Effective Properties of Viscoelastic Composite with Piezoelectric Fibers

M. Al-Ajmi[1], P. Muthusamy[2]
1. Associate Professor, Mechanical Engineering Department, Kuwait University, Kuwait
2. Research Associate, Mechanical Engineering Department, Kuwait University, Kuwait

Introduction

Numerical homogenization of fiber reinforced composites has become a valuable design tool that utilizes the power of modern Finite Element Analysis (FEA). Piezoelectric materials are used extensively as sensors and actuators. Piezoelectric composites are more desirable than the homogeneous layers since they relatively overcome the brittleness disadvantage of piezoelectric material and can be tailored for a better performance in specific applications. In this work, a visco-elastic matrix is reinforced with piezoelectric fibers and the overall electrovisco-elastic homogenized properties are computed using a Representative Volume Element (RVE). The elastic, piezoelectric and dielectric constants will be extracted by applying the proper loading and periodic boundary conditions on the RVE. Since the viscoelastic modulus is time (and frequency) dependent, the overall homogenized properties will also be time and frequency dependent. The frequency dependent properties are determined by frequency response studies for a combination of loading and boundary conditions. A parametric analysis will be performed to study the effect of frequency and piezoelectric fiber volume fraction of the homogenized properties. The problem is covered by the MEMS module, which has the piezoelectric effect and solved in the frequency domain. The main advantage of COMSOL® will be the flexibility to input frequency-dependent material properties and the direct calculation of the homogenized complex material coefficients. The results will set a new benchmark for validating new concepts in the field of piezoelectric composites.

Theory

Piezoelectric fiber composites (PFC) were developed to increase the conformability of piezocomposites. The unique coupling property of piezoceramics is widely used in developing the electromechanical actuators and sensors [1]. Though bulk piezoceramic materials are favored for sensing and actuation, due to their advantages like, large actuation force, short response time and insensitive to magnetic fields, they are often limited by their weight and brittle characteristics. To overcome these limitations, piezocomposites have been developed by embedding brittle piezoceramic fibers in a relatively ductile polymer matrix. For linear homogeneous piezoelectric materials, the constitutive equations that relate the electric and elastic fields are given by:

$$
\begin{aligned}
T &= CS - e^T E \\
D &= eS + \varepsilon E
\end{aligned}
\qquad \textbf{(1)}
$$

Where T, S, E and D are the stress, strain, electric field and the electric displacement respectively. C, e and Ɛ are the elastic stiffness tensor, piezoelectric tensor and the dielectric tensor respectively.
This work deals with the viscoelectroelastic behavior of polymer piezoelectric composites. This kind of composites shows frequency dependent properties. The frequency dependent constitutive model of viscoelectroelastic homogeneous material is given by:

$$
\begin{aligned}
G(\omega) &= G'(\omega) + iG''(\omega) \\
G(\omega) &= G'(\omega)(1 + i\eta) \\
\eta &= \frac{G''(\omega)}{G'(\omega)}
\end{aligned}
\qquad \textbf{(2)}
$$

The constitutive equation of the piezocomposites that depends on frequency is given below

$$
\begin{aligned}
\bar{T} &= \hat{C}(\omega)\bar{S} - \hat{e}(\omega)^T \bar{E} \\
\bar{D} &= \hat{e}(\omega)\bar{S} + \hat{\varepsilon}(\omega)\bar{E}
\end{aligned}
\qquad \textbf{(3)}
$$

Numerical Model

The numerical model was developed by COMSOL FE package and compared against existing results from the literature. The elastic properties of the matrix are replaced by viscoelastic complex moduli and the homogenized coefficients are calculated using the volume average method (steps for evaluating the effective coefficients are given below). Composite materials can be envisaged as a periodical array of unit-cells. The elastic matrix of PZT fiber reinforced with polymer matrix is given in equation (1). In order to represent the continuous physical body of the arranged unit-cells, continuity conditions must be satisfied at the boundaries of the adjacent unit cells. The first condition is that the displacements must be continuous, i.e., after deformation the adjacent unit cells cannot be separated or overlapped at the boundaries. The second condition implies that the traction distributions at the opposite parallel boundaries of a unit cell must be the same. To ensure the above said conditions, periodic boundary conditions (PBC) are imposed on the unit-cells. A unit cell with PZT fiber reinforced in elastic matrix is taken for analysis (the geometry is given in Figure.1).

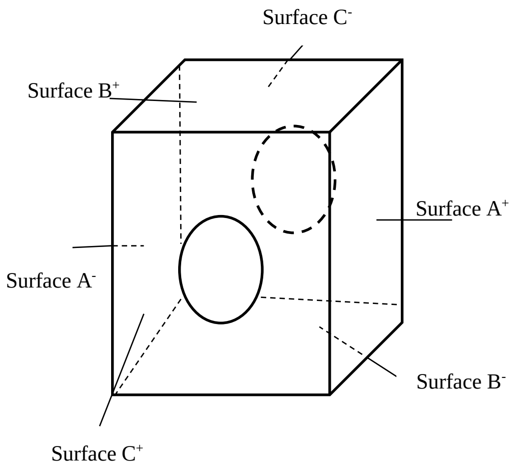

Figure.1 Geometry of representative volume element.

The unit-cell configuration is considered by assuming that the fibers are continuous and uniformly embedded in a matrix along with periodicity in all directions. The macroscopic average fields such as stress, strain, the electric field and electric displacement are calculated based on the volume average method as shown below

$$\overline{S}_{ij} = \frac{1}{V}\int_V S_{ij}dV$$
$$\overline{E}_{ij} = \frac{1}{V}\int_V E_{ij}dV$$
$$\overline{T}_{ij} = \frac{1}{V}\int_V T_{ij}dV$$
$$\overline{D}_{ij} = \frac{1}{V}\int_V D_{ij}dV \quad \textbf{(4)}$$

Using the FEM, the average values can be calculated By using equation 5. The effective electromechanical constants of PZT fiber reinforced in an elastic matrix are evaluated by imposing the periodic boundary conditions on the RVE [2]. Note that A± denote opposite faces normal to the 1 direction at both ends of the RVE, while B± and C± are those normal to the 2 and 3 directions, respectively.

$$\overline{S}_{ij} = \frac{\sum_{e=1}^{N} S_{ij}^{e} V^{e}}{\sum_{e=1}^{N} V^{e}}, \quad \overline{T}_{ij} = \frac{\sum_{e=1}^{N} T_{ij}^{e} V^{e}}{\sum_{e=1}^{N} V^{e}}$$

$$\overline{E}_{ij} = \frac{\sum_{e=1}^{N} E_{ij}^{e} V^{e}}{\sum_{e=1}^{N} V^{e}}, \quad \overline{D}_{ij} = \frac{\sum_{e=1}^{N} D_{ij}^{e} V^{e}}{\sum_{e=1}^{N} V^{e}} \quad \textbf{(5)}$$

For example, to obtain the effective coefficient $\hat{C}_{11}$, the displacement in 2 and 3 directions is constrained in B± and C± surfaces, respectively. A unit normal strain is applied at face A+ while face A- is constrained. Also, zero electric potential difference across the electrodes is maintained. The effective elastic constants $\hat{C}_{11}$, $\hat{C}_{22}$ and $\hat{C}_{33}$ are obtained by applying the boundary conditions in such a way that, except strain in direction 1 (S_1) all other independent fields are equal to zero. The same can be achieved by applying the constraint equations, where a unit macro strain is applied on the opposite surfaces of A± and the displacement in y and z directions are constrained in B± and C± surfaces, respectively. A zero-voltage difference across the electrodes and boundary surfaces are applied to obtain short circuit condition (E = 0). The induced stresses and electric displacements for the applied strain are calculated using the linear constitutive equation. The effective coefficients can be obtained by applying the boundary conditions in the unit cell in such a way that, apart from one component of the strain all other components can be made equal to zero. Some of the equations are given below for simple calculation of effective coefficients with appropriate boundary conditions.To evaluate the effective coupling and electrical coefficients, a potential difference is applied across the electrodes such a way that a unit value of the electric field is generated along the direction (3). The macroscopic strain conditions are applied on the opposite surfaces (A±, B± and C±) such a way that the strains in all directions are approximately zero. For instance, the next equation shows sample calculations of constants

$$C_{11} = \frac{\overline{T}_{11}}{\overline{S}_{11}}, \quad C_{12} = \frac{\overline{T}_{22}}{\overline{S}_{11}}$$
$$\hat{e}_{31} = \frac{\overline{T}_{11}}{\overline{E}_{3}}, \quad \varepsilon_{33} = \frac{\overline{D}_{3}}{\overline{E}_{3}} \quad \textbf{(6)}$$

For practical calculations of the elastic coefficients, a unit strain is applied, which makes the elastic coefficient equal to the induced stress. In COMSOL the volume averaged stresses are directly obtained, which makes the calculation process efficient.

Simulation Results

The PZT-5A (material property shown in Table 1) inclusions embedded in a viscoelastic matrix LaRC-SI (material property taken from literature [3] is shown in Table 2) is modelled using COMSOL.

Property	PZT-5A
C_{11} (GPa)	120.3
C_{12} (GPa)	75.2
C_{13} (GPa)	75.1
C_{33} (GPa)	11.1
C_{44} (GPa)	2.11
e_{31} (C/m^2)	-5.4
e_{33} (C/m^2)	15.8
e_{15} (C/m^2)	12.3
ε_{11}	919.1
ε_{33}	826.6

Table 1. Material properties of PZT-5A.

Property	LaRC-SI
D_0 (GPa^{-1})	0.375
D_1 (GPa^{-1} h^{-1})	0.051606
n	0.4130
γ_o	0.367

Table 2. Material properties of LaRC-SI.

The used viscoelastic material LaRC-SI is modeled using the compliance of the matrix given as follows.

$$M_0(s) = D_0 + \frac{D_1 n!}{s^n} \quad \textbf{(7)}$$

where D_0 is the initial elastic compliance, and D_1 and n are experimentally determined parameters and $s=i\omega$. with ω denoting the frequency. The Young modulus $E(s)$ is taken as the inverse of $M_0(s)$. The resulting new composite will have a viscopiezoelectric behavior. In Figs. 2(a and b) and 3(a and b), the storage and loss part of the effective elastic constants and piezoelectric modulus C_{11} and e_{31} are presented with respect to the different volume fraction and the frequency ω (1/h). The effective coefficients obtained in the frequency domain are complex and frequency dependent. The frequency strongly affects the modulus of the material.

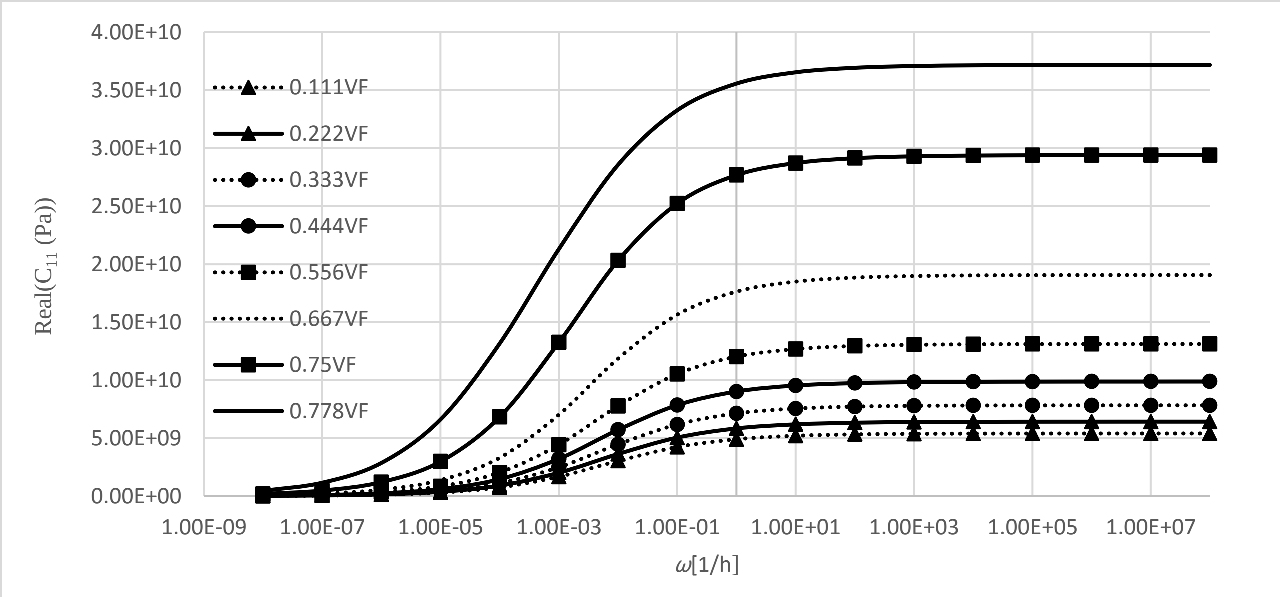

Figure 2(a). Effective storage elastic modulus (C_{11}) for a viscoelectroelastic composite

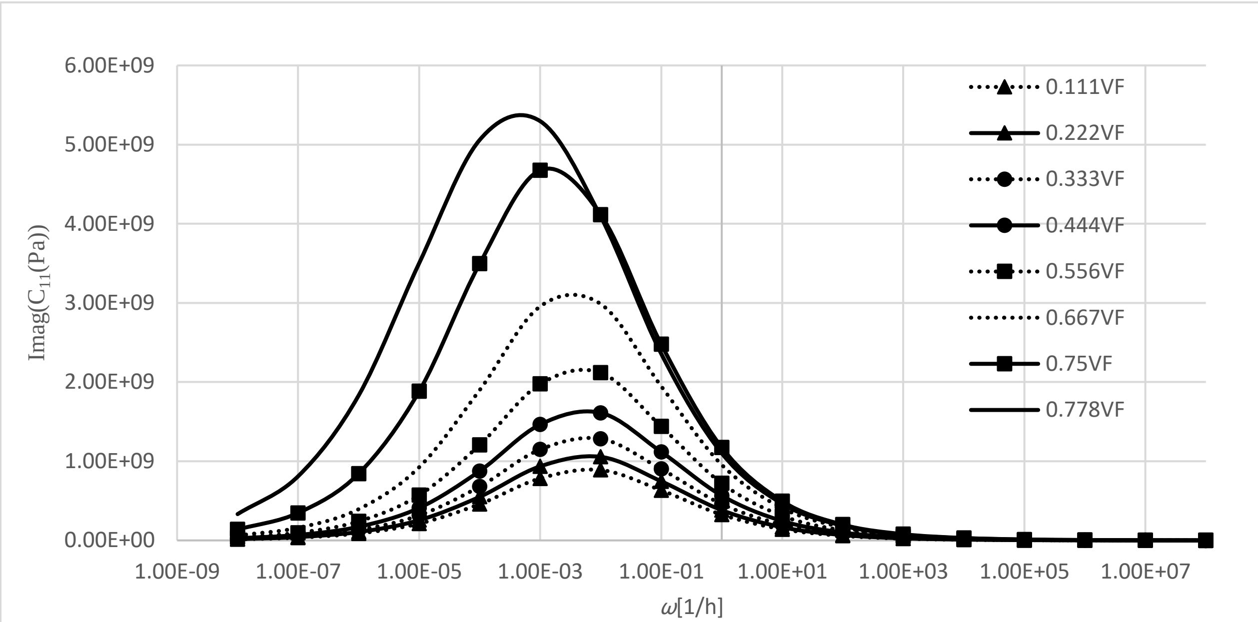

Figure 2(b). Effective loss elastic modulus (C_{11}) for a viscoelectroelastic composite

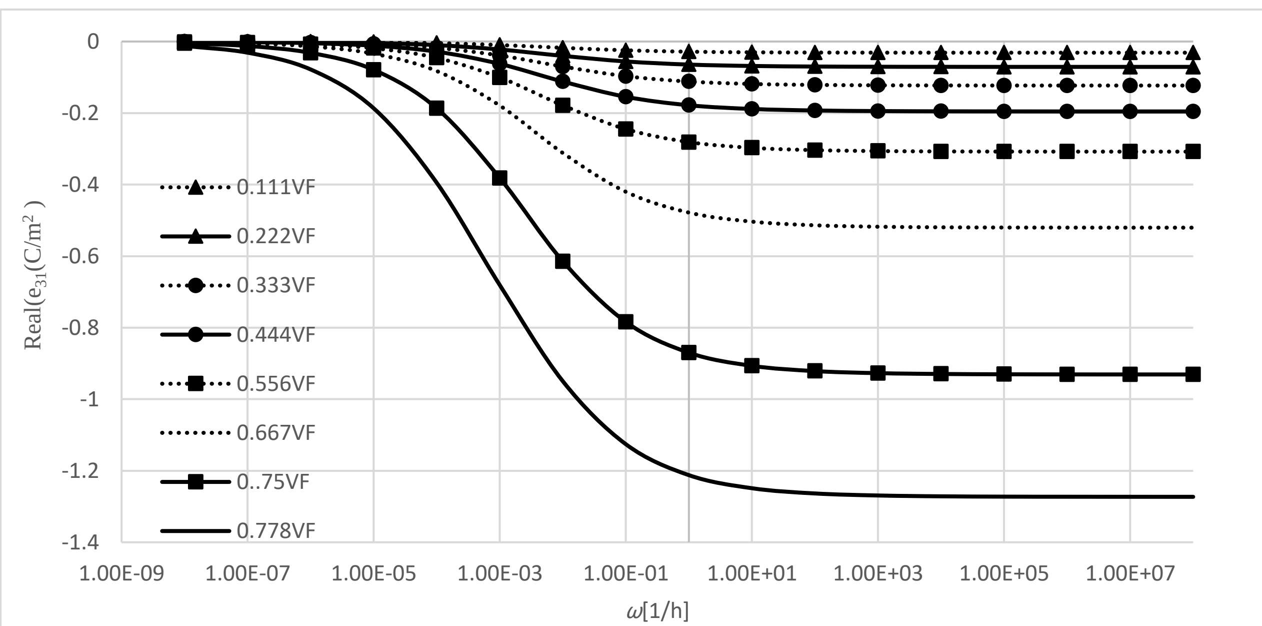

Figure 3(a). Effective storage piezoelectric modulus (e_{31}) for a viscoelectroelastic composite

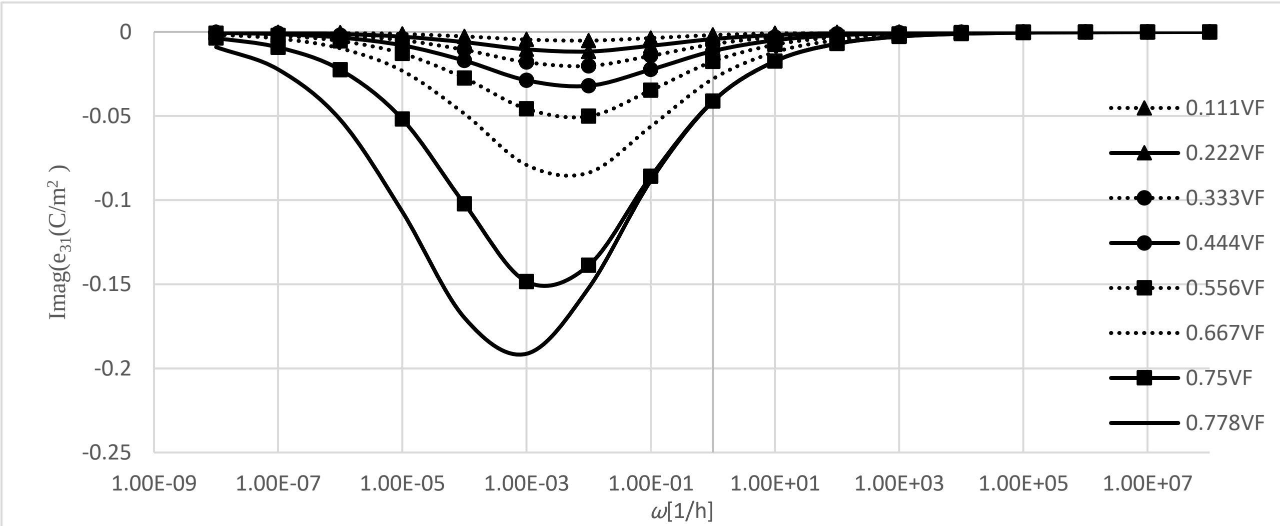

Figure 3(b). Effective loss piezoelectric modulus (e_{31}) for a viscoelectroelastic composite

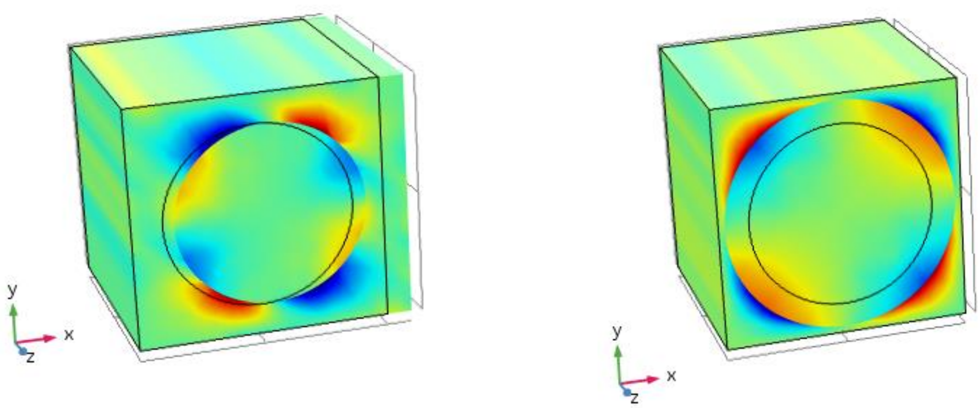

Figure.4(a) and (b). Unit cell deformations to obtain C_{11} and e_{31} coefficients

Conclusions

The frequency dependent material properties under different boundary conditions for different volume fractions are obtained. The results show that the material properties are strongly depend on the applied frequency. The direct calculation of the homogenized complex material coefficients is done using Comsol, which saved more time and made the whole process easier.

References

1. Leo, D. J., *Engineering analysis of smart material systems*. John Wiley & Sons, (2007).
2. Berger, H., Kari, S., Gabbert, U., Rodriguez Ramos, R., Bravo-Castillero, J., Guinovart Diaz, R., Sabina, F., Maugin, G., "*Unit cell models of piezoelectric fiber composites for numerical and analytical calculation of effective properties*", Smart Materials and Structures, 15, pp. 451-458(2006).
3. Azrar L., Bakkali A., Aljinaidi A.A., "*Frequency and time viscoelectroelastic effective properties modeling of heterogeneous and multi coated piezoelectric composite materials*", Composite Structures, 113, pp. 281-297(2014).