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SIMULATION OF INCOMPRESSIBLE FLOW THROUGH RHOMBOHEDRIC PORES

Rossella Viola, Ezio Mesini

Environmental & Mining Engineering, Univ. of Bologna

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Fabiana Zama

Dept. of Mathematics , Univ. of Bologna

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Markus Tuller

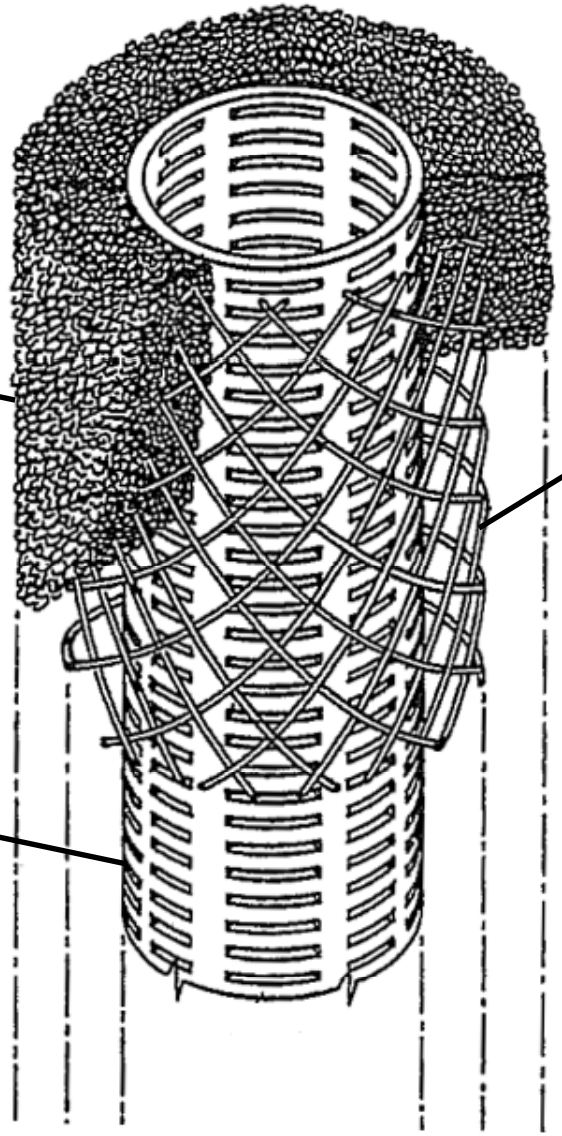
Dept. of Soil, Water and Environmental Science,
The University of Arizona

THE UNIVERSITY OF ARIZONA

Introduction

- **Re difficult to define for complex porous media with intricate pore geometries**
- **In packed beds Re calculated based on average pore dimensions is several times lower than for flow through a straight pipe.**
- **Microscopic flow pattern reveal deviations from the linear relationship between pressure drop and flow rate.**
- **Only a few studies focused on three-dimensional (3-D) eddies at the pore-scale, e.g in a single pore throat between cubic-packed spherical and ellipsoidal grains (1–2 mm diameter).**

Gravel pack for petroleum or water wells



Gravel of a predetermined particle size, or mixture of particle sizes

Wire cage, as reinforcement

Well screen: perforated or slotted pipe

aim

To investigate flow through rhombohedric unit pore cells:

- effects of pore geometry on flow behavior, with both water and oil as permeating fluids

- preferential flow paths

- relation between Re and velocity, pressure gradient and velocity to quantitatively define flow patterns

Methods- Equations

Numerical simulations in case of laminar flow for a single pore between rhombohedric packed grains (low Re < 3)

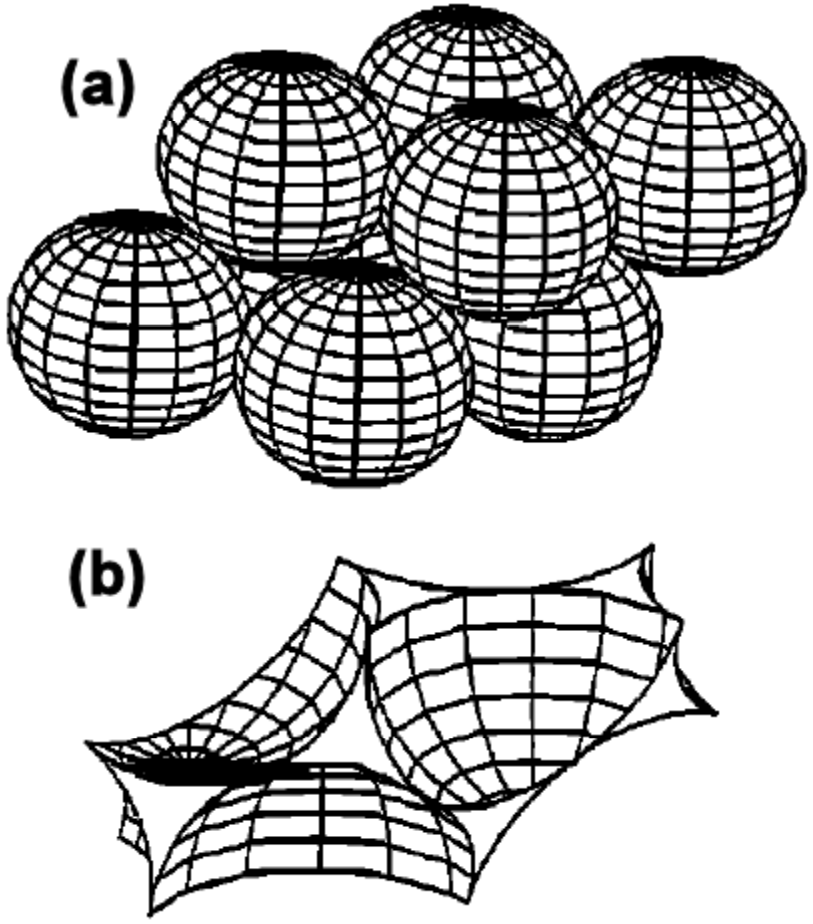
+

3D NAVIER-STOKES equations

$$\rho \mathbf{u}.\nabla \mathbf{u} + \nabla p = \mu \nabla^2 \mathbf{u},$$

$$\nabla . \mathbf{u} = 0$$

ρ fluid density: 1000(800) kg m^{-3},
$\mathbf{u}$ =[u,v,w] velocity vector,
μ dynamic viscosity 0.001(0.01) Pa s,
p total pressure



0.1 mm space between grains (1.9-mm diameter)

Methods-Boundary conditions

(a) **Wall no-slip (u=0)** at the grains surfaces.

(b) **Slip symmetry** at the interfaces between different rhombohedric cells.

(c) Pressure ($p = p_0$) together with a Dirichlet condition at the inlet surface (defined on the (x,y) plane at z=0) . The condition of no viscous stress was added to assure stability with low Reynolds numbers.

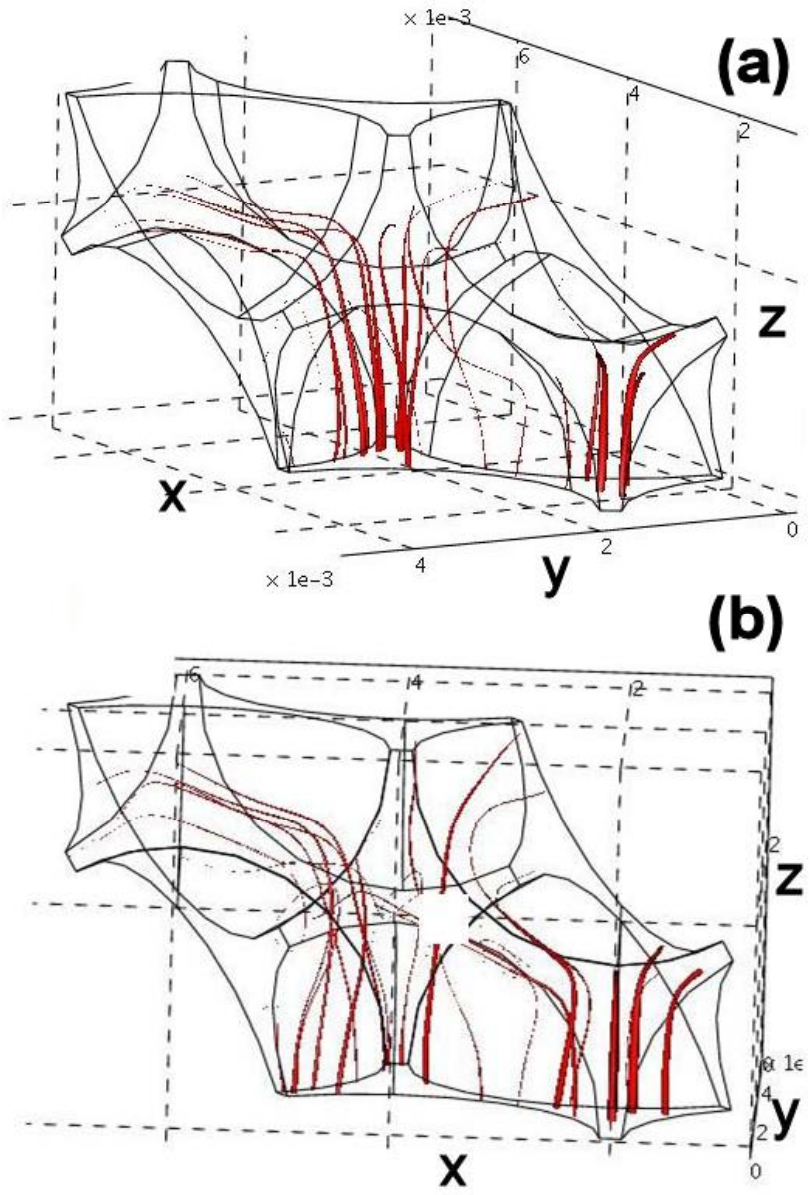
$$(\eta(\nabla u + (\nabla u)^T)n=0 .$$

(d) Homogeneous pressure ($p=0$) at the outflow surface (defined at $z=1.E^{-3}$ on the (x,y) plane).

Methods- fem solution

- ✓ 1.5E+5 to 5.0E+5 tetrahedral MESH elements.
- ✓ Lagrange p2-p1 elements and streamline diffusion (Galerkin least-squares) to assure stability.
- ✓ 1.E+5 to 4.E+5 Degrees of freedom (DOF)
- ✓ Parallel Direct Solver (PARDISO) for small dimension problems; iterative BiCGstab solver coupled with geometric multigrid otherwise.
- ✓ In most cases between 4 and 8 iteration steps with the non linear solver for the solution to converge.
- ✓ 4-processor workstation with 16 GB of shared RAM for computations.
- ✓ 5 minutes to 1 hour computation times.

Results. Streamlines and velocity field



Rhombohedric unit pore cell: inlet pressure at the bottom and zero pressure boundary at outlet faces.

(a) 1 Pa **(b) 5 Pa**

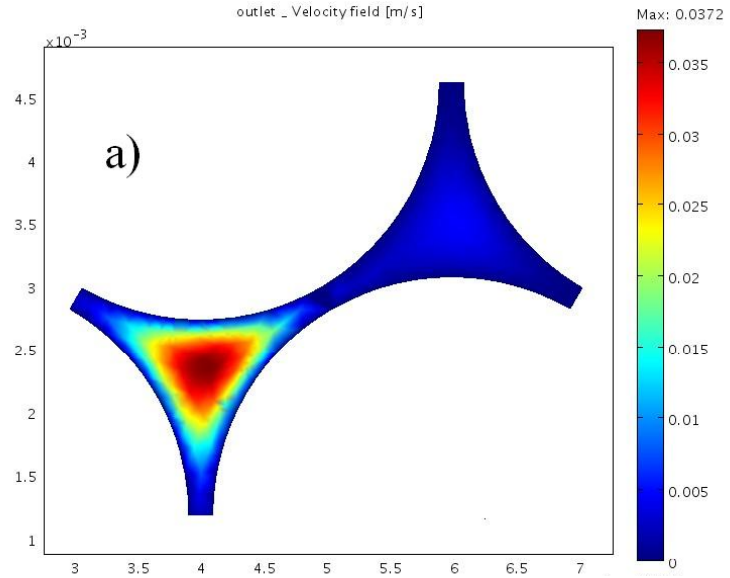
Higher velocities can be observed close to grain surfaces, while flow velocities between spheres remain relatively low.

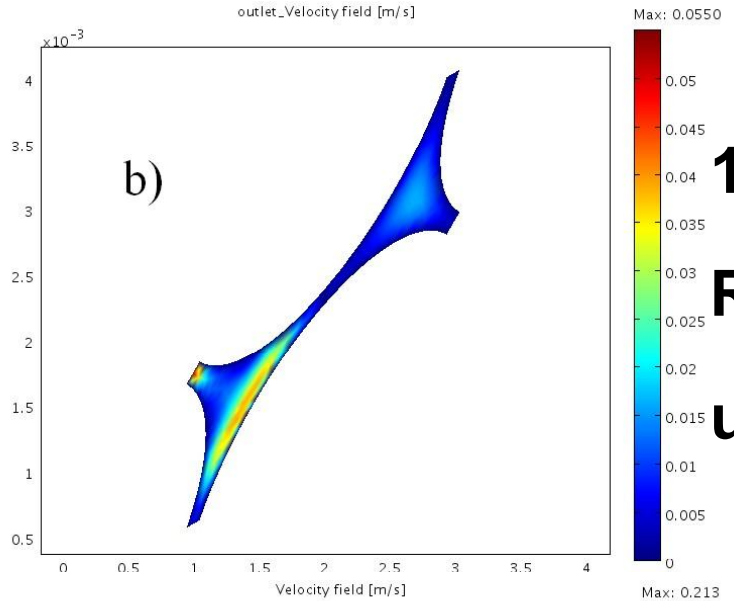
Streamlines reach more distant regions, hence highest velocities occur further away from the solid pore walls.

Evidence of preferred flow paths on the left side of the cell. This is due to the rhombohedric geometry of the pore space with more accessible volume towards the left (path of least resistance).

Results. Pore scale Re and velocity fields

Outlet faces

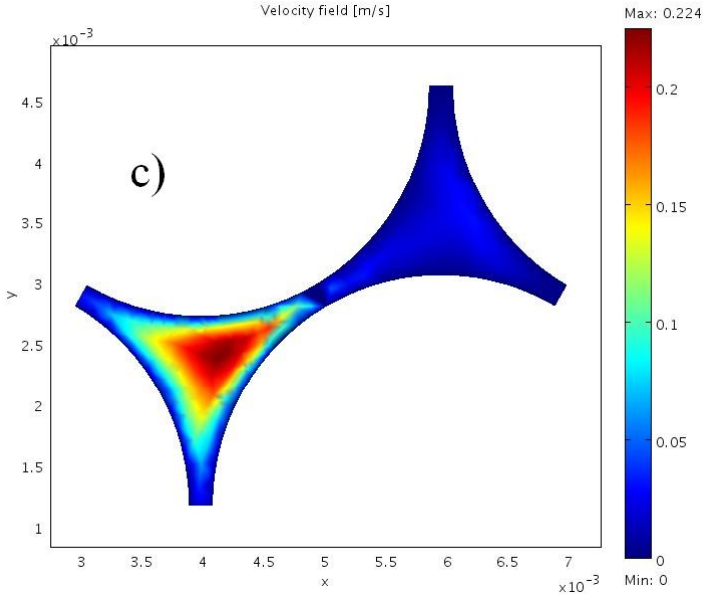


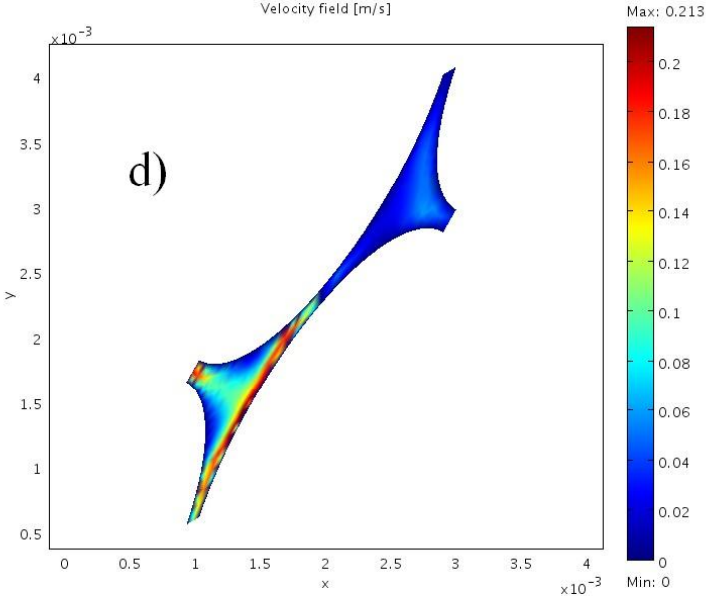


1 Pa

Re_{ps}=10

u_{max}=0.04





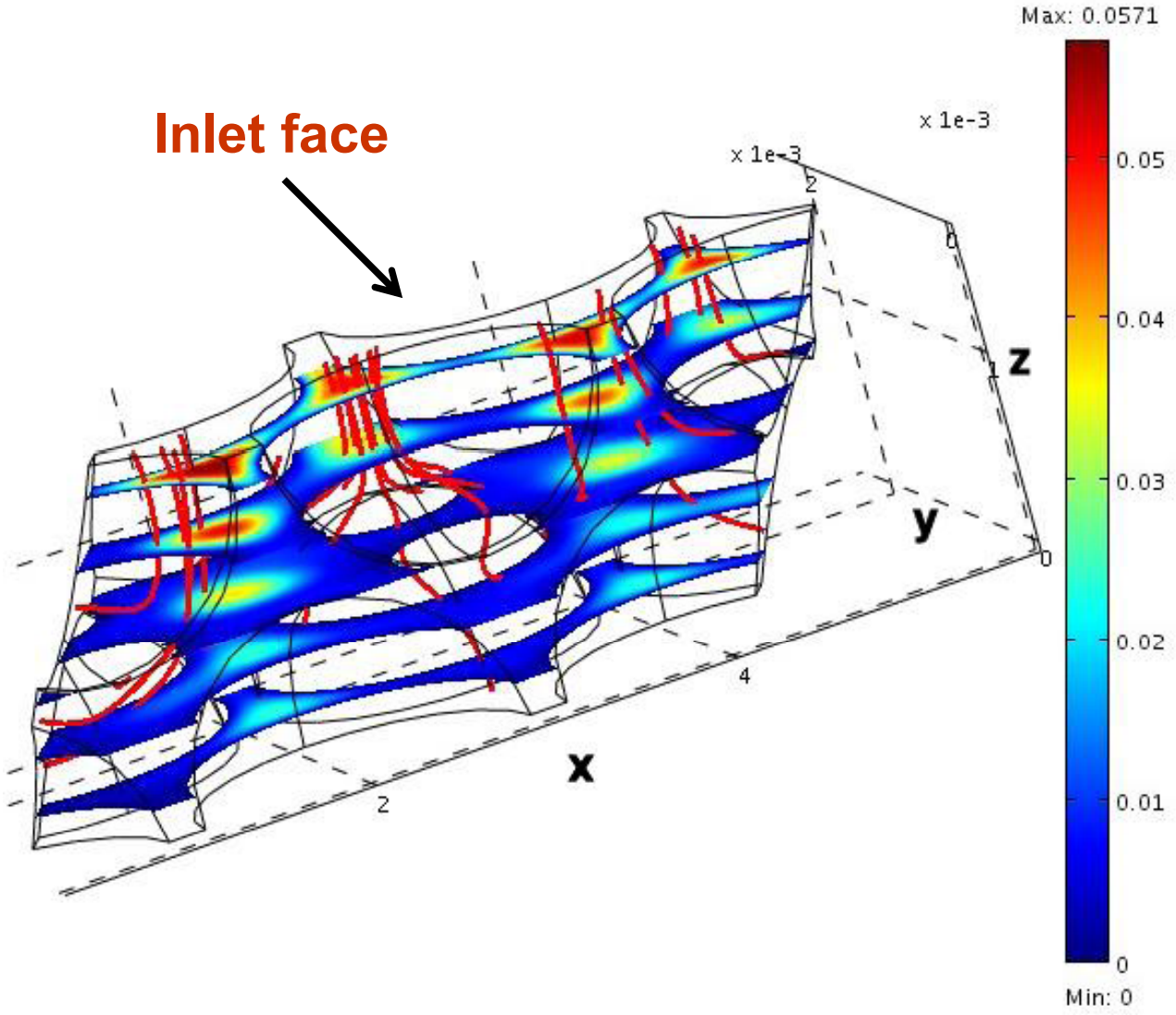
5 Pa

Re_{ps}=50

u_{max}=0.2

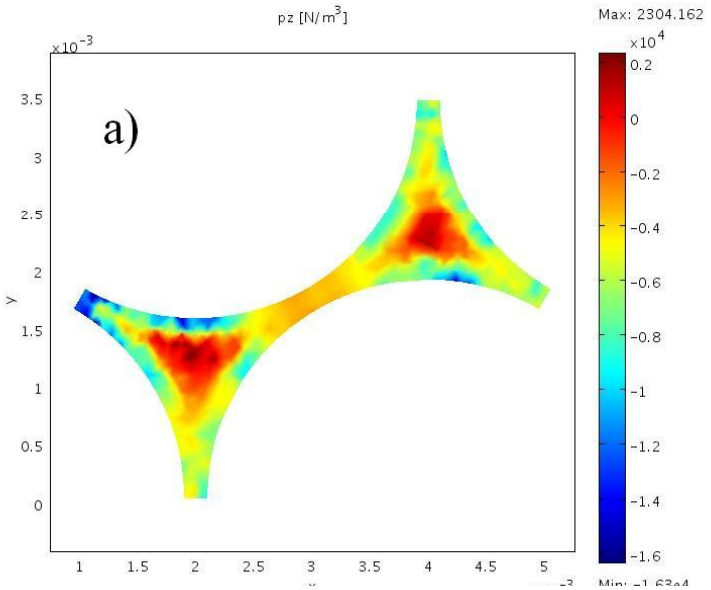
Results. Pore scale Re and velocity fields

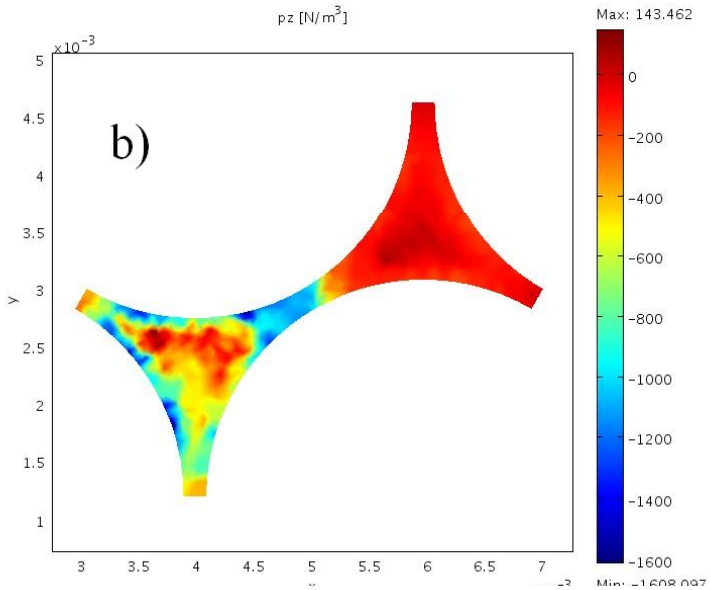
Two joint rhombohedric cells

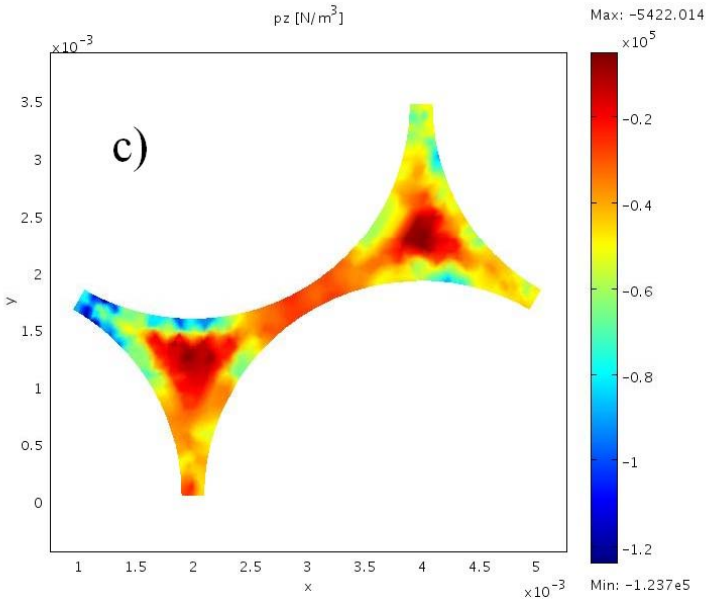


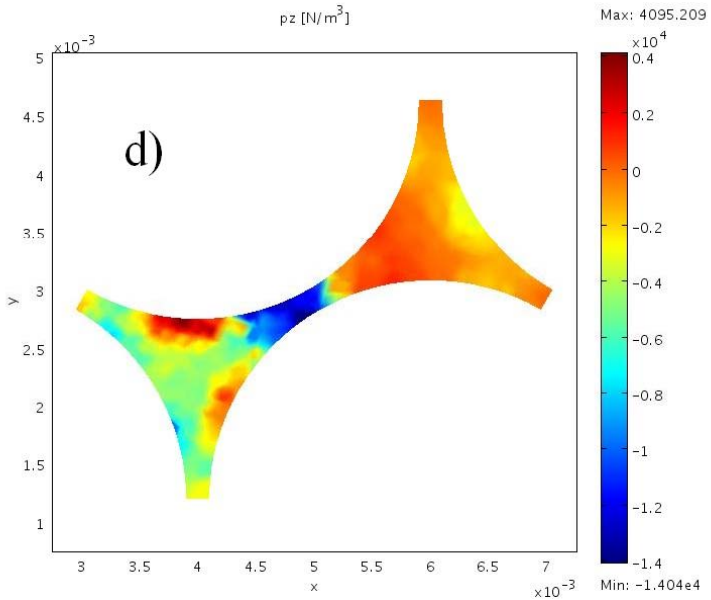
Even for this case, velocity remains zero just on one side of the outlet face

Results. Pressure gradients









Re_{ps}=10

Inlet (a)

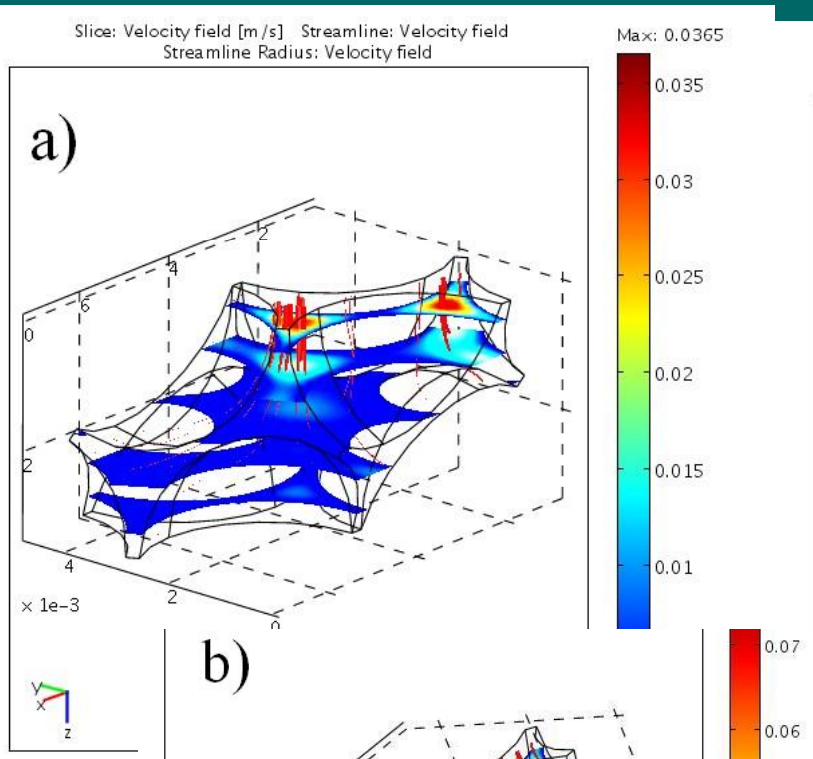
Outlet (b)

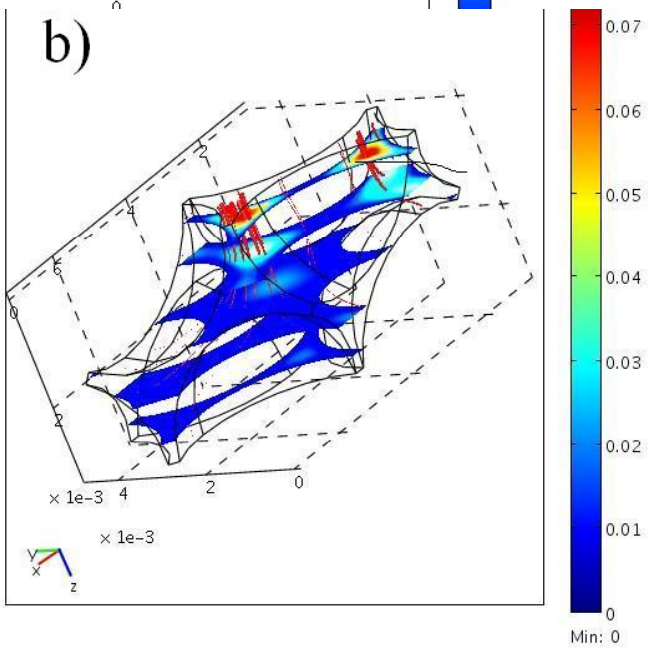
Re_{ps}=50

Inlet (c)

Outlet (d)

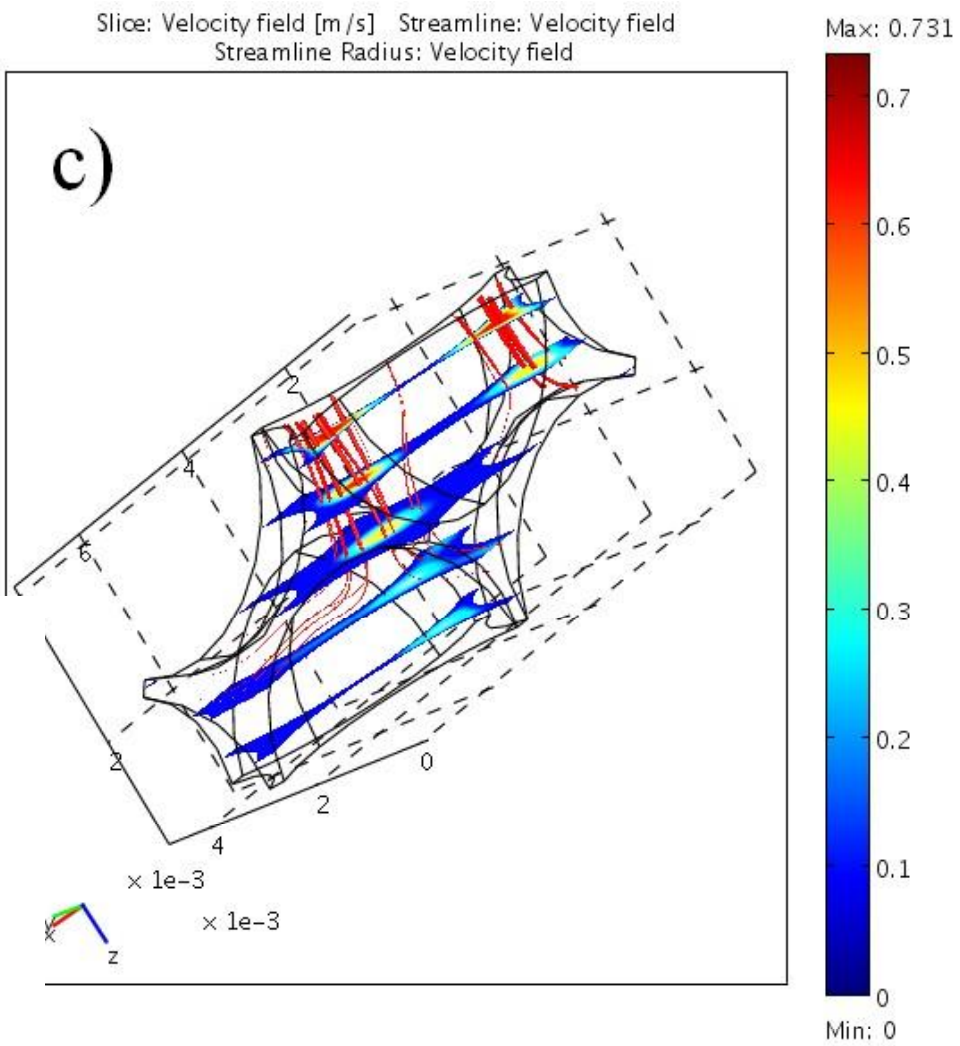
Results. Case of Oil





(a) 5 Pa

(b) 10 Pa



(c) 100 Pa

Conclusions

- proportionality between gradients and velocities for pore-scale Reynolds numbers ranging from 10 to 50, indicating laminar flow behavior.

- preferred flow paths within rhombohedric pores. The presence of a boundary layer near the solid-liquid interface and the development of an inertial core in the center of the pore are visible only on one side of the outlet face. In addition, higher velocity (higher Re) is confirmed when the pore-neck and the lateral faces are reached.

While for all simulated cases apparent streamline distortion points to inapplicability of Darcy's law, application of incompressible Navier-Stokes equations seems to realistically represent flow phenomena in idealized pore space.