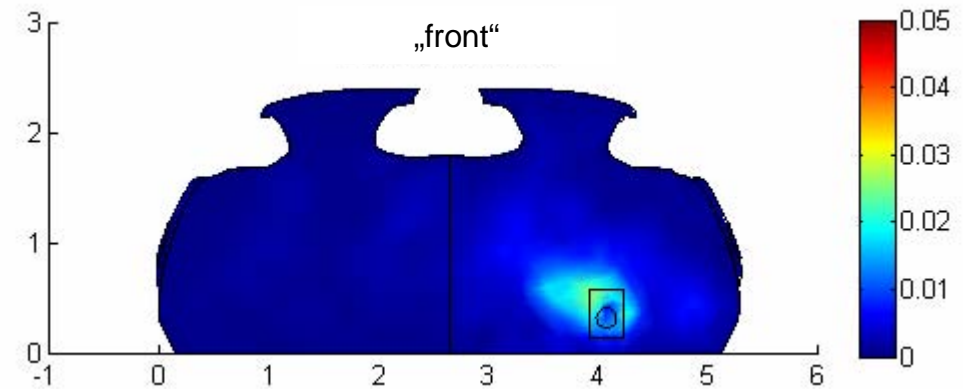
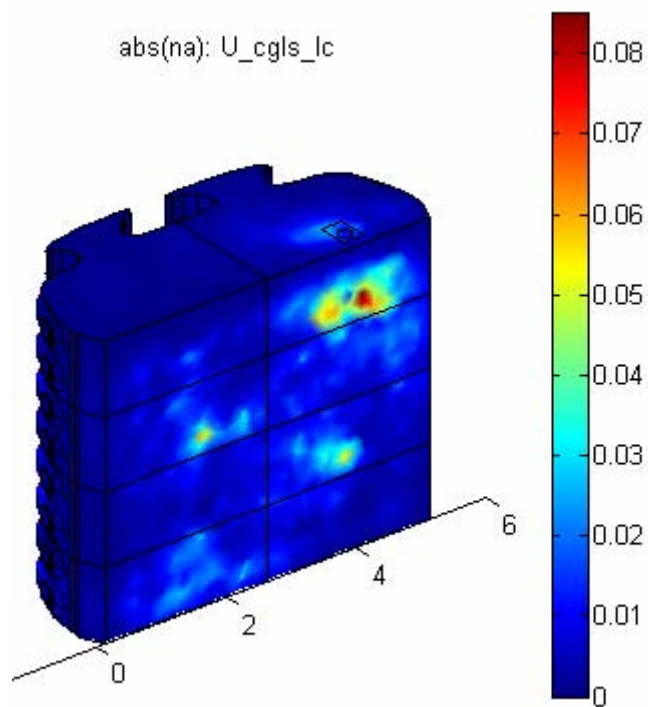
max. element size:  $\lambda/30$



# Inverse Calculation: Normal Acceleration (Magnitude) (Inner Loudspeaker)

**f = 200Hz, coarse mesh**

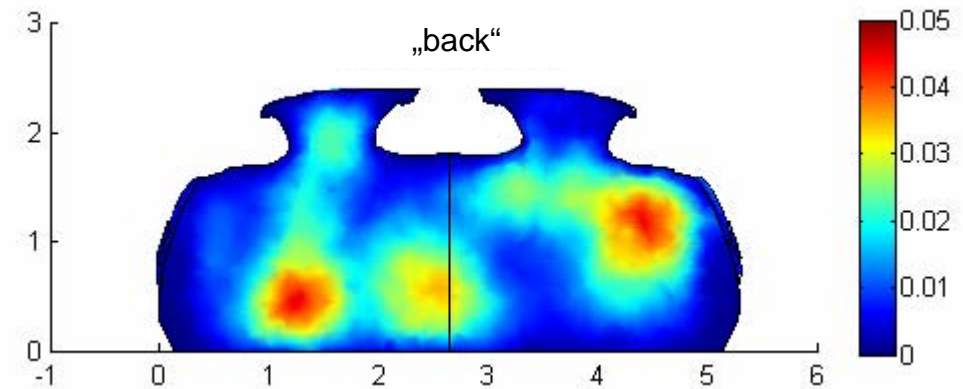
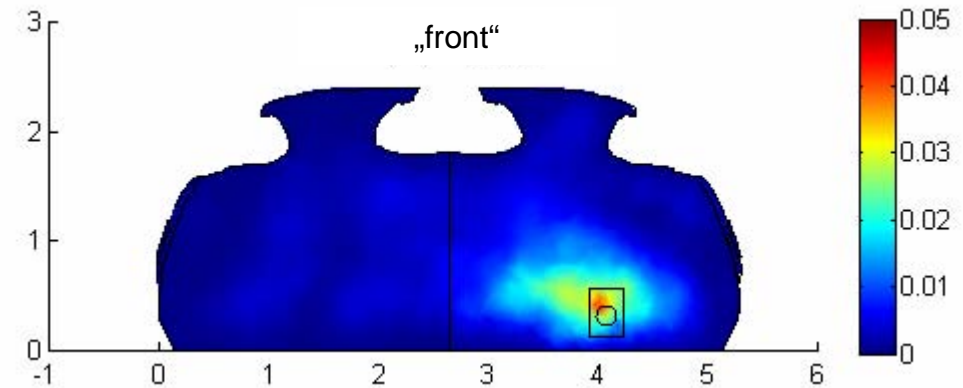
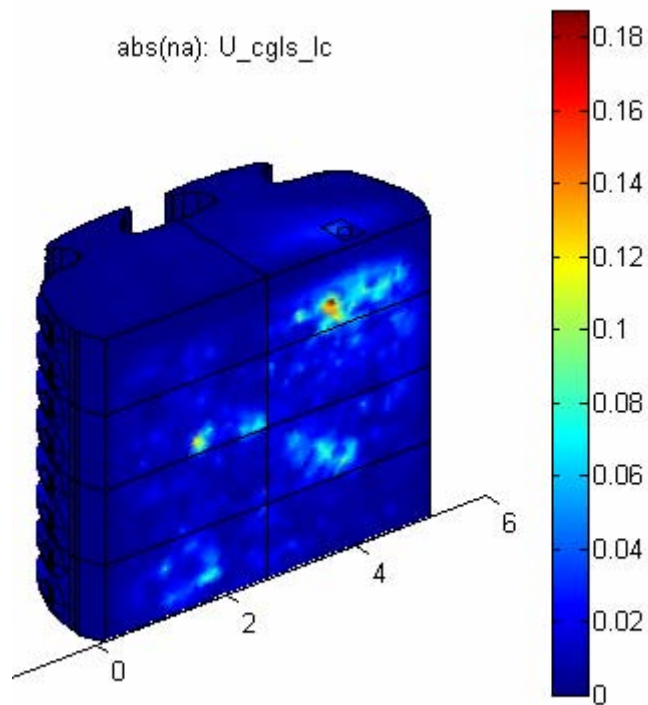
max. element size:  $\lambda/9$



# Inverse Calculation: Normal Acceleration (Magnitude) (Inner Loudspeaker)

**f = 200Hz, fine mesh**

max. element size:  $\lambda/13$



## Conclusion

- The primary source and possible other sources and sinks could be identified.
- The absorbing quality of the foam wedges was confirmed for high frequencies.
- Obviously there is an optimal mesh density dependent on the frequency.

## Next Steps

- determine optimal mesh density
- thin out measurement grid / narrow inner sub-domain  
=> change determinedness of the equation system
- enforce nodes at measurement positions to minimize interpolation error

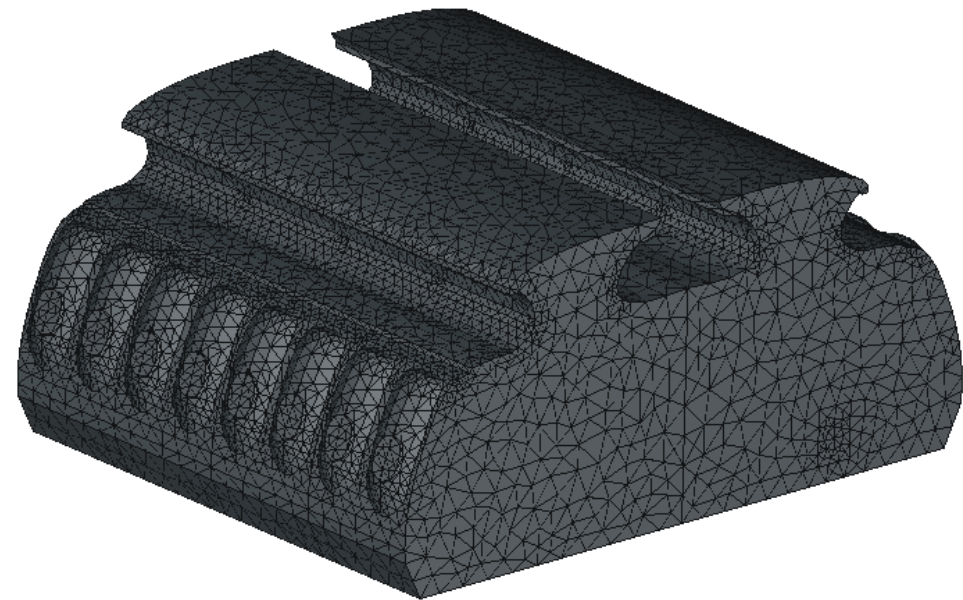
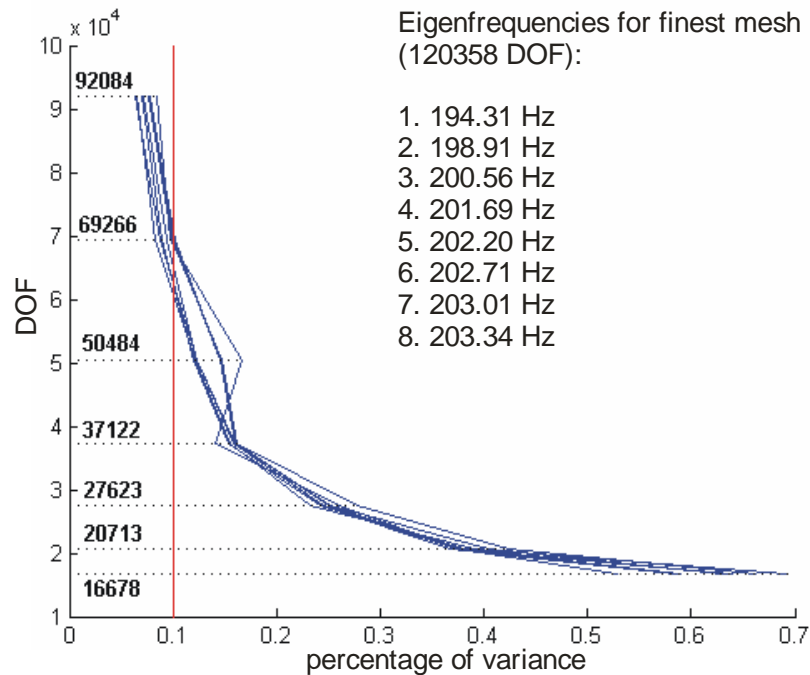


### **Acknowledgement:**

The authors gratefully acknowledge funding by the City of Hamburg in the framework of LuFoHH in cooperation with Airbus.

# Validation of the FE Model

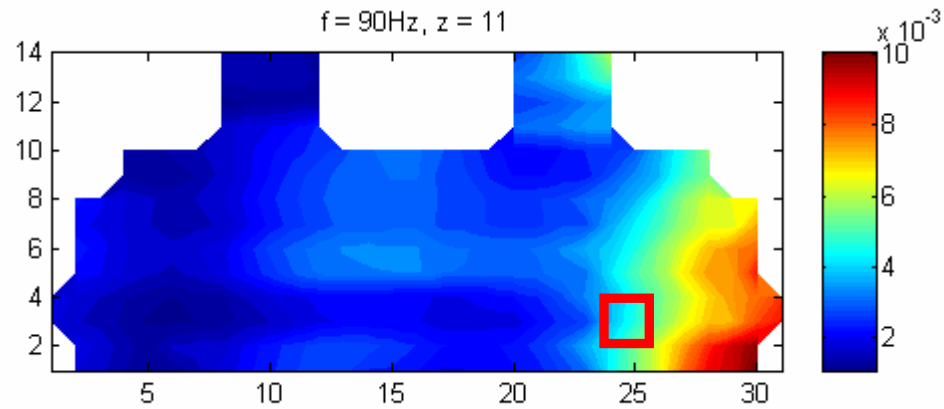
- Model convergence:**
- calculate some eigenfrequencies around 200Hz
  - increase number of DOF and re-calculate
  - repeat until eigenfrequencies vary < 0.1% (identification via MAC)



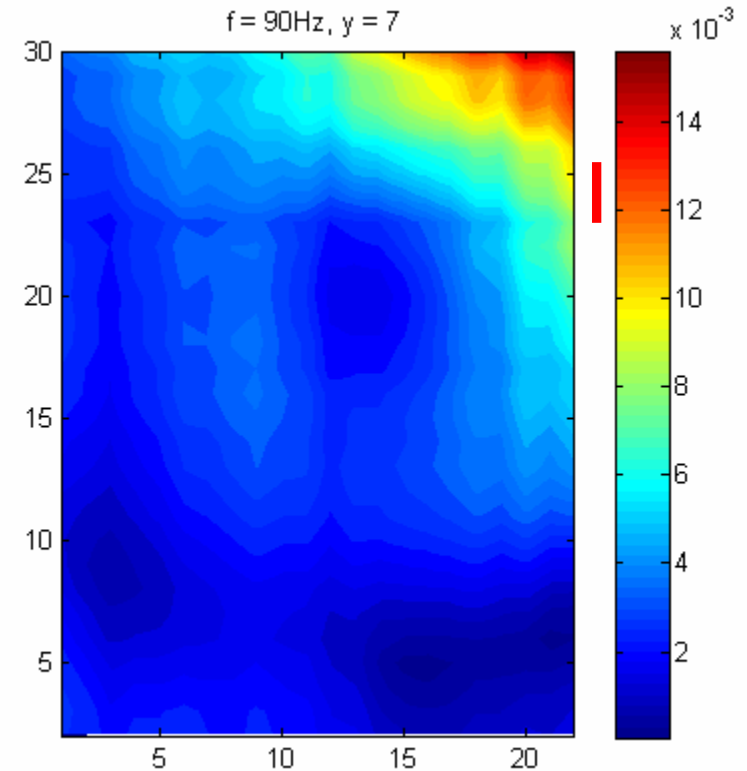
~70 000 DOF equals  $\lambda/9$  for  $f_{\max} = 300\text{Hz}$

# Mapping: Slice Plots, Sound Pressure (Magnitude), $f = 90\text{Hz}$

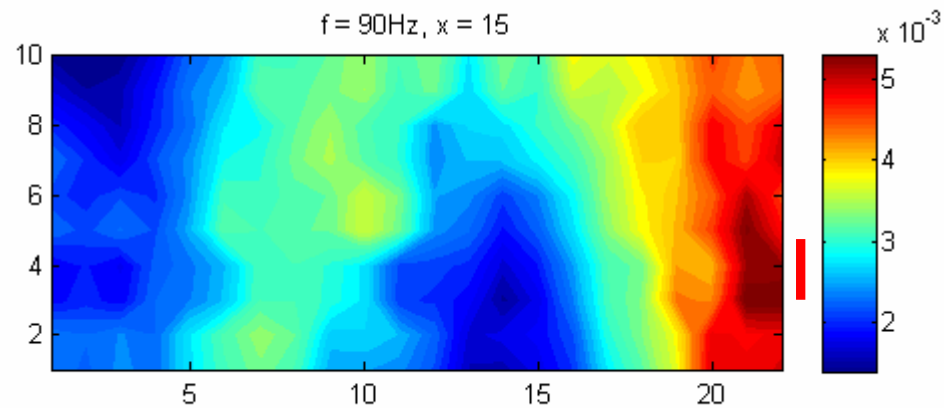
Front view:



Top view:

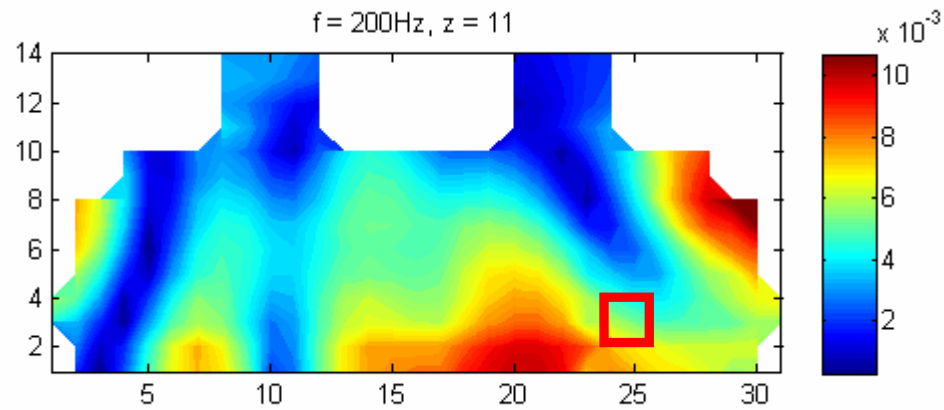


Side view:

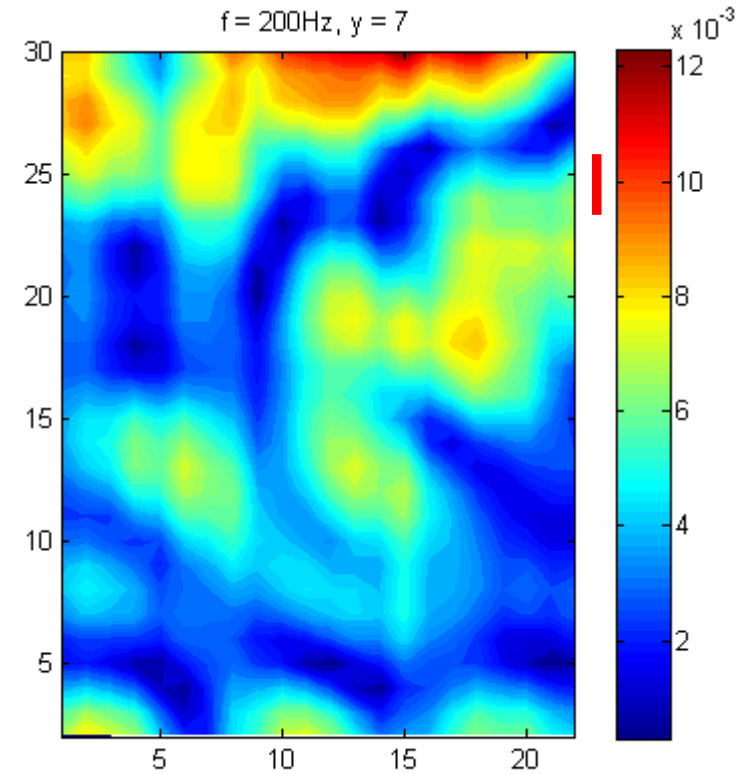


# Mapping: Slice Plots, Sound Pressure (Magnitude), $f = 200\text{Hz}$

Front view:



Top view:



Side view:

