

Numerical Homogenization of Viscoelastic Composites with Piezoelectric Fibers

Mohammed Al-Ajmi, Ph.D.

Associate Professor

Prabha Muthusamy, Ph.D.

Research Assistant

Mechanical Engineering Department Kuwait University



Layout

○ **Introduction**

○ **Theoretical Formulation**

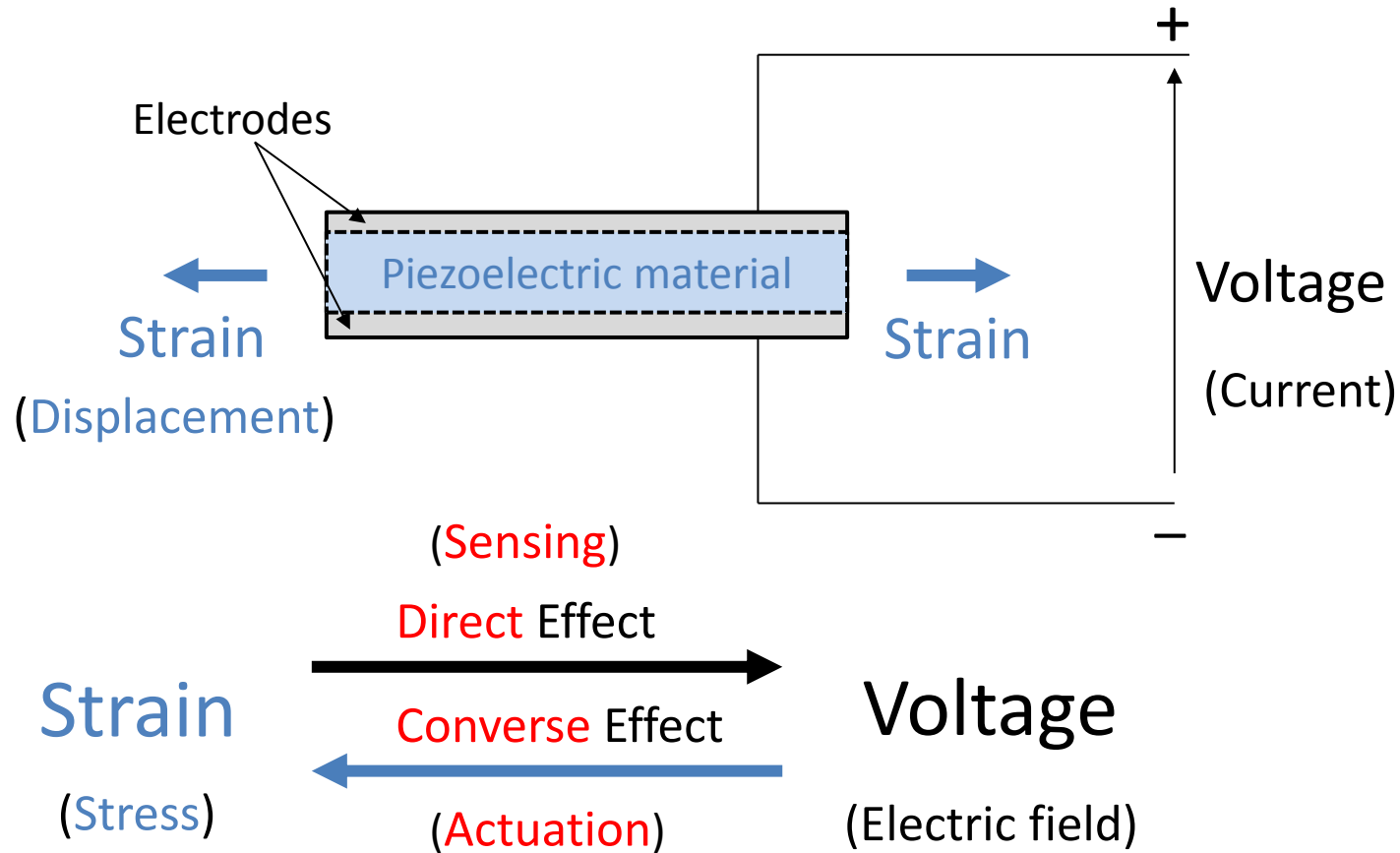
○ **Numerical Analysis**

○ **Conclusion**



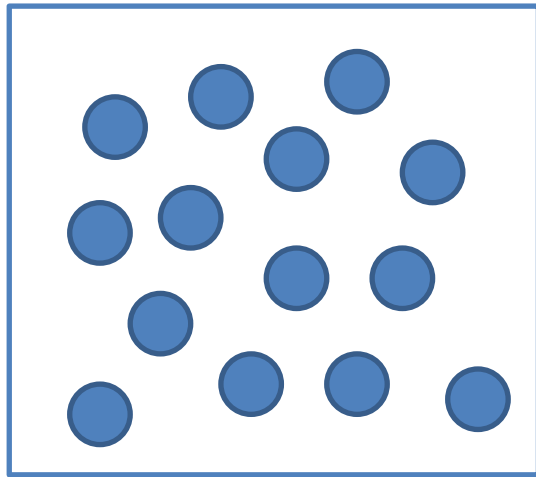
Introduction

Piezoelectric effects:

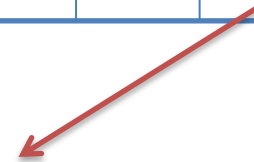
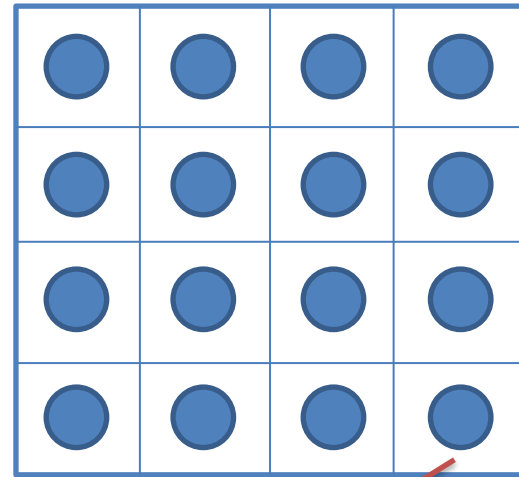


Introduction

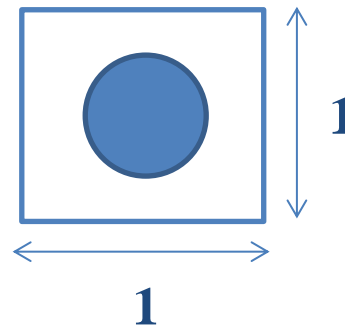
Real



Assumed

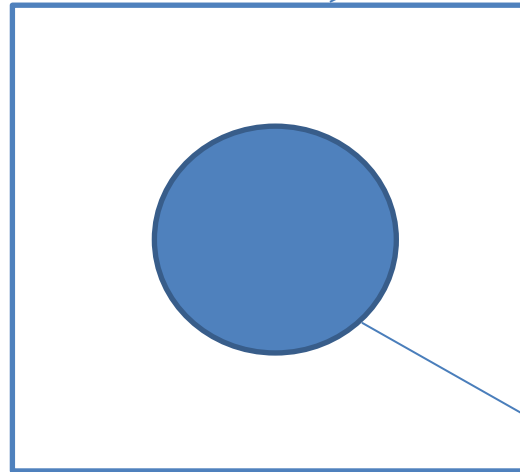


Representative Unit Cell



Introduction

**Piezo-Visco
Composite**



Matrix

-Square

-Linear Visco-elastic

Fiber

-Circular

-Linear Piezo-electric

-Poled in Fiber direction

Theoretical Formulation

Linear Piezoelectric Constitutive Equations:

e-form (used in FEA):

$$\mathbf{T} = \mathbf{C}^E \mathbf{S} - \mathbf{e}^t \mathbf{E}$$

$$\mathbf{D} = \mathbf{e} \mathbf{S} + \epsilon^S \mathbf{E}$$

Theoretical Formulation

Linear Piezoelectric Constitutive Equations:

Stress

Elastic Coefficients
At constant Electric Field

Strain

Electric Field

$$\mathbf{T} = \mathbf{C}^E \mathbf{S} - \mathbf{e}^t \mathbf{E}$$

Electrical Displacement

Stress Piezoelectric
Coupling Coefficients

Permittivity
At constant strain

$$\mathbf{D} = \mathbf{e} \mathbf{S} + \epsilon^S \mathbf{E}$$

The diagram illustrates the linear piezoelectric constitutive equations. It features two equations: $\mathbf{T} = \mathbf{C}^E \mathbf{S} - \mathbf{e}^t \mathbf{E}$ and $\mathbf{D} = \mathbf{e} \mathbf{S} + \epsilon^S \mathbf{E}$. Arrows point from descriptive labels to the corresponding terms in the equations. Labels include 'Stress' pointing to \mathbf{T} , 'Elastic Coefficients At constant Electric Field' pointing to \mathbf{C}^E , 'Strain' pointing to \mathbf{S} , 'Electric Field' pointing to \mathbf{E} in the first equation, 'Electrical Displacement' pointing to \mathbf{D} , 'Stress Piezoelectric Coupling Coefficients' pointing to \mathbf{e} , and 'Permittivity At constant strain' pointing to ϵ^S .

Theoretical Formulation

Linear Isotropic Viscoelastic Material:

$$E(\omega) = E'(\omega) + iE''(\omega)$$

$$h = \frac{E''(\omega)}{E'(\omega)} \quad \longrightarrow \quad E(\omega) = E'(\omega)(1 + ih)$$

Theoretical Formulation

Homogenization:

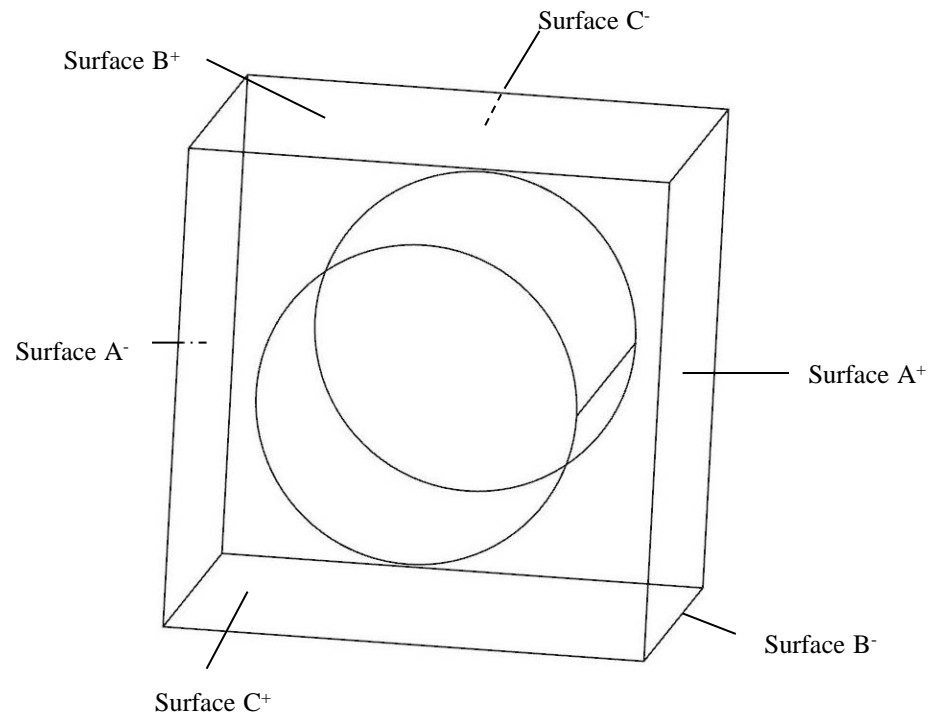
$$\bar{T}_{ij} = \frac{1}{V} \int_V T_{ij} dV \quad \bar{D}_{ij} = \frac{1}{V} \int_V D_{ij} dV$$
$$\bar{S}_{ij} = \frac{1}{V} \int_V S_{ij} dV \quad \bar{E}_{ij} = \frac{1}{V} \int_V E_{ij} dV$$

$$\bar{T} = \hat{C}(w)\bar{S} - \hat{e}(w)^T \bar{E}$$

$$\bar{D} = \hat{e}(w)\bar{S} + \hat{e}(w)\bar{E}$$

Numerical Analysis

Geometry of representative volume element:



Numerical Analysis

- Viscoelastic material is modeled using:

$$M_0(s) = D_0 + \frac{D_1 n!}{s^n}$$

D_0 is the initial elastic compliance,
 D_1 and n are experimentally determined parameters, and
 $s=i\omega$. with ω denoting the frequency.

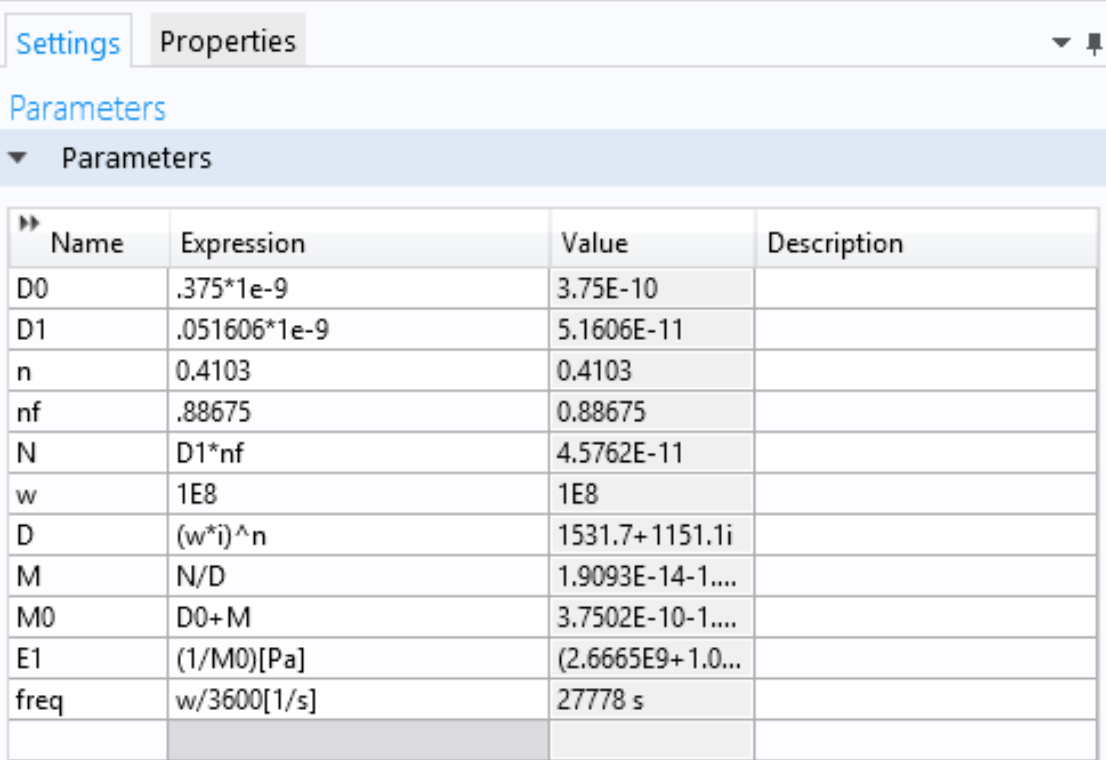
The Young modulus $E(s)$ is taken as the inverse of $M_0(s)$

- Piezoelectric Material used: PZT-7A



Numerical Analysis

Frequency-dependent viscoelastic material implementation in COMSOL



| Name | Expression | Value | Description |
|------|--------------|-------------------|-------------|
| D0 | .375*1e-9 | 3.75E-10 | |
| D1 | .051606*1e-9 | 5.1606E-11 | |
| n | 0.4103 | 0.4103 | |
| nf | .88675 | 0.88675 | |
| N | D1*nf | 4.5762E-11 | |
| w | 1E8 | 1E8 | |
| D | (w*i)^n | 1531.7+1151.1i | |
| M | N/D | 1.9093E-14-1.... | |
| M0 | D0+M | 3.7502E-10-1.... | |
| E1 | (1/M0)[Pa] | (2.6665E9+1.0.... | |
| freq | w/3600[1/s] | 27778 s | |

Youngs modulus in terms of frequency



Numerical Analysis

Settings Properties

Material

Label: Material 2

Geometric Entity Selection

Geometric entity level: Domain

Selection: Manual

1
Active

Override

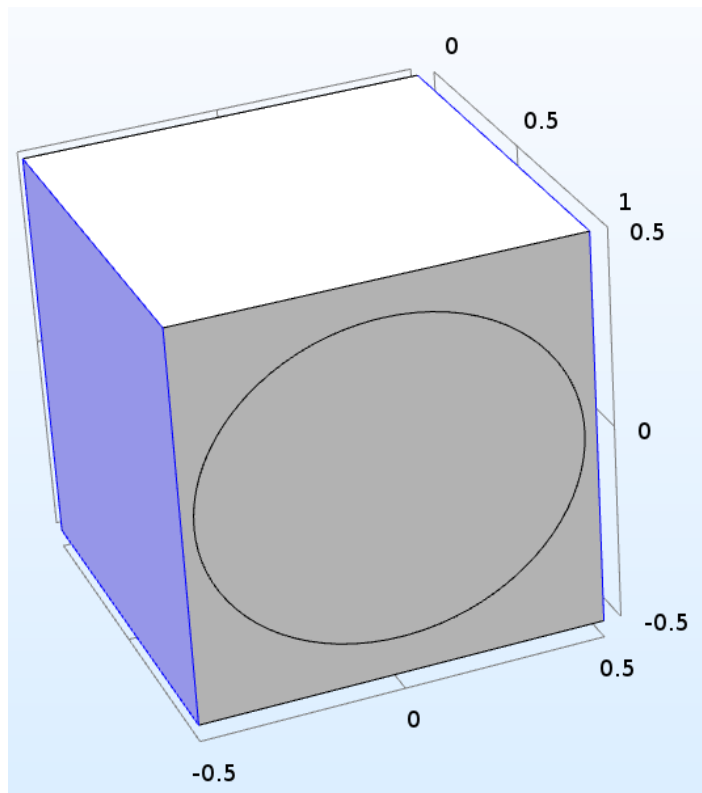
Material Properties

Material Contents

| Property | Name | Value | Unit | Property group |
|---|----------------------|-------|-------------------|--------------------|
| <input checked="" type="checkbox"/> Density | rho | 1370 | kg/m ³ | Basic |
| <input checked="" type="checkbox"/> Relative permittivity | epsilon _r | 2.8 | 1 | Basic |
| <input checked="" type="checkbox"/> Young's modulus | E | E1 | Pa | Young's modulus an |
| <input checked="" type="checkbox"/> Poisson's ratio | nu | 0.367 | 1 | Young's modulus an |

Numerical Analysis

Periodicity Condition



Settings

Periodic Condition

Label: Periodic Condition 3

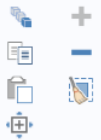
Boundary Selection

Selection: Manual



Active

1
12



Override and Contribution

Equation

Show equation assuming:

Study 1, Stationary

Periodicity Settings

Type of periodicity:

User defined

- Periodic in u
- Periodic in v
- Periodic in w

Numerical Analysis

Deformed Viscoelastic material under normal and shear load

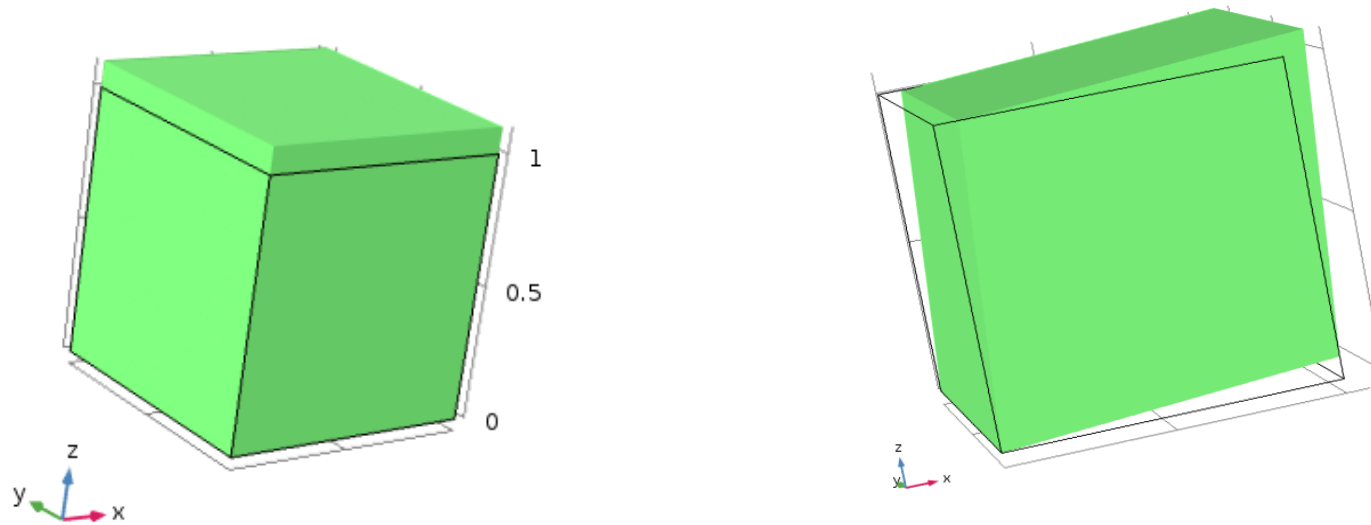
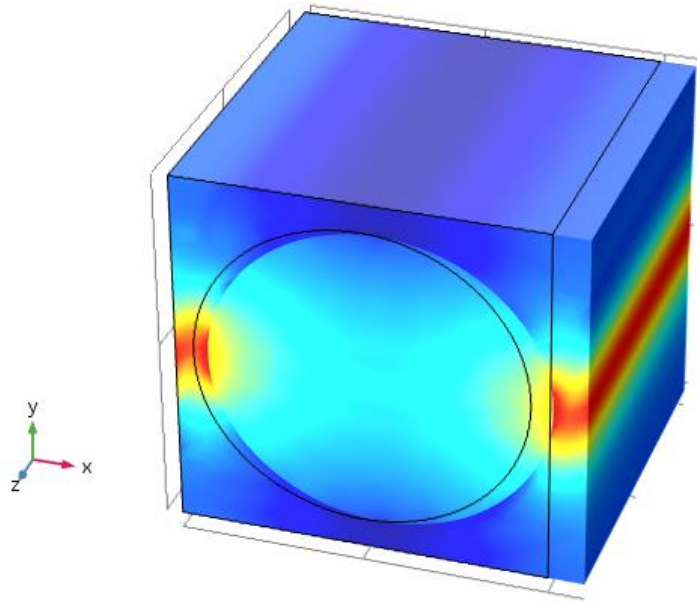


Figure 3: Deformed unit cell of viscoelastic material

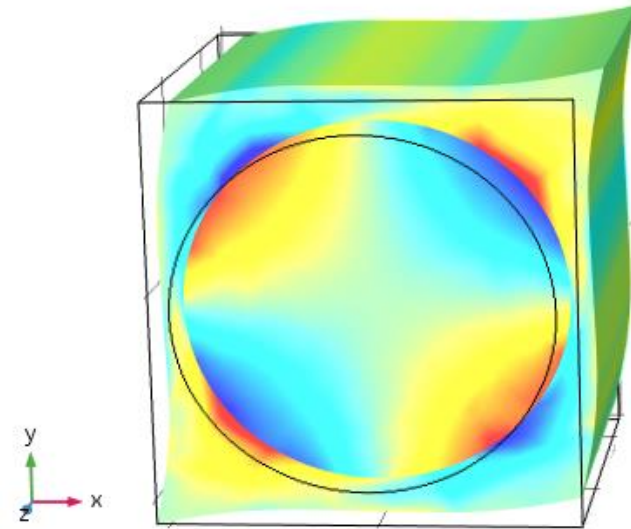
Numerical Analysis

Sample Mechanical Calculation:

-Apply normal and shear on the cell cross section (all potentials =0)



$$C_{11} = \frac{\bar{T}_{11}}{\bar{S}_{11}}$$



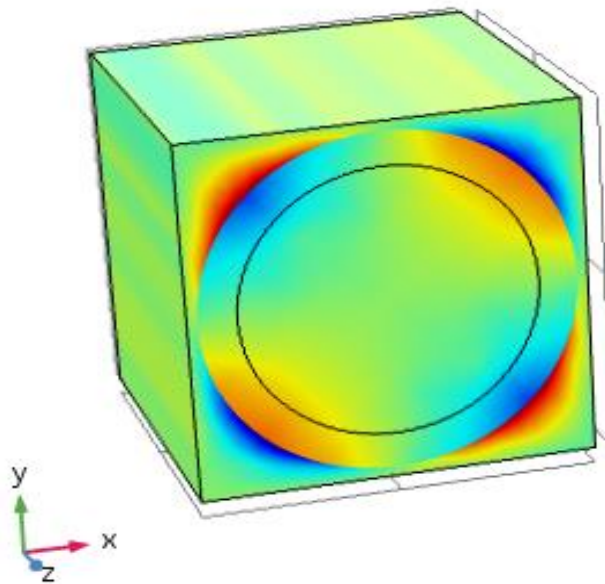
$$C_{66} = -\frac{\bar{T}_{11}}{\bar{S}_{12}}$$

Figure 4: Deformed unit cell of viscoelastic matrix reinforced with PZT fiber

Numerical Analysis

Sample Piezoelectric Calculation

-Apply potential difference across fiber (all displacements =0)



$$\hat{e}_{31} = \frac{\bar{T}_{11}}{\bar{E}_3}$$

Figure 4: Deformed unit cell of viscoelastic matrix reinforced with PZT fiber

Results

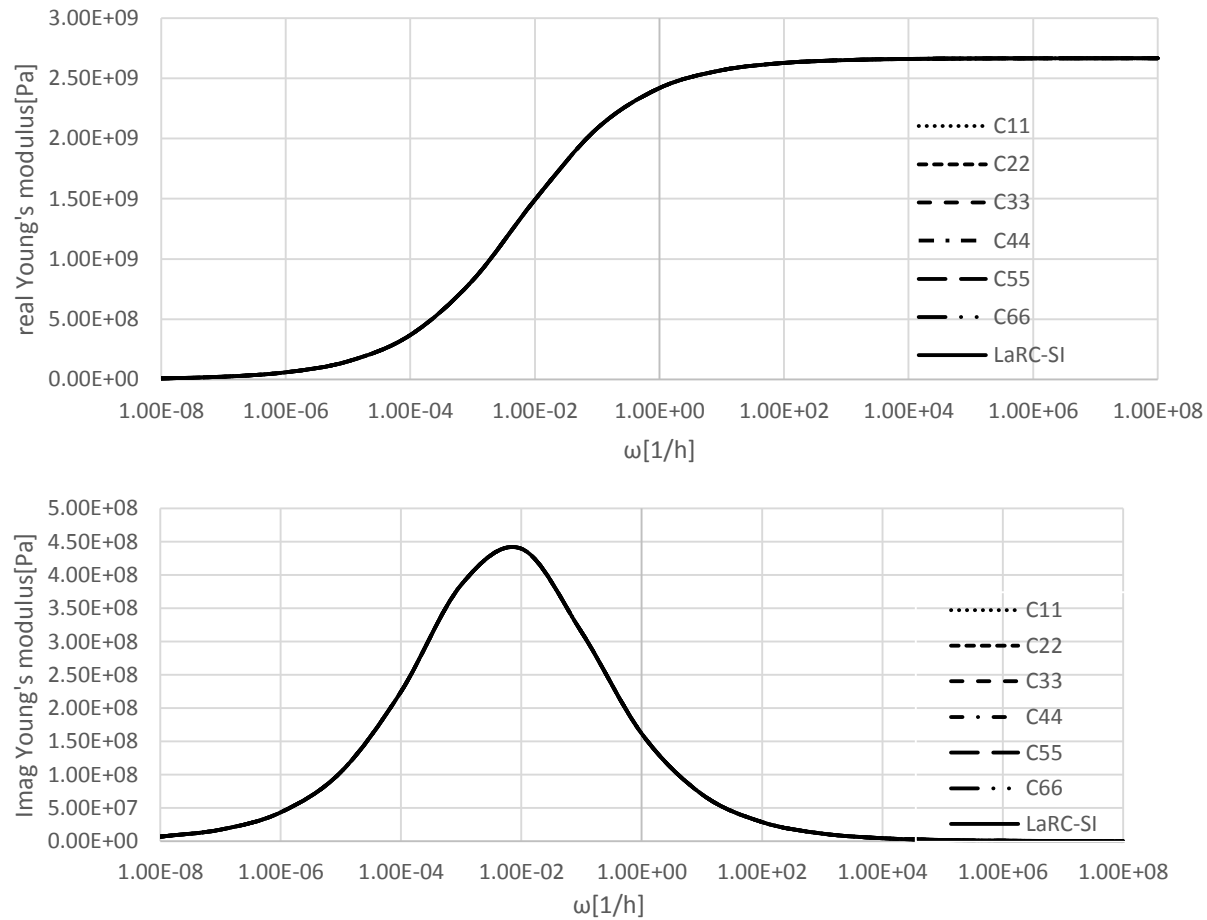
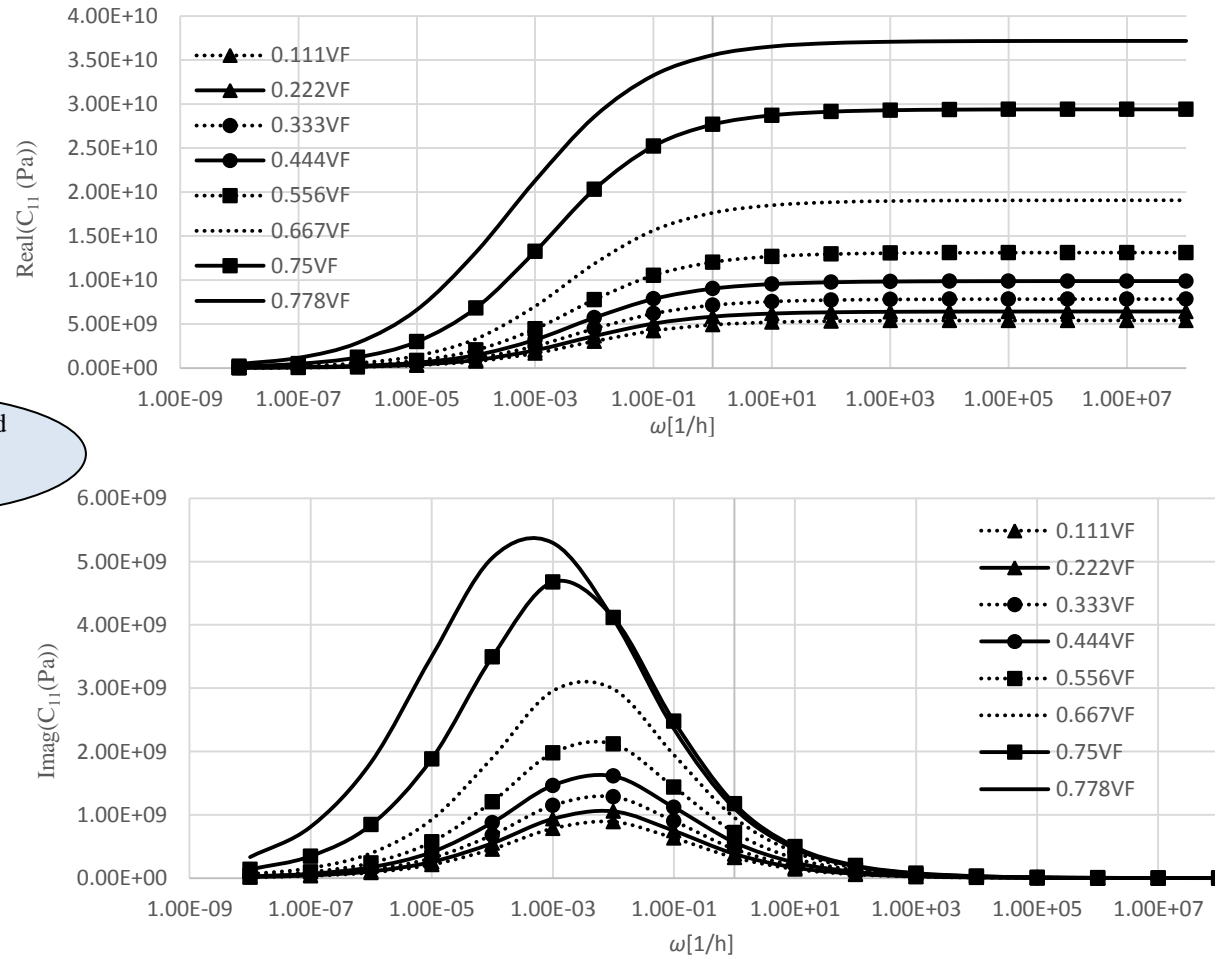


Figure 5. Real and Loss Modulus LaRC-SI with respect to frequency for different boundary condition.

Results



PZT fiber reinforced
in viscoelastic
matrix.

Figure 6 . Effective storage and loss elastic modulus (C_{11}) for a viscoelectroelastic composite for different volume fraction.

Results

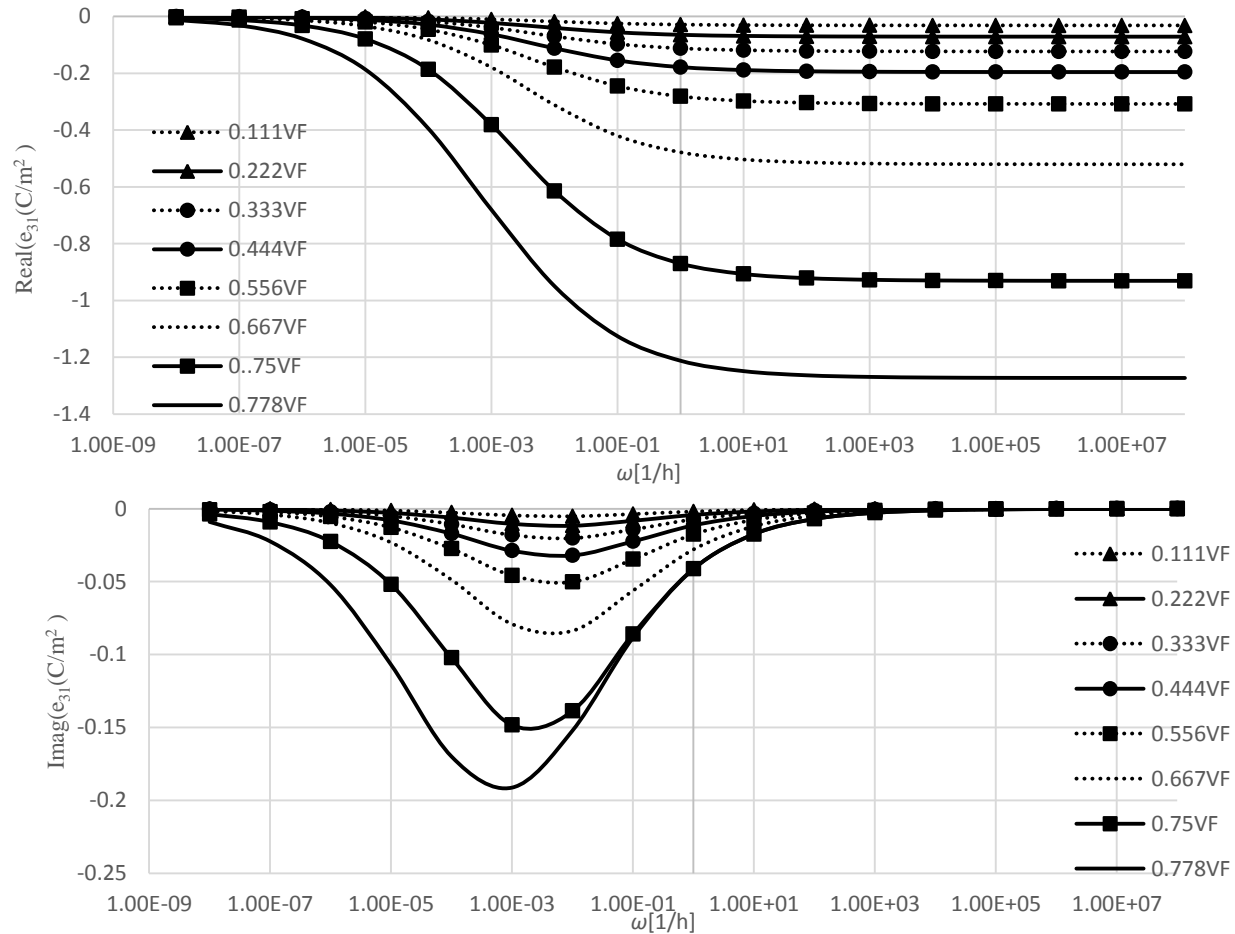
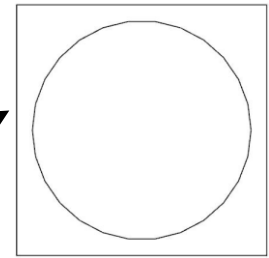
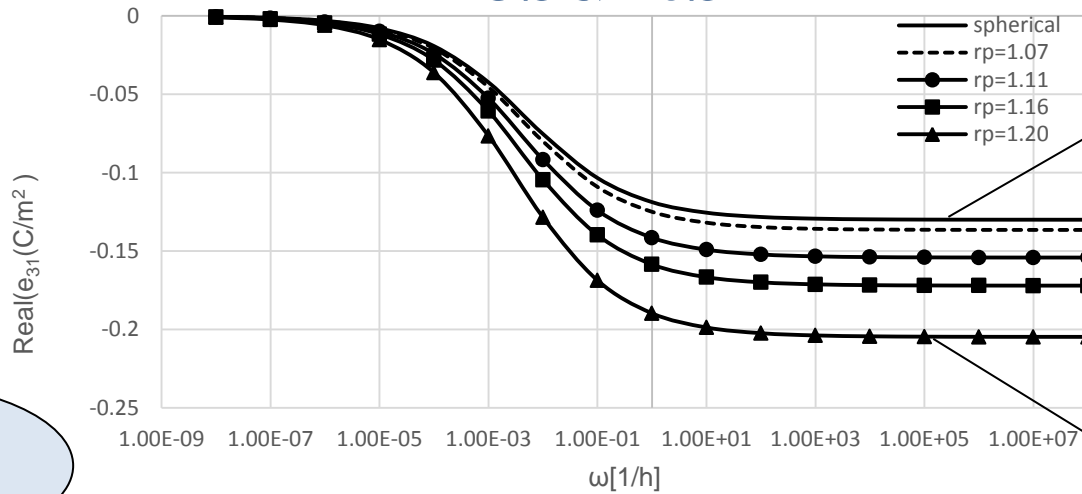
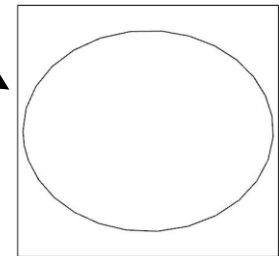
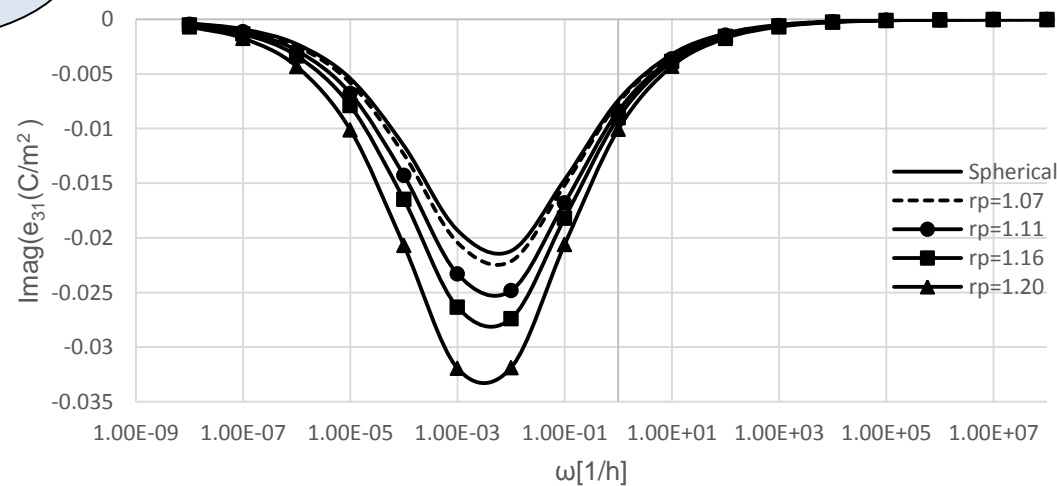


Figure 7. Effective storage and loss piezoelectric modulus (e_{31}) for a viscoelectroelastic composite for different volume fraction.

Results



spherical fiber unit cell



ellipsoidal fiber unit cell

PZT fiber reinforced in viscoelastic matrix with different fiber cross section.

Figure . Effective storage and loss piezoelectric modulus (e_{31}) for a viscoelectroelastic composite.

Major to minor axis ratio of the ellipse is rp .

Conclusions

- **The results shows that the material properties strongly depend on:**
 - **Frequency-dependent viscoelastic properties**
 - **Piezoelectric fiber volume fraction**
- **COMSOL directly calculates the frequency-dependent properties with no need for customized functions/subroutines for material properties or periodic BCs**