FSI Analysis of Microcantilevers Vibrating in Fluid Environment

A. Ricci, E. Giuri

Materials and Microsystems Laboratory

Politecnico di Torino
Outline

• Brief Presentation of Materials and Microsystems Laboratory

• Fluid Structure Interaction Problem (FSI)
  - Analytical Model
  - FSI in Time Domain
  - FSI in Frequency Domain

• Results

• Conclusion and Future Works

• References
Materials and Microsystems Laboratory, is managed by Politecnico di Torino and works on the design and realization of micro and nano systems prototypes with a specific focus on technological transfer.

http://www.polito.it/micronanotech

MEMS simulation activity is required for the design of microstructures or for their performance prevision. F.E.M. Simulations of microstructures behaviour is carried out by Comsol Multiphysics™

European COMSOL Conference 2009
Fluid Structure Interaction (FSI) of Microcantilevers Vibrating in Fluid Environment for Biosensing Applications

Lab–on-Chip (LOC) for genomic and proteomic detection

Dynamic Measurement conducted evaluating

Q factors and Resonance Frequency
FSI Analysis of Microcantilevers Vibrating in Fluid Environment

A. Ricci, E. Giuri

FSI Problem

Analytical Model

Assumptions

- it is exact for a beam of infinite length vibrating in an incompressible viscous fluid
- thickness should be negligible compared to the length
- cantilever should have a constant cross section along the length
- modal cross talk is not taken in account

\[
\frac{\omega_{R,n}}{\omega_{\text{vac},n}} = \left[ 1 + \frac{\pi \rho b}{4 \rho_c h} \Gamma^f_r(\omega_{R,n}, n) \right]^{-1/2}
\]

\[
Q_n = \frac{(4 \rho_c h / \pi \rho b) + \Gamma^f_r(\omega_{R,n}, n)}{\Gamma^f_i(\omega_{R,n}, n)}
\]

FSI Analysis of Microcantilevers Vibrating in Fluid Environment

A. Ricci, E. Giuri

FSI Problem in Time Domain

- They hold only when length >> thickness and just for low mode numbers

2D Models:
- Time domain analysis
- Fitting step (possible source of inaccuracy)


FSI Analysis of Microcantilevers Vibrating in Fluid Environment

A. Ricci, E. Giuri

FSI Problem in Time Domain

3D Models:

- CFD-ACE+ Simulation
- ADINA Simulation


European COMSOL Conference 2009
FSI Analysis of Microcantilevers Vibrating in Fluid Environment

A. Ricci, E. Giuri

FSI Problem in Time Domain: drawbacks

- Initial displacement applied on the cantilever free end [Lee]
- Vacuum eigenfrequency analysis results are the input for the time dependent analysis in fluid environment [Basak]
- Time dependent analysis
- Data fitting and filtering steps (Prony analysis) possible sources of inaccuracy
- Modal cross talk in not taken into account
FSI Problem in Frequency Domain

Our method (Comsol Multiphysics) → Equations are written in frequency domain → An eigenvalue problem is obtained

Frequency Domain FSI Analysis vs Time Domain FSI Analysis

• On equal mesh density, an eigenfrequency analysis is certainly less time consuming than a time domain one

• The convergence study regards just the mesh density and not, as in the time domain approach, both mesh density and time parameters

• Mode shapes and frequency in fluid are directly calculated so that no curve fitting step is needed. A possible source of inaccuracy is therefore eliminated
Our method
(Comsol Multiphysics)

Equations are written in frequency domain

An eigenvalue problem is obtained

Equations in frequency domain

\[ \frac{\partial}{\partial t} = j \omega \]

Approximation 1
If the amplitude of structure vibration is far smaller than any other length scale the non-linear convective inertial term could be dropped.

Solid

\[ \nabla \cdot \sigma = -\rho_s \omega^2 \bar{u} \]

Fluid

\[ -\nabla p + \mu \nabla^2 \bar{v} - \bar{v} \nabla \bar{v} = \rho_f j \omega \bar{v} \]
\[ \nabla \cdot \bar{v} = 0 \]
FSI Analysis of Microcantilevers Vibrating in Fluid Environment
A. Ricci, E. Giuri

FSI Problem in Frequency Domain

Our method (Comsol Multiphysics)

Stokes equations are written in frequency domain

An eigenvalue problem is obtained

Approximation 2
Fluid vorticity, plays a significant role just in proximity of the vibrating structure; it is possible to further simplify Stokes equations in the region of fluid domain sufficiently far from the cantilever.

Equations in frequency domain

\[- \nabla p + \nabla^2 \vec{v} = \rho_f j \omega \vec{v} \]
\[\nabla \cdot \vec{v} = 0\]

\[\nabla^2 \phi = 0\]

\[\nabla^2 \vec{v} = 0\]

\[\vec{v} = \nabla \phi\]

\[\Phi = \text{Scalar Velocity Potential}\]
FSI Problem in Frequency Domain

Use of Comsol Multiphysics

Fluid Domain can be subdivided in:

**Fluid Far Field**

\[
\begin{align*}
\nabla^2 \phi &= 0 \\
p &= -j \omega \text{smsld} \cdot \rho f \phi \\
\tilde{v} &= \nabla \phi
\end{align*}
\]

**Fluid Near Field**

\[
\begin{align*}
-\nabla p + \mu \nabla^2 \tilde{v} &= -j \omega \text{smsld} \cdot \rho f \tilde{v} \\
\nabla \cdot \tilde{v} &= 0
\end{align*}
\]

**Solid - Cantilever**

\[
\nabla \cdot \tilde{\sigma} = (j \omega \text{smsld})^2 \cdot \rho_s \tilde{u}
\]

\(\Phi\) is the only dependent variable of this subdomain.

European COMSOL Conference 2009
FSI Analysis of Microcantilevers Vibrating in Fluid Environment

A. Ricci, E. Giuri

FSI Problem in Frequency Domain

Boundary Conditions

The Model contains two type of interfaces

\[ p = p_{irr} \]
\[ \frac{\partial \phi}{\partial n} = \bar{\nu} \cdot \bar{n} \]
\[ \bar{\nu} = j \omega \bar{u} \]
\[ f_{ext}^{sm} = \bar{T}_{mm}^{glf} \]

Near Field – Far Field

Solid – Near Field

\[ \bar{u} = 0 \]

Clamp boundary condition is assigned to the fixed end of the cantilever.

Dirichlet boundary condition is assigned external surface of the fluid far field to mimics an “open” condition since the value of the pressure is constrained to zero.
FSI Analysis of Microcantilevers Vibrating in Fluid Environment

A. Ricci, E. Giuri

FSI Problem in Frequency Domain

Symmetry Conditions

Since the model is symmetrical with respect to xz plane, symmetry conditions are required both for the solid and the fluid domains.

\[ u_y = 0 \]

\[ \begin{align*}
  v_y &= 0 \\
  T(x,z) &= 0 \\
  \frac{\partial \phi}{\partial n} &= 0
\end{align*} \]

Solid - Cantilever
Fluid Near Field
Fluid Far Field
**FSI Analysis of Microcantilevers Vibrating in Fluid Environment**

A. Ricci, E. Giuri

**FSI Problem in Frequency Domain**

Our method (Comsol Multiphysics) - Stokes equations are written in frequency domain

\[
Q_{\text{fluid}} = \left| \frac{\Im(\lambda)}{2\Re(\lambda)} \right| \quad f_{\text{fluid}} = \left| \frac{\Im(\lambda)}{2\pi} \right|
\]

\(\lambda\) is a complex eigenvalue representing a complex angular frequency.

European COMSOL Conference 2009
Results: benchmark with the analytical model

**Results: model validation**

<table>
<thead>
<tr>
<th>mode n.</th>
<th>$f^e$ data (KHz)</th>
<th>err%</th>
<th>$f^a$ data (KHz)</th>
<th>err%</th>
<th>$f^B$ data (KHz)</th>
<th>err%</th>
<th>$f^{pw1}$ data (KHz)</th>
<th>err%</th>
<th>$f^{pw2}$ data (KHz)</th>
<th>err%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>69.87</td>
<td>-</td>
<td>70.61</td>
<td>1.06</td>
<td>70.49</td>
<td>0.89</td>
<td>70.49</td>
<td>0.89</td>
<td>69.52</td>
<td>-0.50</td>
</tr>
<tr>
<td>2</td>
<td>438.5</td>
<td>-</td>
<td>443.50</td>
<td>1.14</td>
<td>441.6</td>
<td>0.71</td>
<td>442.45</td>
<td>0.90</td>
<td>436.23</td>
<td>-0.52</td>
</tr>
</tbody>
</table>

**Table 1a.** Comparison between experimental (e) [9, 14], analytical (a) [16] and computational (B “Basak et al. [9]”, pw1, 2 “present work”) results about the first two mode resonance frequencies in air environment of cantilever C2 [9, 14].

<table>
<thead>
<tr>
<th>mode n.</th>
<th>$Q^e$ data</th>
<th>err%</th>
<th>$Q^a$ data</th>
<th>err%</th>
<th>$Q^B$ data</th>
<th>err%</th>
<th>$Q^{pw1}$ data</th>
<th>err%</th>
<th>$Q^{pw2}$ data</th>
<th>err%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>136</td>
<td>-</td>
<td>130.7</td>
<td>-3.89</td>
<td>144.8</td>
<td>6.47</td>
<td>131.4</td>
<td>-3.38</td>
<td>130.4</td>
<td>-4.12</td>
</tr>
<tr>
<td>2</td>
<td>395</td>
<td>-</td>
<td>396.8</td>
<td>0.45</td>
<td>367</td>
<td>-7.09</td>
<td>397.7</td>
<td>0.68</td>
<td>394.4</td>
<td>-0.15</td>
</tr>
</tbody>
</table>

**Table 1b.** Comparison between experimental (e) [9, 14], analytical (a) [16] and computational (B “Basak et al. [9]”, pw1, 2 “present work”) results about the first two mode $Q$ factors in air environment of cantilever C2 [9, 14].
**FSI Analysis of Microcantilevers Vibrating in Fluid Environment**

**A. Ricci, E. Giuri**

**Results: squeeze film damping simulation**

![Image](image-url)

*Figure 9. Detail of the 3D FSI model about a cantilever vibrating near a surface at distance $g_0$ [15]. Only the near field is showed.*

<table>
<thead>
<tr>
<th>$Q_{air}^{e}$</th>
<th>$Q_{air}^{a}$</th>
<th>$Q_{air}^{s1}$</th>
<th>$Q_{air}^{s2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>data</td>
<td>data</td>
<td>data</td>
</tr>
<tr>
<td>5.7</td>
<td>6.0</td>
<td>5.1</td>
<td>5.5</td>
</tr>
<tr>
<td>6.0</td>
<td>5.1</td>
<td>5.5</td>
<td>6.0</td>
</tr>
<tr>
<td>5.5</td>
<td>6.0</td>
<td>5.1</td>
<td>5.5</td>
</tr>
<tr>
<td>-3.0</td>
<td>-5.0</td>
<td>-6.0</td>
<td>-3.0</td>
</tr>
<tr>
<td>12.7</td>
<td>12.7</td>
<td>12.7</td>
<td>12.7</td>
</tr>
<tr>
<td>123.81</td>
<td>123.81</td>
<td>123.81</td>
<td>123.81</td>
</tr>
</tbody>
</table>

*Table 2b. Comparison between experimental (e) [15], analytical (a) [18] and computational results (subdivided in the ones calculates through the full 3D FSI model, “s1” superscript, and those obtained by the “Solid, stress-strain with film Damping” application mode, “s2” superscript) about the first mode Q factor of cantilever A [15].*

<table>
<thead>
<tr>
<th>$f_{vac}^{e}$</th>
<th>$\text{shift}_{air}^{e}$</th>
<th>$f_{vac}^{a}$</th>
<th>$\text{shift}_{air}^{a}$</th>
<th>$f_{vac}^{s1}$</th>
<th>$\text{shift}_{air}^{s1}$</th>
<th>$f_{vac}^{s2}$</th>
<th>$\text{shift}_{air}^{s2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>data</td>
<td>data</td>
<td>data</td>
<td>data</td>
<td>data</td>
<td>data</td>
<td>data</td>
</tr>
<tr>
<td>(KHz)</td>
<td>(KHz)</td>
<td>err %</td>
<td>data</td>
<td>err %</td>
<td>data</td>
<td>err %</td>
<td>data</td>
</tr>
<tr>
<td>18.33</td>
<td>-2.10</td>
<td>18.45</td>
<td>0.68</td>
<td>-0.74</td>
<td>-64.91</td>
<td>18.54</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Table 2a. Comparison between experimental (e) [15], analytical (a) [18] and computational results (subdivided in the ones calculates through the full 3D FSI model, “s1” superscript, and those obtained by the “Solid, stress-strain with film Damping” application mode, “s2” superscript) about the first mode resonance frequency of cantilever A [15].*
Conclusions

**Frequency Domain Approach**

\[ \frac{\partial}{\partial t} = j \omega \]

**Subdivision of the fluid domain in a Near and a Far Field**

**Strong reduction of the computation time**

+ **high degree of accuracy of results**

European COMSOL Conference 2009
Future Works

Design and optimization of a fluid cell containing a vibrating **Cantilever Plate**

- Eigenfrequency Analyses in Fluid Environment
- Frequency Response Analyses in Fluid Environment with Magnetic Excitation
FSI Analysis of Microcantilevers Vibrating in Fluid Environment

A. Ricci, E. Giuri

References

13. Comsol Multiphysics 3.5a Manuals
Thanks For Your Attention
FSI Analysis of Microcantilevers Vibrating in Fluid Environment

A. Ricci, E. Giuri

FSI Problem in Frequency Domain

Domain Optimization

Near Field