# Helical Coil Flow: a Case Study

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# Outline

- Motivations
- Fluid dynamics background
- Geometry details
- Model implementation
- Toroidal path
- Helical path
- Concluding remarks

# The REET Unit at FBK

REET: Renewable Energies and Environmental Technologies

*Topics*:

- Solar thermal energy
- Non ionizing radiations
- Biomass
- Geothermal energy

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Applied research
in collaboration with
local companies

*Methods*:

- Experimental activity
- Numerical simulations
- Partner collaborations

# Heat Exchange Applications

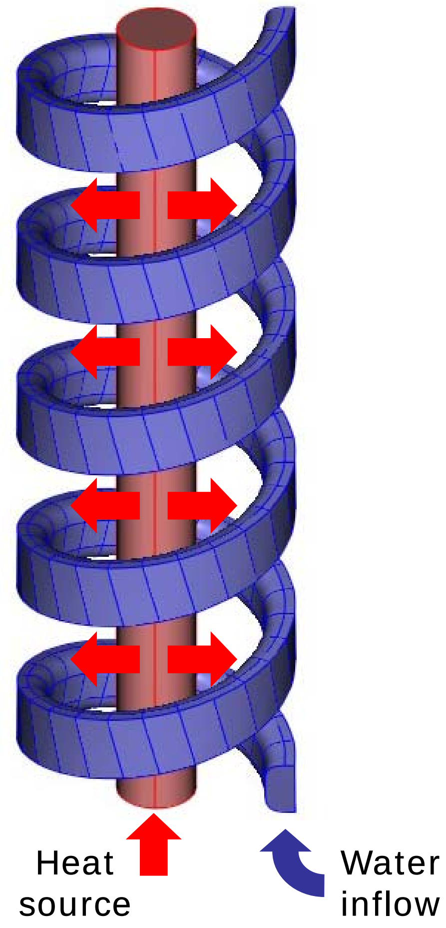
Crucial aspects:

- Vector fluid
- Piping system
- Heat source
- Flow regime

In general: non-isothermal flow

Here: fluid dynamics *independent*
of heat transfer

Water, small temperature variation
→ **Purely incompressible flow**



# Flow in Curved Pipes

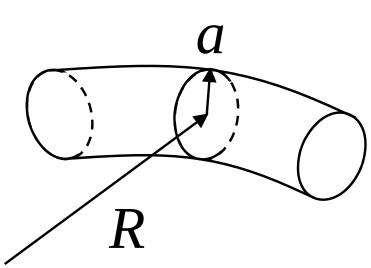
Flow in curved circular pipes: Dean (1927)

- Small velocity (laminar regime)
- Negligible torsion
- Small curvature ($\delta \ll 1$)

**Dean number**

$$\boxed{\mathrm{De} = \mathrm{Re}\sqrt{\delta}}$$

$$\delta = a / R$$




Investigations found in literature:

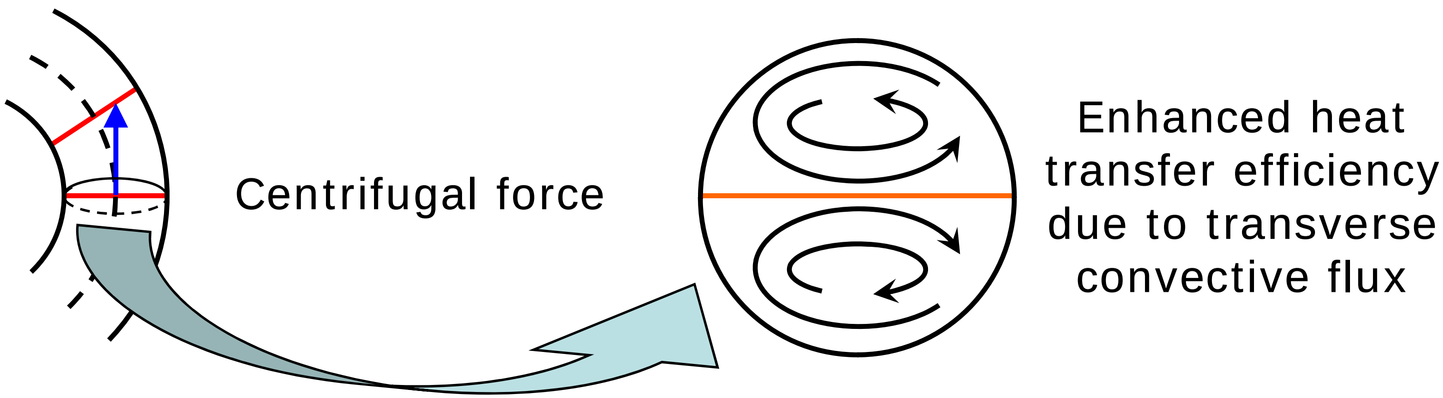
- Cross section (circle, square, ...)
- Pipe path (torus, helix, ...)
- Regime (laminar, turbulent, ...)

# Flow in Curved Pipes

Pipe curvature gives rise to *secondary flow* (flow perpendicular to the main flow direction).

Typical flow pattern at small Dean numbers:

- **Main flow**: slightly modified with respect to straight tubes due to *centrifugal force*
- **Secondary flow**: recirculation structures (Dean flow)



# Helical Coil Flow

Helical channel with non trivial cross section
Large number of turns → infinite coil approximation

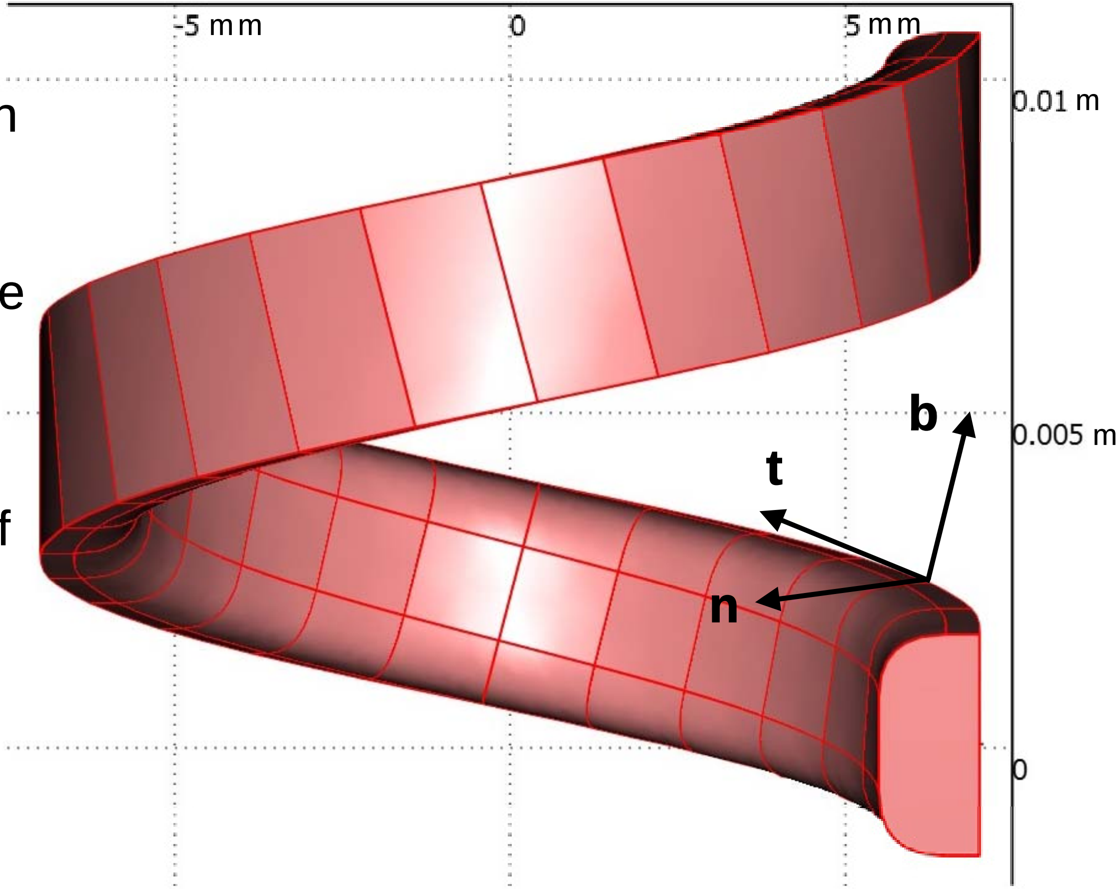
Translational invariance with respect to curvilinear coordinates (Frenet frame)
→ possible dimensional reduction (3D → 2D)

Here: 3D finite geometry with *periodic-like* boundary conditions

Helical path: non negligible role of *torsion*
Put in evidence by comparison with toroidal path

# Frenet Frame

- **t**, tangent unit vector: tangent to curvilinear path
- **n**, normal unit vector: pointing towards curvature radius
- **b**, binormal unit vector: constant in the absence of torsion



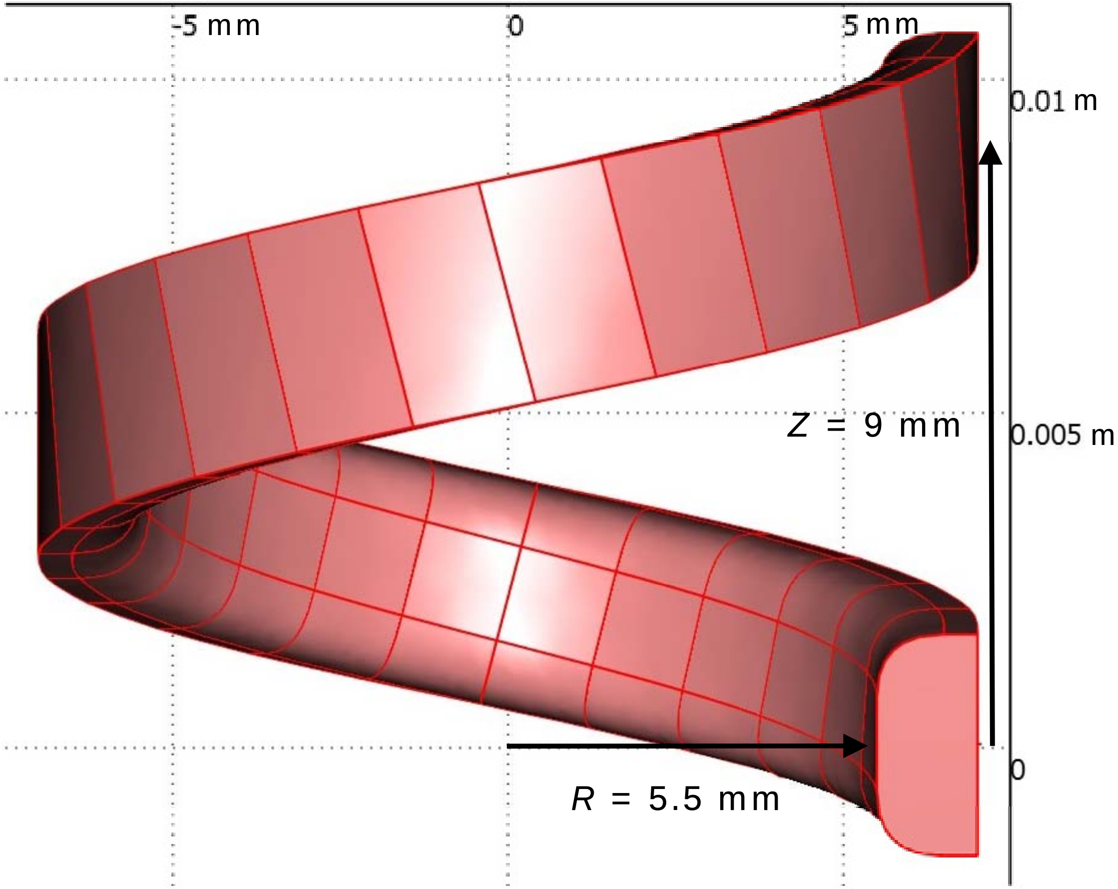
# Helical Path

$$\begin{cases} x(\varphi) = R\cos\varphi \\ y(\varphi) = R\sin\varphi \\ z(\varphi) = \dfrac{Z}{2\pi}\varphi \end{cases}$$

$$\begin{cases} \text{curvature}: \kappa = \dfrac{R}{R^2 + (Z/2\pi)^2} \\ \text{torsion}: \tau = \dfrac{Z/2\pi}{R^2 + (Z/2\pi)^2} \end{cases}$$

$R$, helix radius

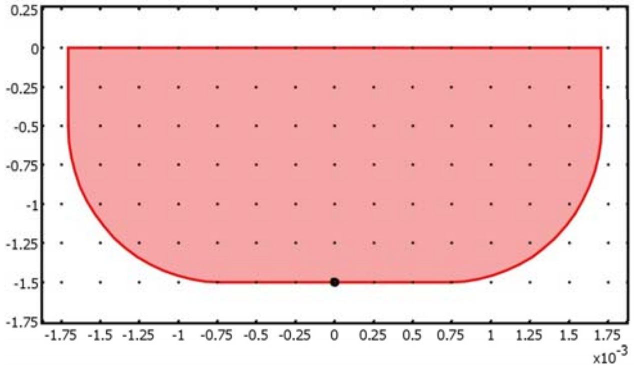
$Z$, helix pitch



# Channel Cross Section

Cross section in the plane orthogonal to the tangent vector **t**

Same cross section for helical and toroidal geometries




$$\mathrm{Re} = \frac{\rho v D_\mathrm{h}}{\eta}$$

$$\begin{cases} \rho = \text{density} \\ \eta = \text{dynamic viscosity} \\ v = \text{average velocity} \\ D_\mathrm{h} = \text{hydraulic diameter} \end{cases}$$

$$D_\mathrm{h} = 4\frac{A_\mathrm{ch}}{P_\mathrm{ch}} \approx 2.1\,\mathrm{mm}$$

$$\begin{cases} A_\mathrm{ch} = \text{channel cross section area} \\ P_\mathrm{ch} = \text{channel cross section perimeter} \end{cases}$$

# Model Implementation

Navier-Stokes equations, incompressible fluid (water).

**Solver**. PARDISO, highly non-linear problem, manual tuning of damping parameters.

**Mesh, element order**. Unstructured tetrahedral mesh elements, swept prism mesh elements, Lagrange – $P_2P_1$ or Lagrange – $P_3P_2$.

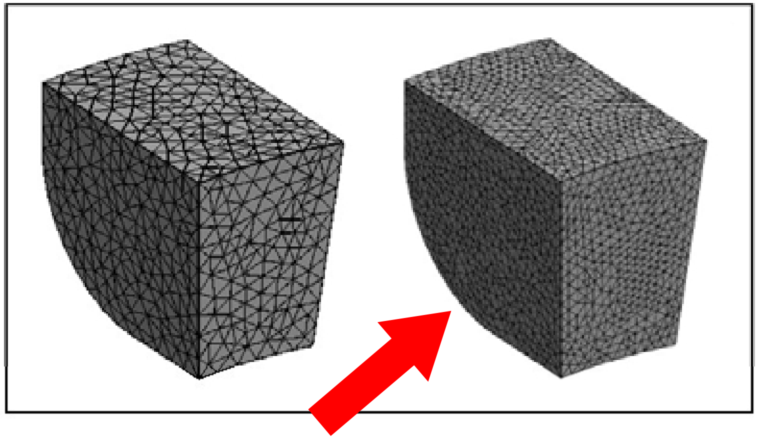
**Artificial diffusion**: crosswind diffusion (0.1), isotropic diffusion occasionally used for intermediate simulations.

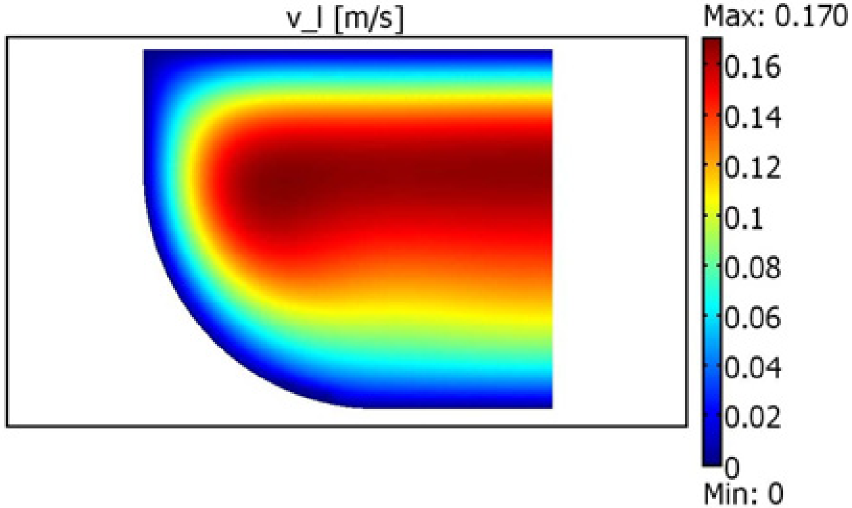
**Boundary conditions**. Walls: no slip b.c.'s. Inlet and outlet: no viscous stress + periodic b.c.'s + pressure at a point.

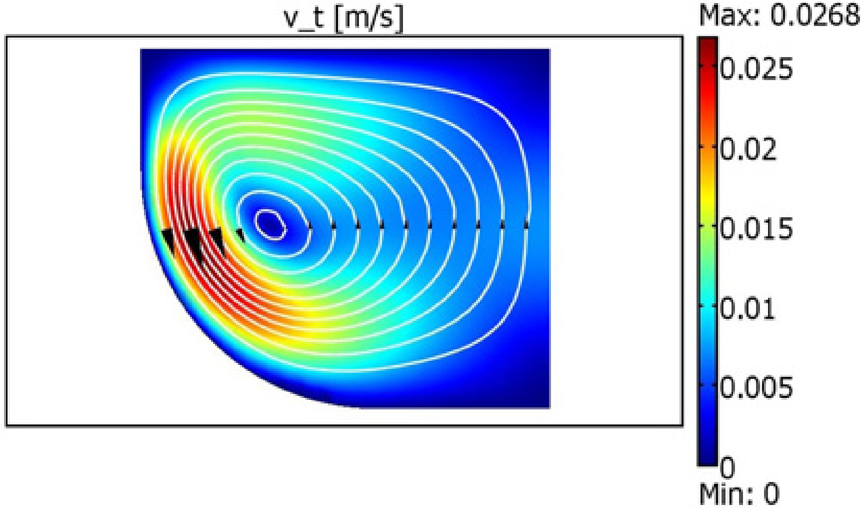
# Torus

Symmetry: half cross section
Arc-length: 10°
Meshes: 9x10, 16x18
Element order: $P_2P_1$
Re = 220, De = 90, Dp = 40 Pa / 360°
B.c.'s:

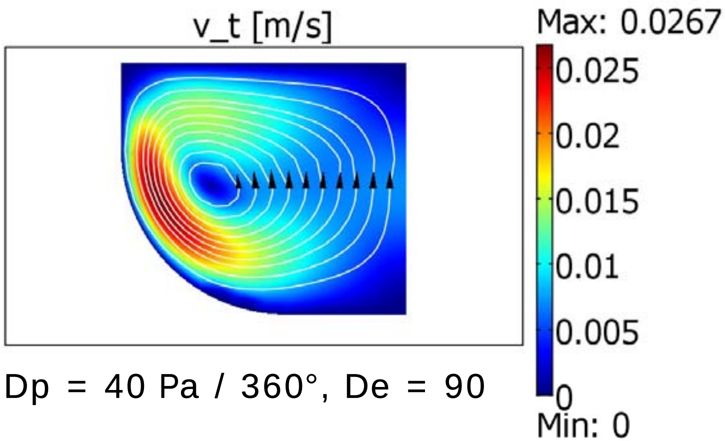
$$\mathbf{v}(\mathbf{r}_{out}) = R_{\mathbf{b}}(\pi/18)\,\mathbf{v}(\mathbf{r}_{in}) \,,$$
$$p(\mathbf{r}_{out}) = p(\mathbf{r}_{in}) - \Delta p \,,$$
$$\mathbf{r}_{in} = R_{\mathbf{b}}^{-1}(\pi/2)\,\mathbf{r}_{out}$$



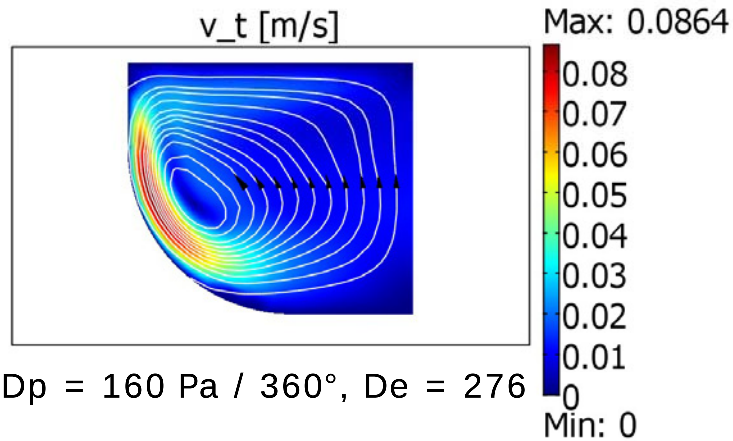


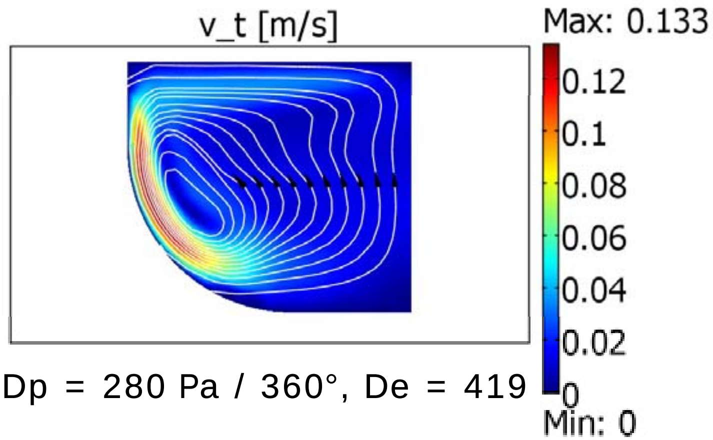
# Torus



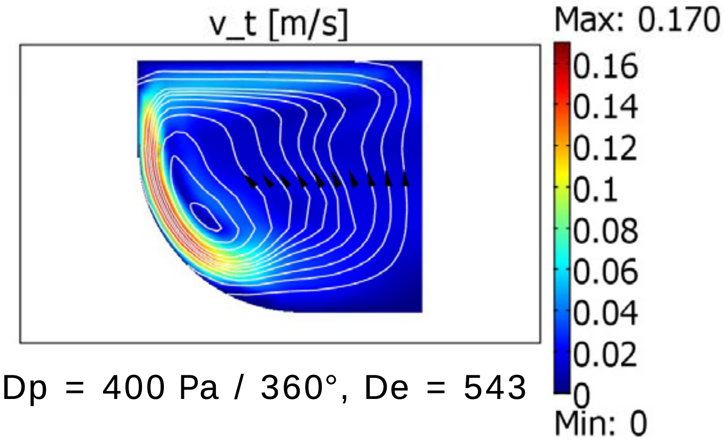

Dp = 40 Pa / 360°, De = 90




Dp = 160 Pa / 360°, De = 276




Dp = 280 Pa / 360°, De = 419

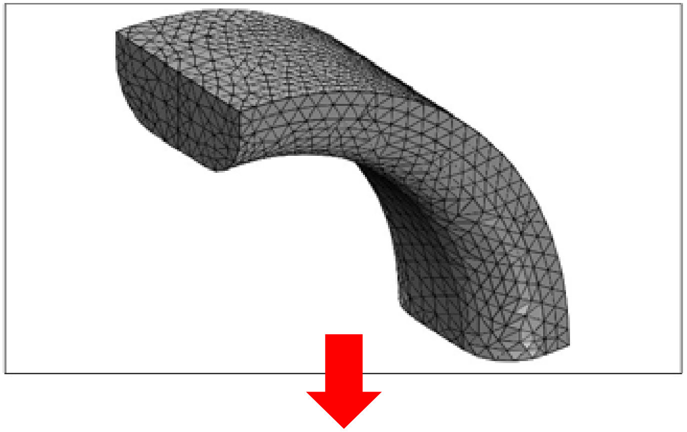


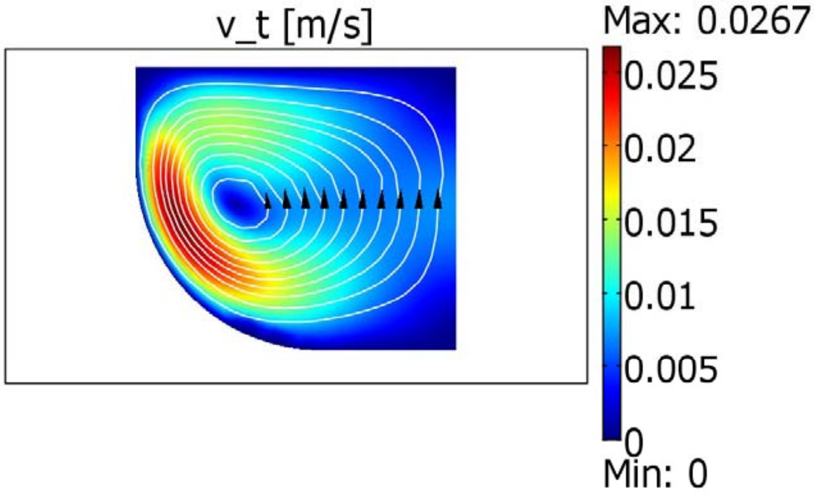

Dp = 400 Pa / 360°, De = 543

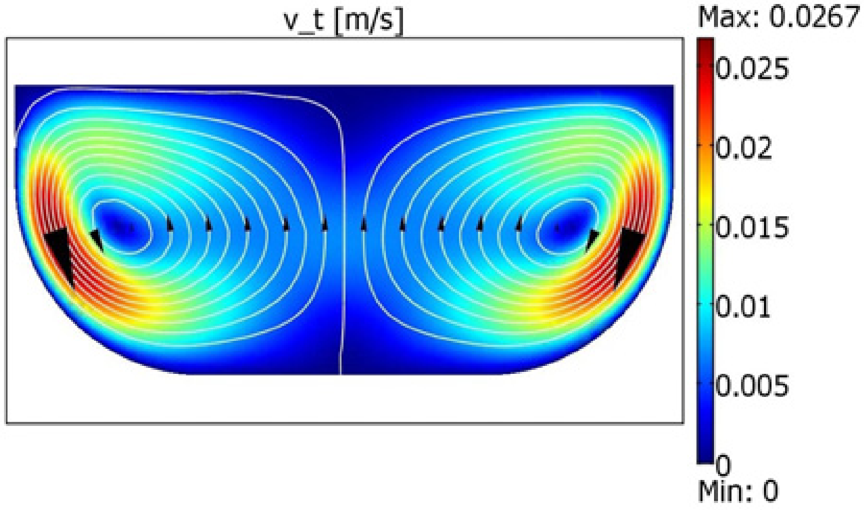
# Torus

Check symmetry: full cross section
Check curvature: 90° arc
Meshes: 4x5 (half section)
Element order: $P_3P_2$
Re = 220, De = 90, Dp = 40 Pa / 360°

Good agreement with 10° half section geometry for similar dof density



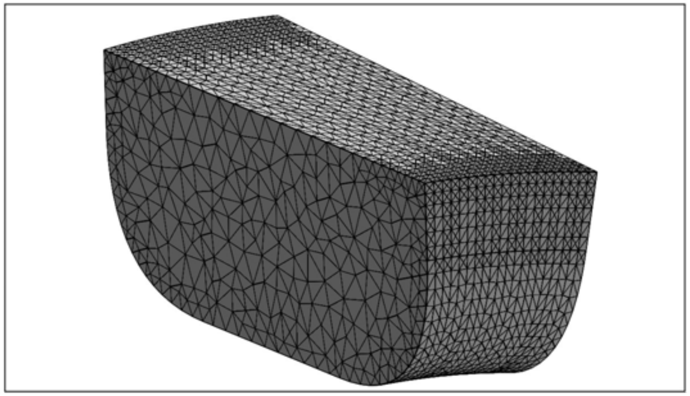


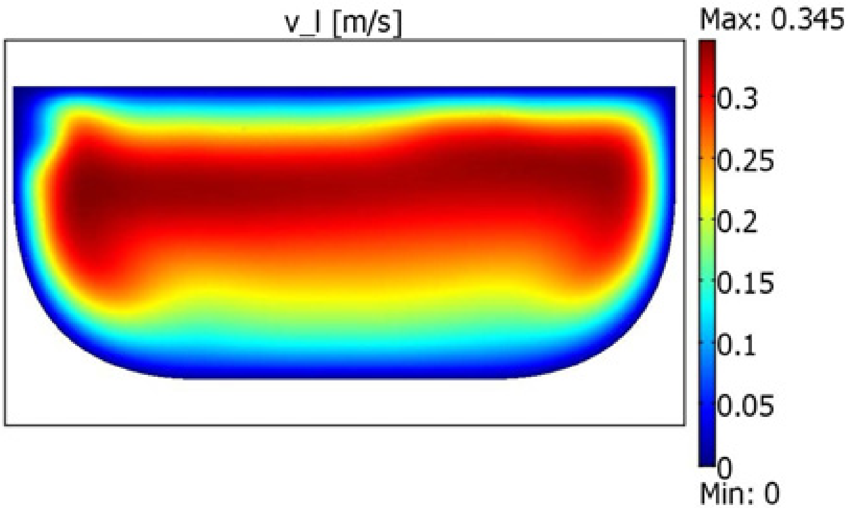
# Helix

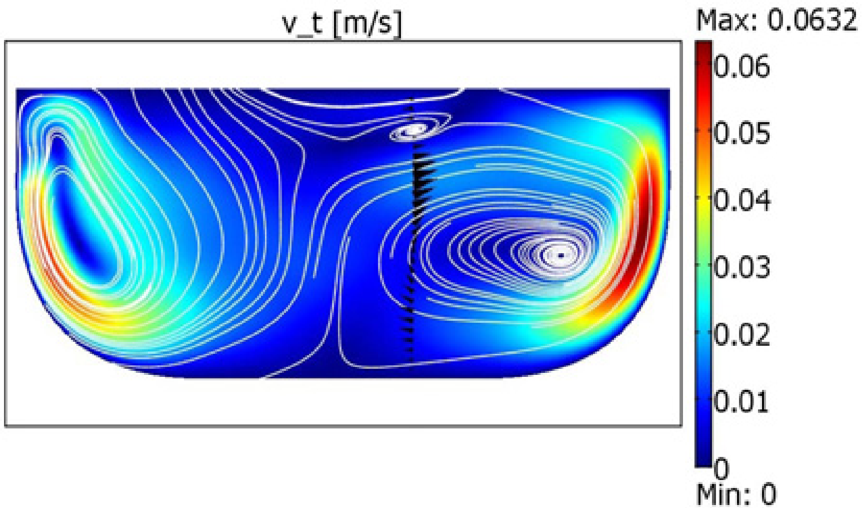
Arc-length: 10°
Mesh: 10x12 (half section)
Element order: $P_3P_2$
Re = 453, De = 181, Dp = 100 Pa / 360°
B.c.'s:

$$\mathbf{v}(\mathbf{r}_{out}) = R_z(\pi/18)\,\mathbf{v}(\mathbf{r}_{in}) \,,$$
$$p(\mathbf{r}_{out}) = p(\mathbf{r}_{in}) - \Delta p \,,$$



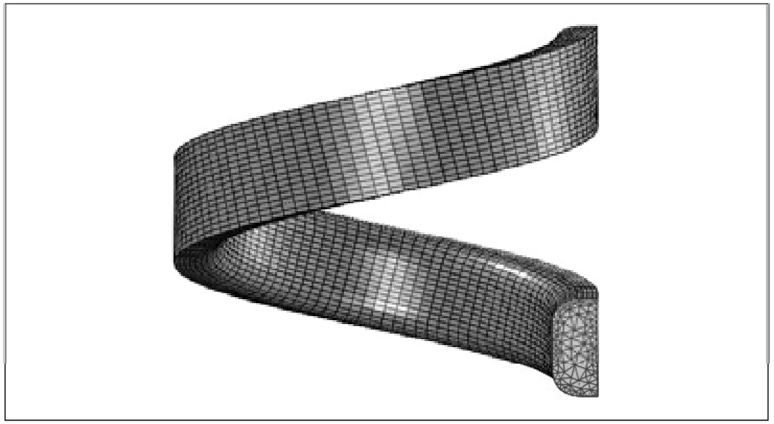
No symmetry, additional Dean structure



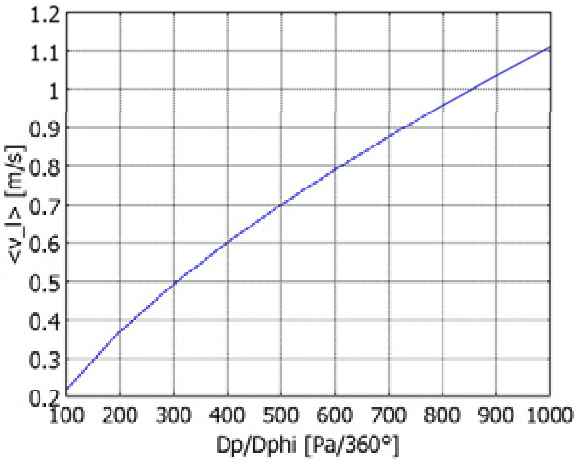



# Helix

Arc-length: 360°
Mesh: swept, prism elements
Element order: $P_2P_1$
Dp = 100, ..., 1000 Pa / 360°



Basically linear velocity - pressure relation, laminar regime (similar results are obtained for the toroidal geometry)



# Conclusions

Periodic boundary conditions:
Convergence issues with rispect to standard inlet-outlet b.c.'s

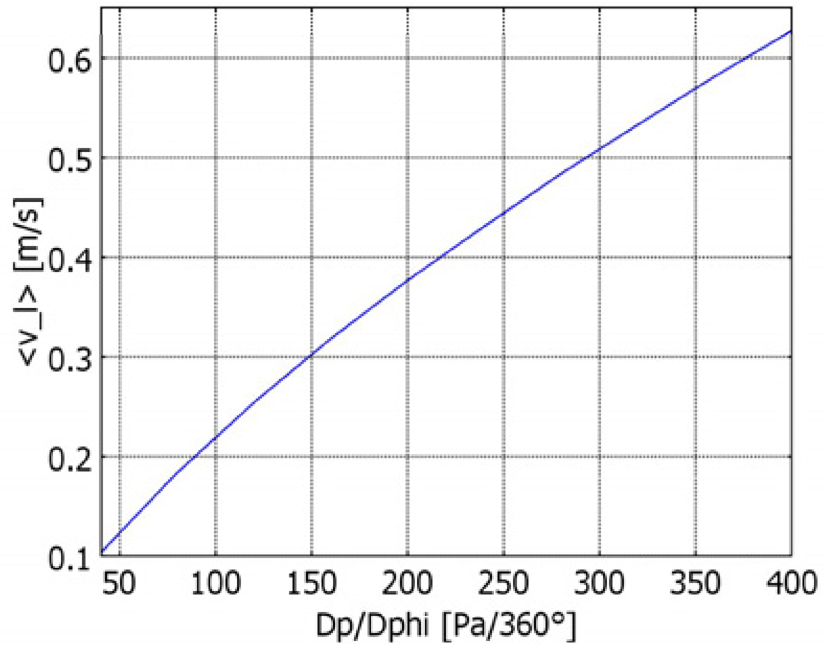
Mesh requirements:
- Identical meshes for coupled boundaries
- High quality elements (in particular close to periodic boundaries)

→ unstructured meshes and higher order elements

Successful observation of non trivial secondary flow structures with full 3D Navier-Stokes simulations

# Torus

Toroidal path:
velocity-pressure relation,
laminar regime



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# Technical Info

**Machine.** Processor: double quad-core, 2GHz. RAM: 16 GB.

**Number of degrees of freedom.**

*Helix*:
• 10° geometry: 360000 dof
• single turn geometry: 250000 dof

*Torus*:
• 10° half section geometry: 42000 dof (9x10), 240000 dof (16x18)
• 90° full section geometry: 220000 dof