Numerical Homogenization of Viscoelastic Composites with Piezoelectric Fibers

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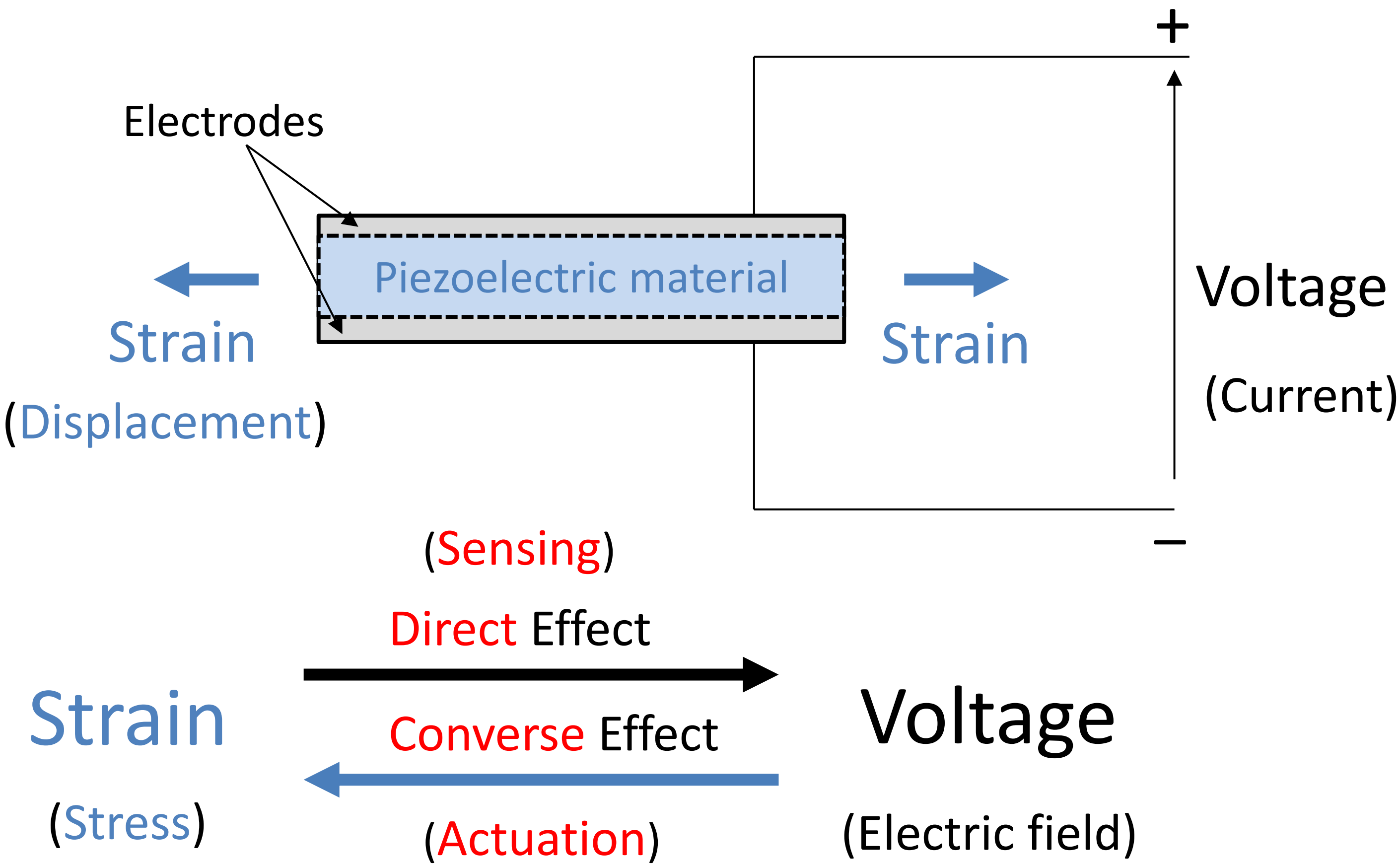
Layout

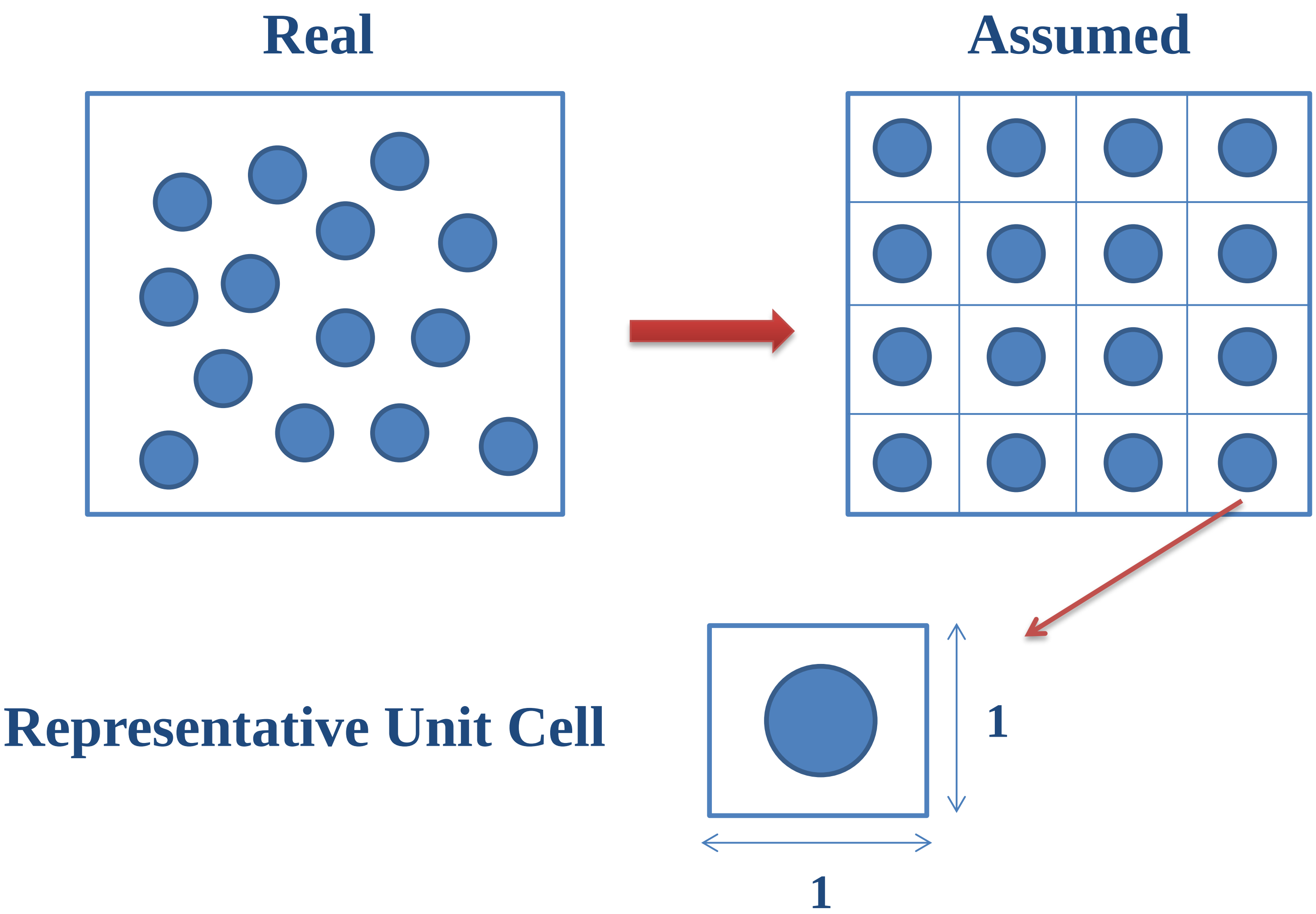
- Introduction
- Theoretical Formulation
- Numerical Analysis
- Conclusion

Introduction

Piezoelectric effects:

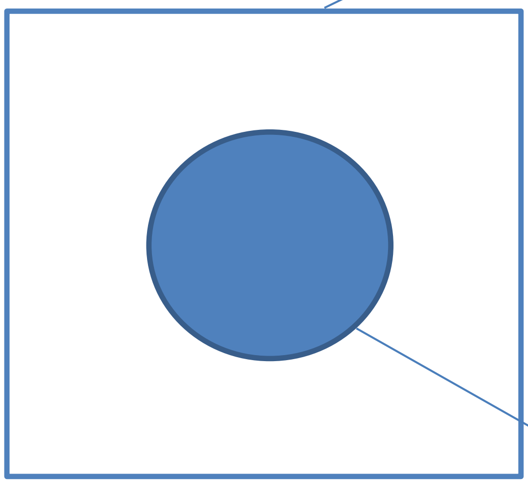


Introduction



Introduction

Piezo-Visco Composite



Matrix

-Square

-Linear Visco-elastic

Fiber

-Circular

-Linear Piezo-electric

-Poled in Fiber direction

Theoretical Formulation

Linear Piezoelectric Constitutive Equations:

e-form (used in FEA):

$$\mathbf{T} = \mathbf{C}^{\mathbf{E}}\mathbf{S} - \mathbf{e}^{\mathbf{t}}\mathbf{E}$$

$$\mathbf{D} = \mathbf{e}\mathbf{S} + \in^{\mathbf{S}} \mathbf{E}$$

Theoretical Formulation

Linear Piezoelectric Constitutive Equations:

$$T = C^{E}S - e^{t}E$$

$$D = eS + \in^{S} E$$

Stress — T

Elastic Coefficients At constant Electric Field — C^{E}

Strain — S

Electric Field — E

Electrical Displacement — D

Stress Piezoelectric Coupling Coefficients — e

Permittivity At constant strain — $\in^{S}$

Theoretical Formulation

Linear Isotropic Viscoelastic Material:

$$E(w) = E'(w) + iE''(w)$$

$$h = \frac{E''(w)}{E'(w)} \longrightarrow E(w) = E'(w)(1 + ih)$$

Theoretical Formulation

Homogenization:

$$\bar{T}_{ij} = \frac{1}{V}\int_V T_{ij}\, dV \qquad \bar{D}_{ij} = \frac{1}{V}\int_V D_{ij}\, dV$$

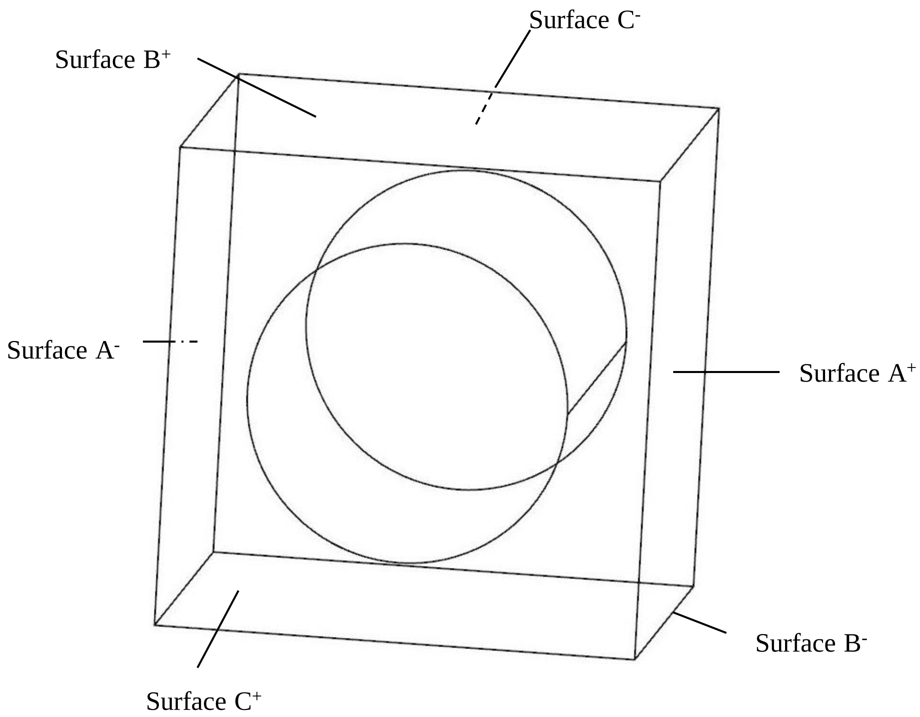
$$\bar{S}_{ij} = \frac{1}{V}\int_V S_{ij}\, dV \qquad \bar{E}_{ij} = \frac{1}{V}\int_V E_{ij}\, dV$$

$$\bar{T} = \hat{C}(w)\bar{S} - \hat{e}(w)^T \bar{E}$$

$$\bar{D} = \hat{e}(w)\bar{S} + \hat{e}(w)\bar{E}$$

Numerical Analysis

Geometry of representative volume element:



Numerical Analysis

- **Viscoelastic material is modeled using:**

$$M_0(s) = D_0 + \frac{D_1 n!}{s^n}$$

D_0 is the initial elastic compliance,
D_1 and n are experimentally determined parameters, and
s=iω. with ω denoting the frequency.

The Young modulus $E(s)$ is taken as the inverse of $M_0(s)$

- **Piezoelectric Material used: PZT-7A**

Numerical Analysis

Frequency-dependent viscoelastic material implementation in COMSOL

Settings | Properties

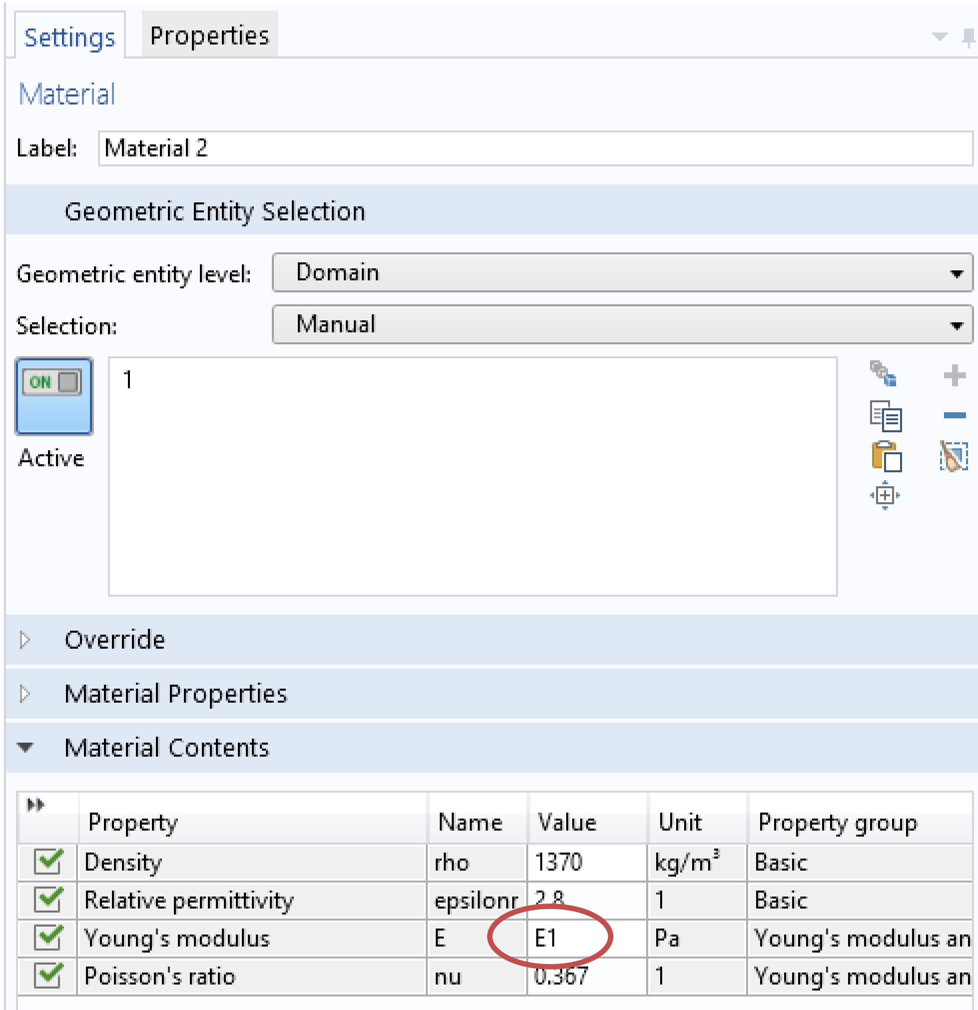
Parameters

▼ Parameters

Name	Expression	Value	Description
D0	.375*1e-9	3.75E-10	
D1	.051606*1e-9	5.1606E-11	
n	0.4103	0.4103	
nf	.88675	0.88675	
N	D1*nf	4.5762E-11	
w	1E8	1E8	
D	(w*i)^n	1531.7+1151.1i	
M	N/D	1.9093E-14-1....	
M0	D0+M	3.7502E-10-1....	
E1	(1/M0)[Pa]	(2.6665E9+1.0...	
freq	w/3600[1/s]	27778 s	

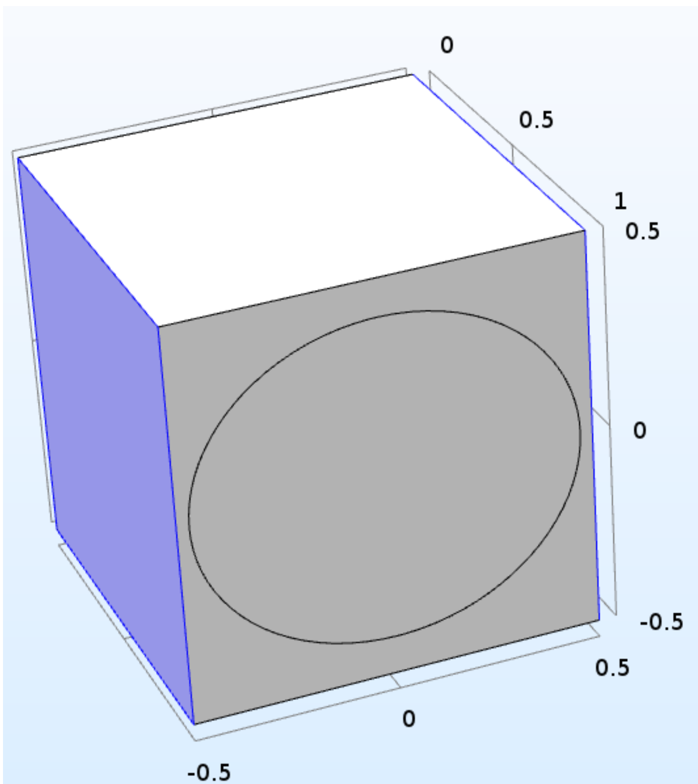
Youngs modulus in terms of frequency →

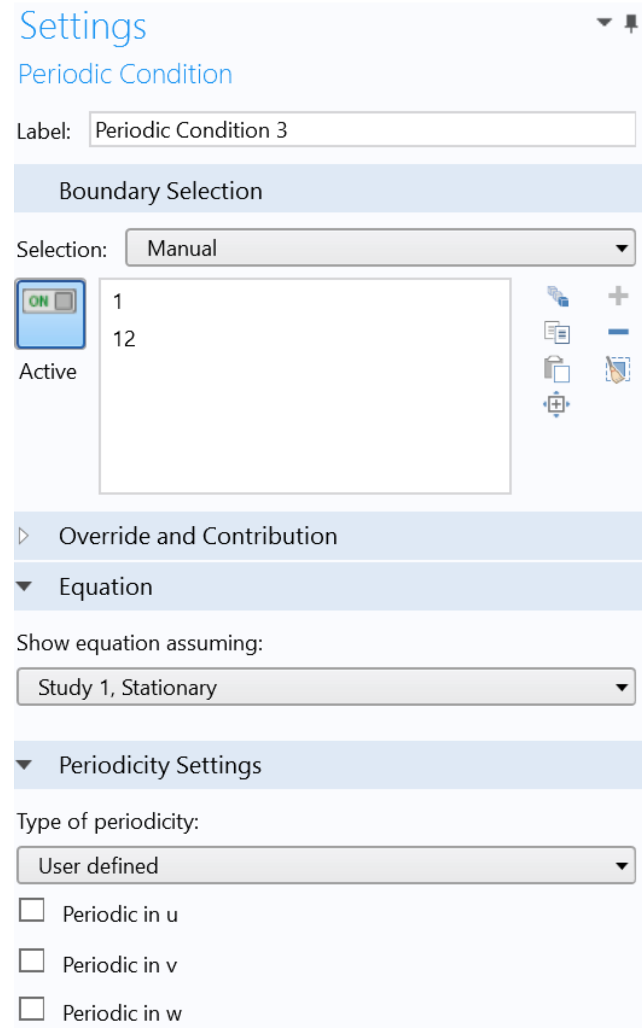
Numerical Analysis



Numerical Analysis

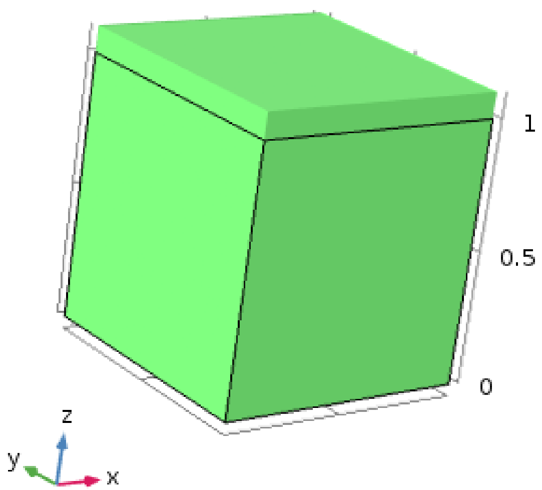
Periodicity Condition





Numerical Analysis

Deformed Viscoelastic material under normal and shear load



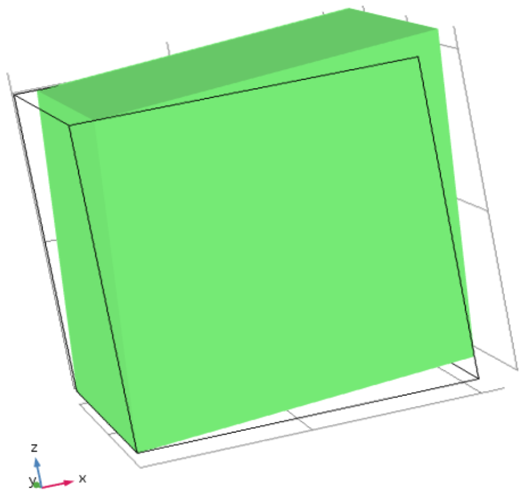
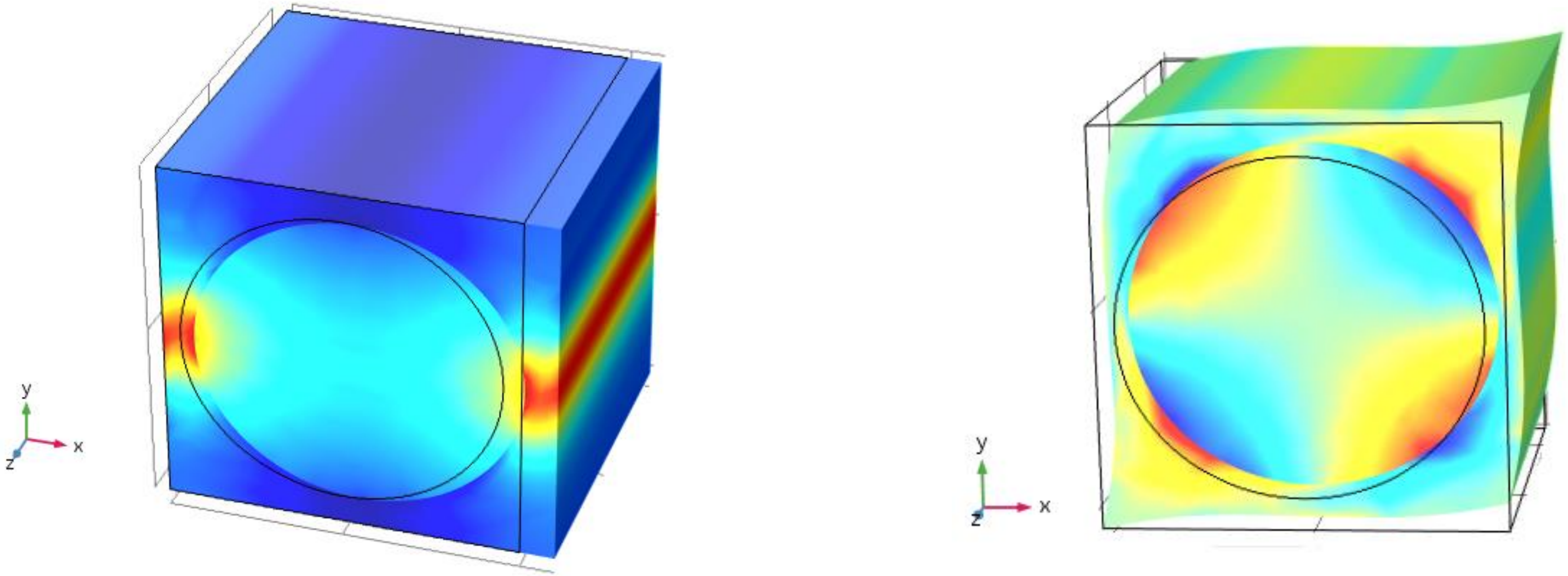


Figure 3: Deformed unit cell of viscoelastic material

Numerical Analysis

Sample Mechanical Calculation:

-Apply normal and shear on the cell cross section (all potentials =0)



$$C_{11} = \frac{\overline{T}_{11}}{\overline{S}_{11}}$$

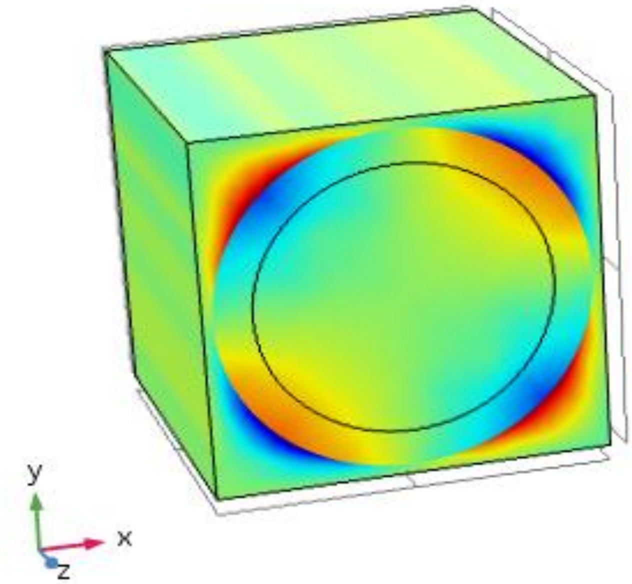
$$C_{66} = -\frac{\overline{T}_{11}}{\overline{S}_{12}}$$

Figure 4: Deformed unit cell of viscoelastic matrix reinforced with PZT fiber

Numerical Analysis

Sample Piezoelectric Calculation

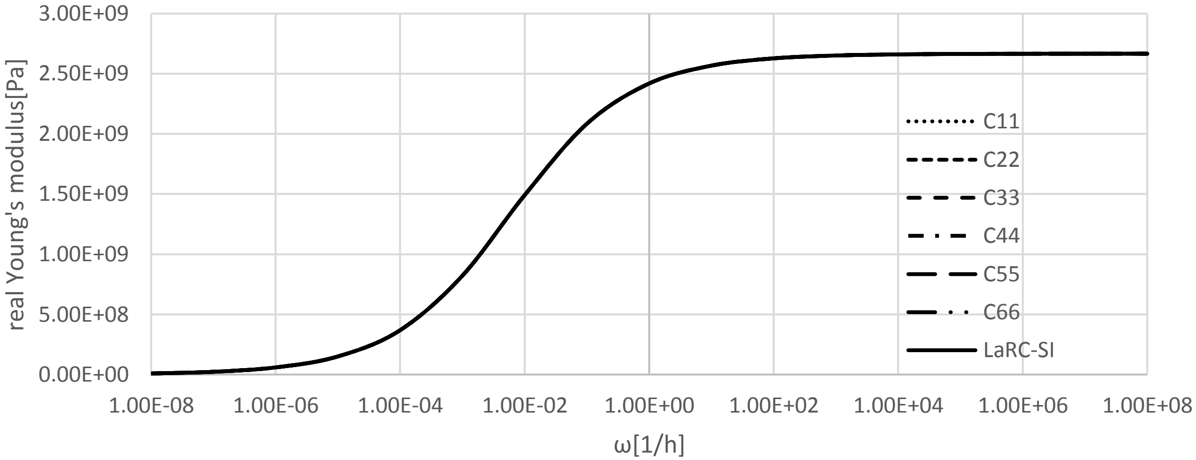
-Apply potential difference across fiber (all displacements =0)



$$\hat{e}_{31} = \frac{\overline{T}_{11}}{\overline{E}_3}$$

Figure 4: Deformed unit cell of viscoelastic matrix reinforced with PZT fiber

Results



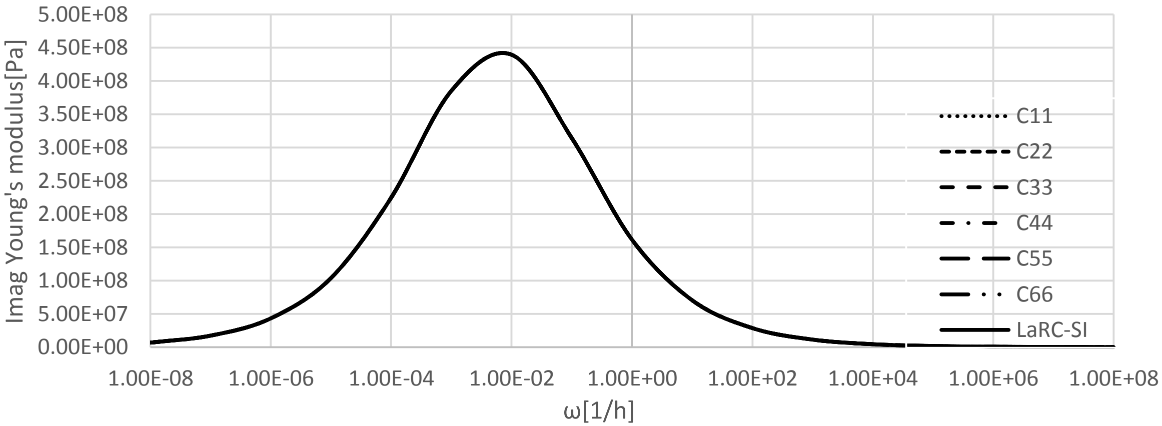
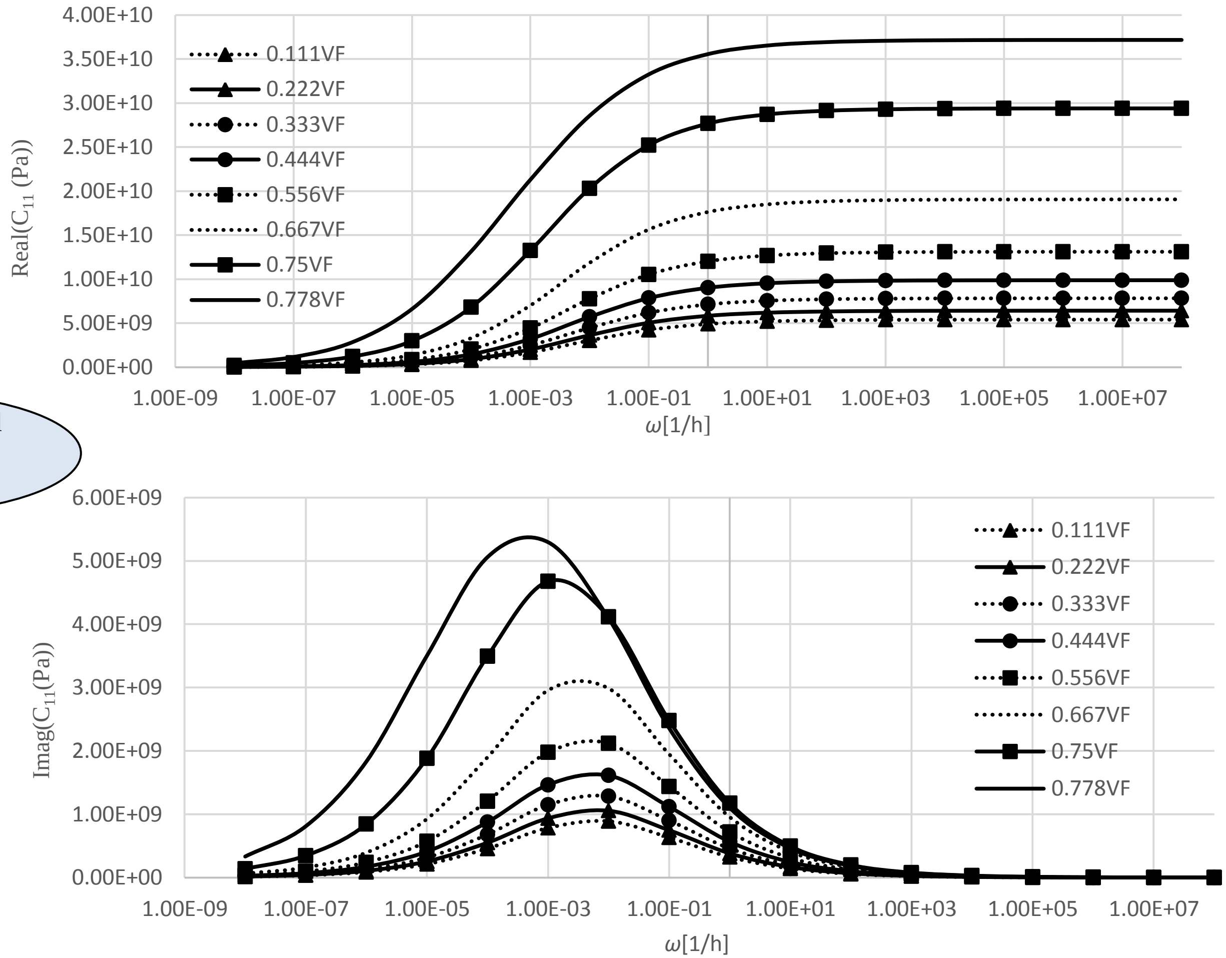


Figure 5. Real and Loss Modulus LaRC-SI with respect to frequency for different boundary condition.

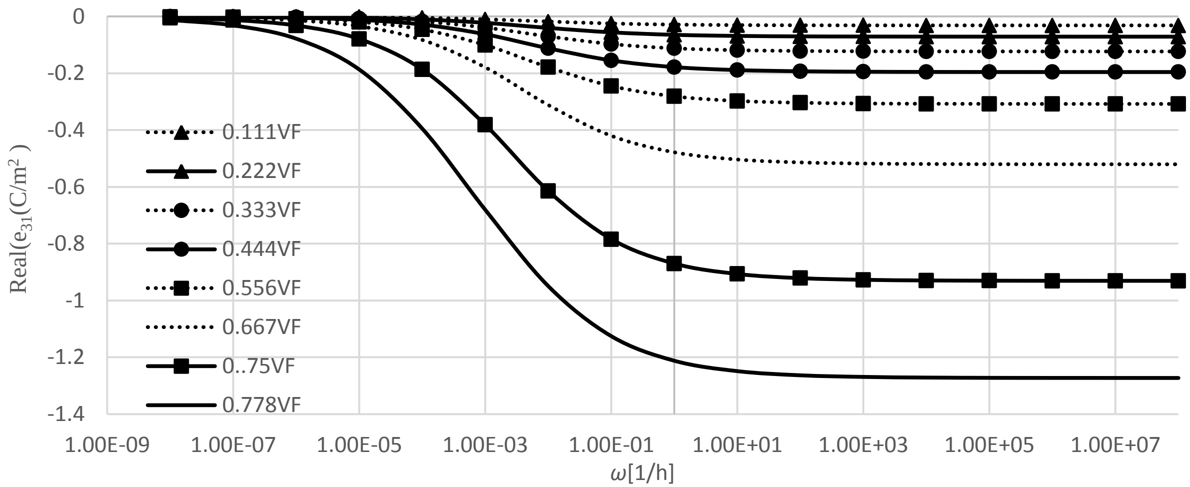
Results



PZT fiber reinforced in viscoelastic matrix.

Figure 6 . Effective storage and loss elastic modulus (C_{11}) for a viscoelectroelastic composite for different volume fraction.

Results



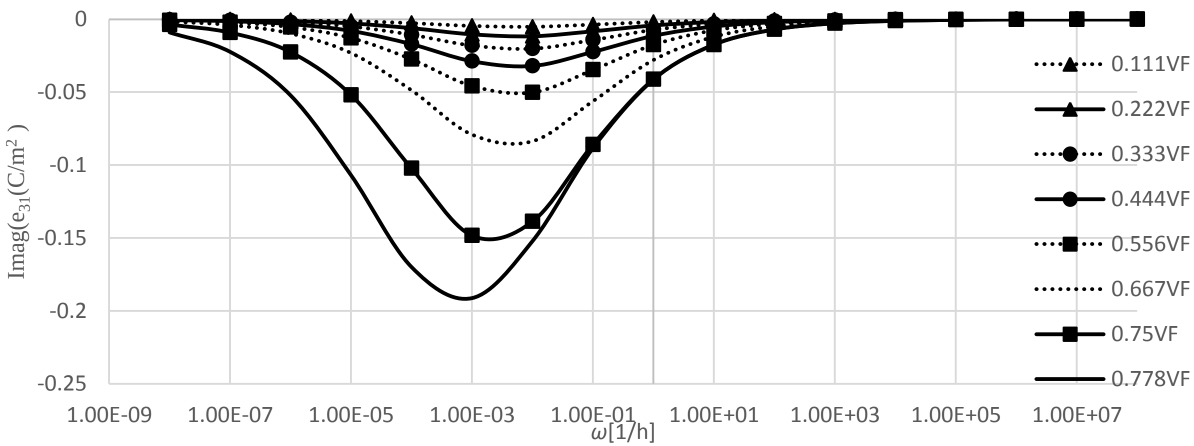
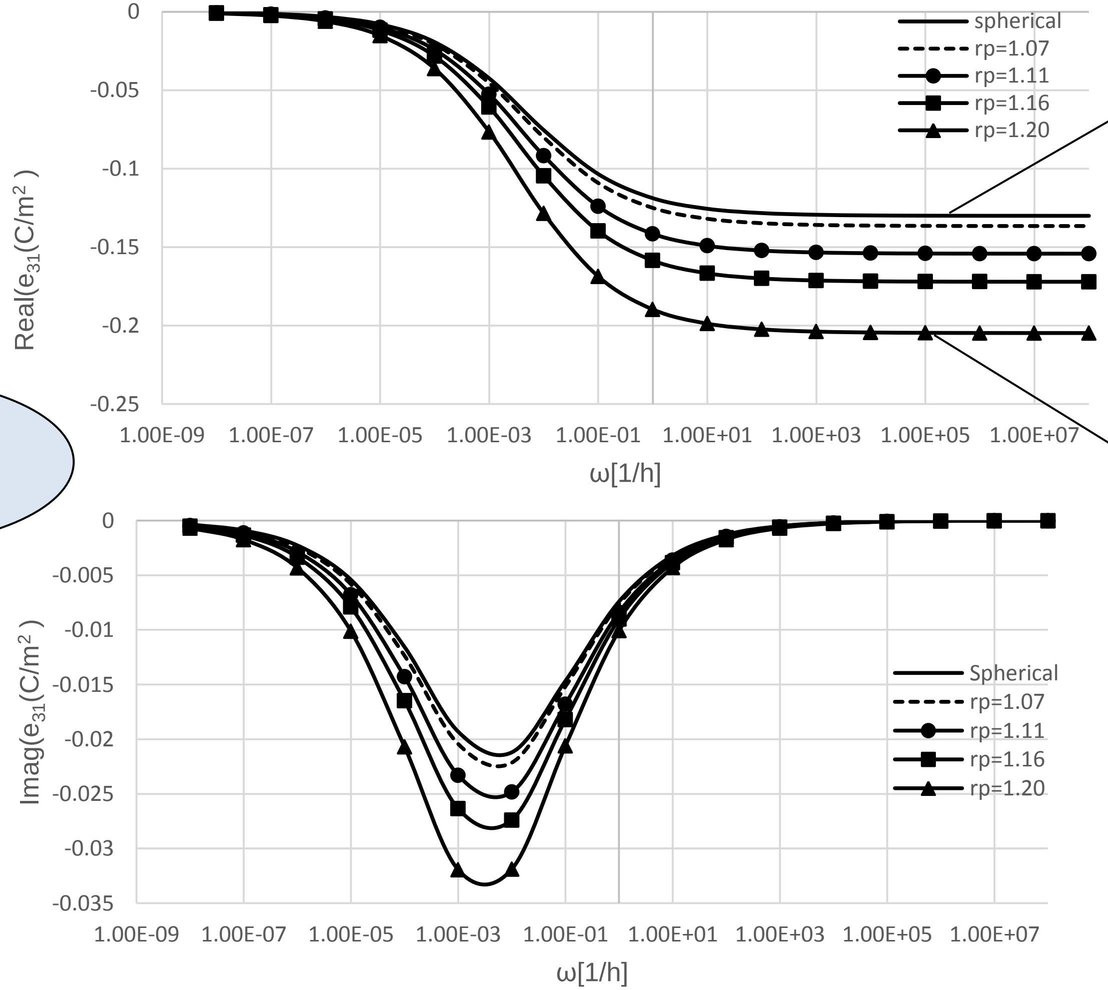
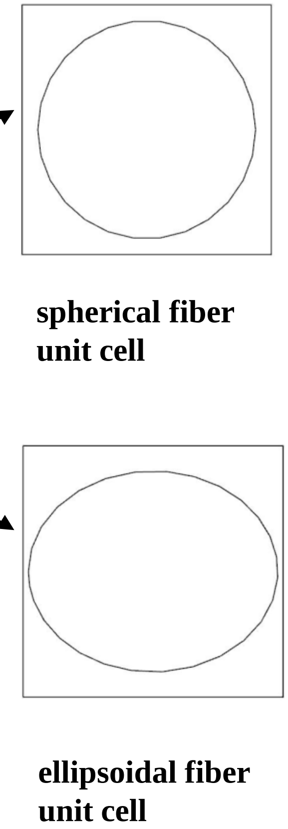


Figure 7. Effective storage and loss piezoelectric modulus (e_{31}) for a viscoelectroelastic composite for different volume fraction.

Results





PZT fiber reinforced in viscoelastic matrix with different fiber cross section.

Figure . Effective storage and loss piezoelectric modulus (e_{31}) for a viscoelectroelastic composite.

Major to minor axis ratio of the ellipse is rp.

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Conclusions

- **The results shows that the material properties strongly depend on:**
  - **- Frequency-dependent viscoelastic properties**
  - **- Piezoelectric fiber volume fraction**
- **COMSOL directly calculates the frequency-dependent properties with no need for customized functions/subroutines for material properties or periodic BCs**