Investigation of Stability of Current Transfer to Thermionic Cathodes

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Introduction

Cathode of an arc discharge in argon. $W, R = 0.75 \text{ mm}, p = 4.5 \text{ bar}, I = 2.5 \text{ A}$. From S. Lichtenberg et al 2002.
Introduction

• The diffuse mode is favorable for operation of cathodes of high-pressure arc devices, however it is difficult to be realized.

• Solutions describing the diffuse mode and different spot modes have been obtained and analyzed in detail.

• This information is not yet sufficient for engineering practice: one needs also to have information on stability of each of these modes in some or other particular conditions.
Equations and boundary conditions

• Non-stationary equation of heat conduction

\[ \rho c_p(T) \frac{\partial T}{\partial t} = \nabla \cdot [\kappa(T) \nabla T] \]

• Boundary conditions

\[ \Gamma_c : T = T_c \]
\[ \Gamma_h : \kappa(T) \frac{\partial T}{\partial n} = q(T,U) \]

• A given value of the arc current

\[ I = \int_{\Gamma_h} j(T,U) dS \]

\( U \): near-cathode voltage

The stationary problem admits multiple solutions describing different modes of current transfer!
Multiple steady-state solutions

- central spot
- central spot and two edge spots
- one edge spot
- two edge spots
- three edge spots
- four edge spots

W, R = 2 mm, h = 10 mm, Ar, 1 bar. ■, ●: bifurcation points.
Formalism of the linear stability theory

Superposition of a steady-state solution and of a perturbation

\[ T(\vec{r}, t) = T_0(\vec{r}) + e^{\lambda t} T_1(\vec{r}) + \ldots \]
\[ U(t) = U_0 + e^{\lambda t} U_1 + \ldots \]
\[ I(t) = I_0 + e^{\lambda t} I_1 + \ldots \]

all \( \lambda \leq 0 \): the state is stable
At least one \( \lambda > 0 \): the state is unstable

Eigenvalue problem for perturbations

\[ \rho c_p(T_0) \lambda T_1 = \nabla \cdot \left( \frac{d\kappa}{dT}(T_0) T_1 \nabla T_0 + \kappa(T_0) \nabla T_1 \right) \]
\[ \Gamma_c : T_1 = 0 \]
\[ \Gamma_h : \frac{d\kappa}{dT}(T_0) T_1 \frac{\partial T_0}{\partial n} + \kappa(T_0) \frac{\partial T_1}{\partial n} = \frac{\partial q}{\partial T}(T_0, U_0) T_1 + \frac{\partial q}{\partial U}(T_0, U_0) U_1 \]
\[ 0 = \int_{\Gamma_h} \left( \frac{\partial j}{\partial T}(T_0, U_0) T_1 + \frac{\partial j}{\partial U}(T_0, U_0) U_1 \right) dS \]
Even and odd perturbations

Steady-state solution is even

Even perturbations

Odd perturbations

\[ y = 0 : \frac{\partial T_0}{\partial y} = 0 \]

\[ y = 0 : \frac{\partial T_1}{\partial y} = 0 \]

\[ y = 0 : T_1 = 0 \]
Stability: COMSOL straight

Heat transfer application mode

Stationary solver for the steady-state solutions

Eigenvalue solver for the perturbations

Problem: Only even perturbations are calculated!
Stability: COMSOL straight

• Axially symmetric steady-state solutions

Even perturbation
\[
\| \quad \text{Odd perturbation}
\]
Same eigenvalue:
Complete spectrum

• 3D steady-state solutions

Even perturbation
\[
\quad \text{Odd perturbation}
\]
Different eigenvalues:
Incomplete spectrum!
Stability: combined approach

A combined approach: to use explicitly the linear stability theory and two modes of COMSOL

Steady-state solution: Heat transfer application mode, Stationary solver

Perturbations: PDE mode, Eigenvalue solver

\[ y = 0 : \frac{\partial T_1}{\partial y} = 0 \quad \rightarrow \quad \text{Even perturbations} \]

\[ y = 0 : T_1 = 0 \quad \rightarrow \quad \text{Odd perturbations} \]
W, R = 2 mm, h = 10 mm, Ar, 1 bar. •: bifurcation points.
Numerical results: examples

Perturbation always with $\lambda > 0$!

W, R = 2 mm, h = 10 mm, Ar, 1 bar. •: bifurcation points. ■: turning point.
Numerical results of stability of 3D spot modes

<table>
<thead>
<tr>
<th>$v$</th>
<th>$T$</th>
<th>Even perturbations</th>
<th>Odd perturbations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2\pi$</td>
<td>$+$ $\rightarrow$ $-$</td>
<td>$0$</td>
</tr>
<tr>
<td>2</td>
<td>$2\pi$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td></td>
<td>$\pi$</td>
<td>$+$ $\rightarrow$ $-$</td>
<td>$0$</td>
</tr>
<tr>
<td>3</td>
<td>$2\pi$</td>
<td>$+$ $+$</td>
<td>$+$ $+$</td>
</tr>
<tr>
<td></td>
<td>$2\pi/3$</td>
<td></td>
<td>$0$</td>
</tr>
<tr>
<td>4</td>
<td>$2\pi$</td>
<td>$+$ $+$</td>
<td>$+$ $+$</td>
</tr>
<tr>
<td></td>
<td>$\pi$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td></td>
<td>$\pi/4$</td>
<td></td>
<td>$0$</td>
</tr>
</tbody>
</table>

$W$, $R = 2$ mm, $h = 10$ mm, Ar, 1 bar.

$v$: number of spots at the edge of the front surface of the cathode. $T$: period.
## Summary of results of stability of 3D spot modes

<table>
<thead>
<tr>
<th>Perturbations</th>
<th>Even</th>
<th>Odd</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Even</strong></td>
<td>Can change sign of their increment along 3D steady-state spot modes.</td>
<td>Do not change sign of their increment along 3D steady-state spot modes.</td>
</tr>
<tr>
<td><strong>Odd</strong></td>
<td>Do not change sign of their increment along 3D steady-state spot modes.</td>
<td>Do not change sign of their increment along 3D steady-state spot modes.</td>
</tr>
</tbody>
</table>

Perturbations of a steady-state mode with \( \nu \) spots at the edge of the front surface of the cathode are periodic with respect to the azimuthal angle with periods between \( 2\pi \) and \( 2\pi/\nu \).

A state with \( \nu \) spots at the edge of the front surface of the cathode is:

- unstable against \( \nu \) modes of even perturbations with period exceeding \( 2\pi/\nu \) in the region between the bifurcation point and the turning point;

- unstable against \( \nu - 1 \) modes of even perturbations with period exceeding \( 2\pi/\nu \) in the region after the turning point or if the mode is supercritical;

- stable against all the others modes of perturbations with such periods.
Application of the numerical results

- Modes with one spot at the center or with multiple spots are always unstable.

- The only modes that can be stable are the diffuse mode and the high-voltage branch of the first 3D spot mode.

- The transition between these two modes is non-stationary and accompanied by hysteresis.

$W, R = 2 \text{ mm}, h = 10 \text{ mm}, \text{Ar, 1 bar.}$
In this experiment, both the diffuse mode and the high/voltage branch of the first 3D spot mode are stable in the whole range investigated (1A - 6A).

• => No reproducible diffuse-spot transition!

W, R = 0.75 mm, h = 20 mm, rounding 100 μm, Ar, 2.6 bar.
Conclusions

• A general pattern of stability of the different modes of current transfer has been established.

• This pattern conforms to trends observed in the experiment:
  - the diffuse-spot transition on arc cathodes is a monotonic process;
  - patterns with more than one spot are not normally observed;
  - the diffuse mode is observed at high currents and the mode with a spot at the edge of the cathode at low currents;
  - the transition between the diffuse mode and the mode with a spot at the edge is non-stationary and is accompanied by hysteresis;
  - this transition is difficult to be reproduced in the experiment.