Fast 2D Simulation of Superconductors: a Multiscale Approach

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Abstract: This work presents a method to calculate AC losses in thin conductors such as the commercially available second generation superconducting wires through a multiscale meshing technique. The main idea is to use large aspect ratio elements to accurately simulate thin material layers. For a single thin superconductor, several standard test cases are simulated including transport current, externally applied magnetic field and a combination of both. The results are in good agreement with recently published numerical simulations and show even faster time to solution. A couple of examples involving several thin conductors in different circumstances are also shown.

Keywords: Superconductors, Multiscale Methods, AC-losses, Mapped Mesh, Coupling Variables.

1. Introduction

The increasing use of superconducting coated conductors and the recent introduction of High Power superconducting machines (1) (2) (3), demand a fast and reliable tool to simulate the performance of electromagnetic devices with superconducting components.

One of the biggest issues regarding 2D simulation of AC loses in superconductors for various power applications is the substantial invested time to obtain realistic results. This includes both the actual computing time and, more importantly, the time spent modeling every single application. Therefore, necessary steps must be taken to reduce both contributors to a minimum and make simulation of the electromagnetic properties of superconductors as easy and fast as possible.

In the last years, several approaches have been proposed to address this problem relying on the use of the finite element simulations a few are found in (4) (5) (6) (7) (8) and references therein. However, all PDE based formulations are still limited by the expensive computation of superconducting materials and the increased number of elements in the case of thin conductors such as the second generation superconducting wires that contain a micron thick superconducting layer.

Here, we present a way to simulate superconductors in a timely effective manner. The simulation is based on previously published models (5) (8) and an existing implementation using COMSOL (9). Use of a multiscale approach allows creating custom made applications rapidly and with ease.

2. Governing Equations

Following a formulation originally proposed in (8), the magnetic field strength is calculated by solving:

$$\nabla \times (\rho \nabla \times \boldsymbol{H}) = -\mu \frac{\partial}{\partial t} \boldsymbol{H}$$

here, ρ is constant for regular conductors but for superconductors:

$$\rho = \frac{E_c}{J_c} \left(\frac{|J|}{J_c} \right)^{n-1}$$

where $\boldsymbol{J} = \nabla \times \boldsymbol{H}$ is the current density, μ is the permeability of the material, E_c is the electric field strength of 1 µV/cm obtained when the current density reaches the critical current density J_c and n characterizes how abruptly the transition from the superconducting to normal state is taking place. Coated conductors are generally anisotropic with $J_c(\theta, |B|)$ and $n(\theta, |B|)$ depending on the angle between the conductor and the magnetic flux density, but here they are assumed isotropic and constant. Finally, following a procedure similar to the one described in (5), an initial condition $\boldsymbol{H} = \boldsymbol{0}$ at $t = 0$ is used to satisfy the uniqueness of the solution $\boldsymbol{H}$ for $t > 0$.

For the case where no external field is applied, Neumann boundary conditions are used in a large enough domain. Dirichlet boundary conditions are used to include the contribution of external magnetic fields. Also, in the case of several conductors, integral constrains are utilized to ensure that currents are limited to their respective conductors. Finally, the AC losses per unit length for a periodic current signal can be calculated following:

$$Q = \oint dt \int \boldsymbol{E} \cdot \boldsymbol{J} \, dS$$

3. Use of COMSOL Multiphysics

All computations take place inside a domain at least 10 times larger than the maximum length of the conductor or array of conductors under consideration.

Recalling the resistivity dependence upon the current density for superconducting materials, it is important to point out that this dependence on the current density, makes it impossible to use COMSOL's AC/DC module, otherwise a circular reference would be created for $\boldsymbol{J}$. Therefore the PDE mode application is used to simulate the above described model:

$$e_a \frac{\partial^2 \mathbf{u}}{\partial t^2} + d_a \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \Gamma = F$$

where:

$$\mathbf{u} = \boldsymbol{H} = \begin{bmatrix} H_x \\ H_y \end{bmatrix},$$
$$e_a = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$d_a = \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix},$$
$$\Gamma = \rho \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \text{and}$$
$$F = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For regular conductors ρ is a positive constant, but for superconducting materials the following relation holds:

$$\rho = \frac{E_c}{J_c} \left(\frac{\left| \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right|}{J_c} \right)^{n-1} \quad \text{and}$$

The initial conditions are as follows:
$$\boldsymbol{H}|_{t=0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

As described in the previous section, for the case of zero external applied Magnetic Field, Neumann boundary conditions can be used. In the case of externally applied field, Dirichlet boundary conditions are applied, such that at the boundary of the overall domain the Magnetic field equals the sum of both self and external contributions. The self contributions $\boldsymbol{H_s} = \left[H_{sx}, H_{sy} \right]$ are calculated using Ampere's law. Whereas the external contributions are simply put as $\boldsymbol{H_e} = \left[H_{ex}, H_{ey} \right]$. The boundary conditions are then as follows:

$$\boldsymbol{H}|_{\partial\Omega} = \begin{bmatrix} H_{sx} + H_{ex} \\ H_{sy} + H_{ey} \end{bmatrix}$$

For the case of conductors carrying externally applied electric currents and following (8), integral constrains are used. An integration coupling variable is defined in every conductor to obtain an expression for the total current passing through, the kth conductor C_k:

$$I_k = \int_{C_k} J_z dS$$

An integral constrain is then defined at a given point (it can also be edge or subdomain) to force a net current.

The most important original contribution of this work refers to the use of structured meshes or mapped meshes as referred to in COMSOL. The most significant drawback in using free meshes for the formulation published in (4-8) is the fact that for thin material layers (either normal or superconducting), the number of elements increases heavily with the growing aspect ratio of the layer. This is because the meshing algorithm builds elements with individual aspect ratio close to unity. Therefore individual elements "resemble" either equilateral triangles or squares. It is this large difference in aspect ratios that makes the number of elements increase heavily. In consequence, the number of degrees of freedom (DOF) is largely augmented and so is the computational time as it depends upon the square of the number of DOF.

This has large consequences for the simulation of second generation superconducting wires such as the commercially available 344 coated conductors manufactured by American Superconductor since the aspect ratio of the superconducting layer in such wires can be higher than 4000. It is important to notice that this big scale difference in the conductor width/thickness could be addressed with a multiscale mesh; such mesh elements should have an aspect ratio between 1 and the aspect ratio of the conductor itself. Although no complete study of the optimal aspect ratio for the mesh elements was carried out, aspect ratios in excess of 100 provided good results.

To cope with this multiscale problem, in this work, structured (mapped) meshes were used inside the thin layers, allowing 75 elements for the width and 2 (or 4) for the thickness of the layer. This gives a mesh element aspect ratio of 80 (or 160). As a mere exercise, as many as 20 elements were used for the thickness with no significant differences in the overall computed AC-losses (data not reported here).

Finally, AC losses are computed directly in COMSOL by means of an integrator i.e. adding one ODE as a global equation to calculate Q as defined in section 2 as follows:

$$\dot{\xi} = f\,P, \quad t \in \left(t_0, t_0 + \frac{1}{f}\right) \text{ and } \xi(t_0) = 0$$

where ξ is a dummy variable, f is the reciprocal of the period of the current signal (or external magnetic field), t_0 is an arbitrary time value such that the transitory behavior of the solutions has already passed and P is a subdomain integration coupling variable defined in the regions of interest as:

$$P = \int E_z J_z dS$$

Use of integrators has proven to be the most efficient way in terms of time as does not add any significant time to the simulation and requires no external postprocessing with its implied manual data export.

For the case of sinusoidal currents (or external magnetic field), only half a cycle is used in the integrator ξ to calculate the losses.

Unless otherwise stated all simulations were performed in complete domains without making symmetry simplifications. All times referred in the following sections correspond to calculations performed using the blade server of the Technical University of Denmark (DTU): dual quad-core Intel(R) Xeon(R) E5440 CPUs running at 2.83Ghz with 32GB memory available (2GB per processor). All simulations were carried out with the UMFPACK solver which essentially uses only one processor, however in practice; use of two processors increases the speed by 40%. No increase in speed was noticed when using more than two processors.

4. Results

Following the procedure mentioned above, the cases of transport current and applied external field were considered as in (10). AC losses were calculated for a conductor of width 3mm and thickness of 1μm. J_c and n were assigned values of 10^{10}A m^{-2} and 25 respectively. A finite resistivity of 1 Ω·m was used for the air surrounding the superconducting material. The permeability was assumed to be equal to its free space value. Finally, all signals were sinusoidal with frequency of 50 Hz. Figure 1 depicts the overall domain, along with its corresponding mesh.
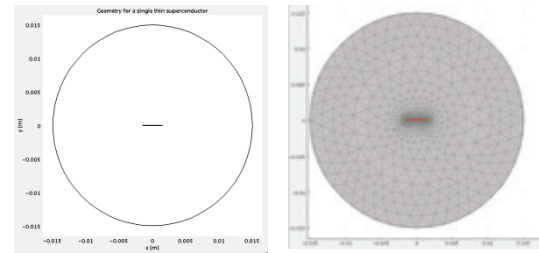

Figure 1. Thin superconductor surrounded by air (axes display longitudes in meters).

As described in the previous section, a mapped mesh was used in the superconducting region Figure 2 shows an overall picture of the mesh and some close ups.
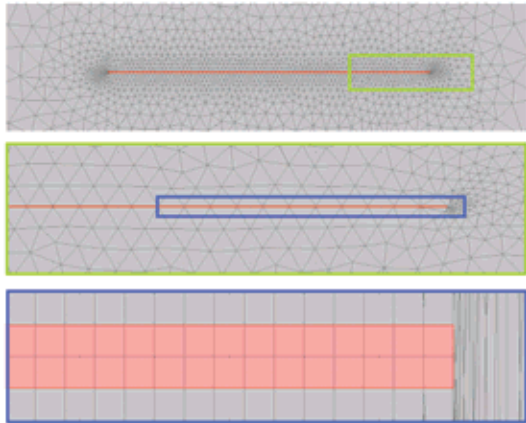
Figure 2. Mesh. The superconductor domain is colored in pink. The two upper figures show details of the mesh close to the superconducting region. The lower figure is deformed to allow the mesh elements in the superconductor to have an apparent aspect ratio of 1 rather than 80.

The instantaneous AC losses for a single tape carrying 90% of its critical current value are shown in Figure 3. Every single peak accounts for half a cycle of the original sinusoidal signal. Notice that after the first one all the following peaks are equal, thus the transitory effects after the first cycle can be disregarded. This allows to calculate the AC losses by simulating only one cycle.
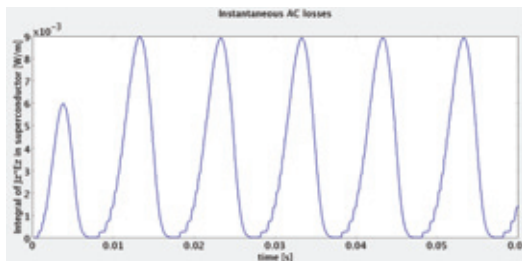
Figure 3. Instantaneous losses.

In order to validate the approach proposed in this work and its relevance, in Figures 4 and 5 the calculated results are compared with the ones published in the one that to the best of our knowledge, offered the fastest strategy to accurately calculate AC-losses in superconducting thin layers (10). Notice that our calculations are in good agreement with the aforementioned previously published numerical calculations.
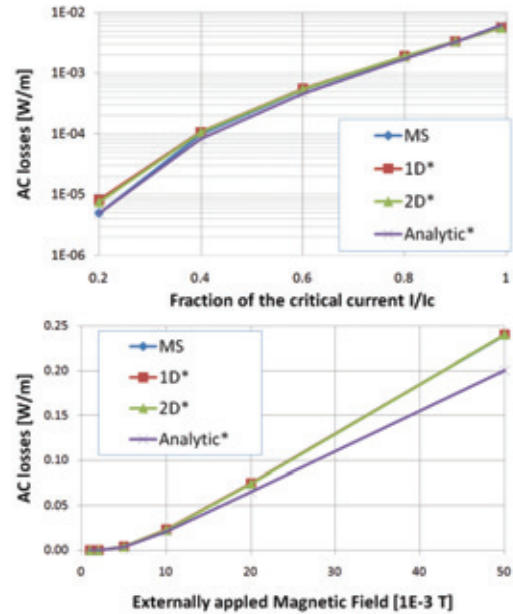

Figure 4. Comparison of calculated AC losses for different values of transport current (top) and for different values of the externally applied Magnetic Field (bottom). The "MS" curve refers to our calculations. * (10)

In the same way, computing time for the same publication is compared with our calculations in Figure 5. Here, it is important to notice that the use of mapped meshes allowed us to perform calculations 2 to 3 orders of magnitude faster than with regular meshing for a full 2-D simulation. Moreover, both our 2 and 4 elements calculations ran up to five times faster than the reported computing time for the 1D formulation proposed in (10).

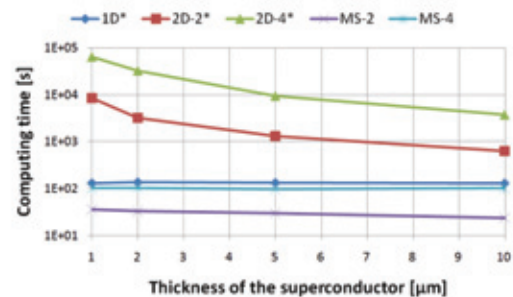Finally, in Figures 6 to 10, we present a few examples of computations involving several conductors.

Figure 5. Comparison of computing times. The "MS-

2(4)" curve refers to our calculations when 2(4) elements were used across the thickness. * (10)
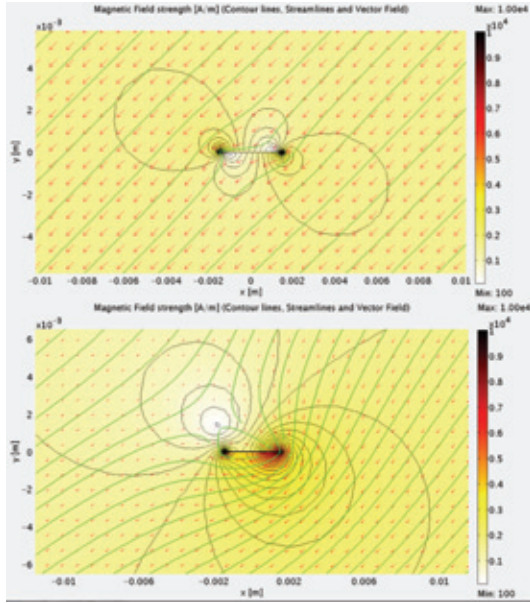

Figure 6. Comparison of the Magnetic Field Intensity for a thin superconductor when a sinusoidal Magnetic Field of 2 mT is applied (peak value shown). Both cases when no current (top) and 80% critical current is applied (bottom).
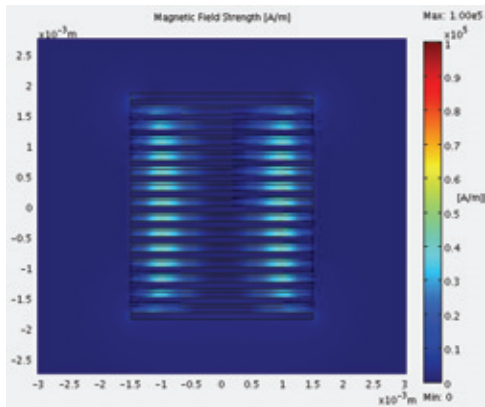

Figure 7. AC electric currents ($0.9Ic$, $50Hz$, in phase) are applied to a stack of 15 coated conductors. Magnetic field strength at 0 phase. For this particular application, a substrate with a relative permeability of 50 was considered.
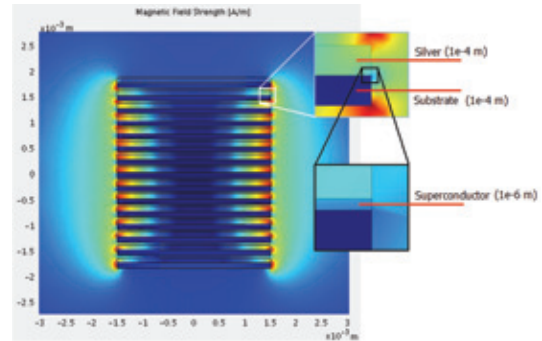

Figure 8. AC electric currents ($0.9Ic$, $50Hz$, in phase) are applied to a stack of 15 coated conductors. Magnetic field strength at peak value. As in Figure 7, a substrate with a relative permeability of 50 was considered. The insert shows the thickness of the different layers.
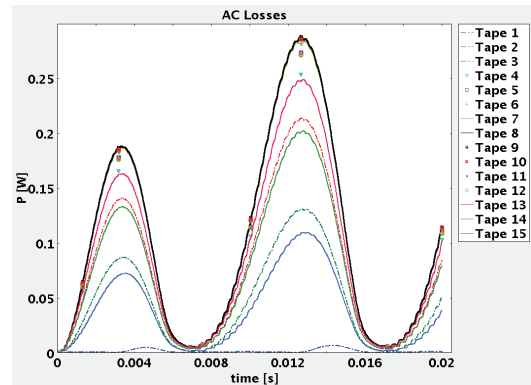

Figure 9. Instantaneous AC losses in the conductors depicted in Fig. 7 and 8 (enumerated from the top). Observe that the higher losses are experienced by the central conductors. Also, notice that the top conductors (tapes 1, 2 and 3) experience less loss than their bottom counterparts (tapes 13, 14 and 15).
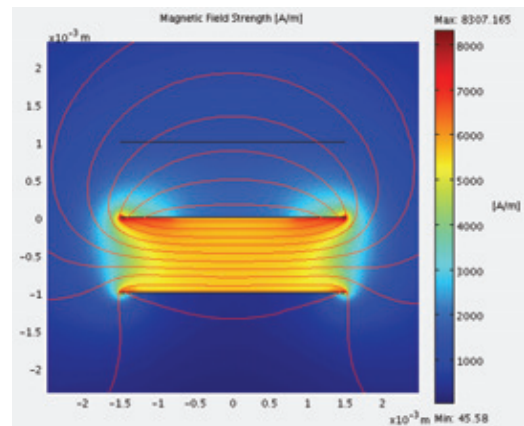

Figure 10. Interaction among several thin conductors. Top: Silver, no external current. Center: superconducting, imposed AC transport current ($0.9Ic$,

50Hz). Bottom: superconducting, no external transport current. Notice how the field is expelled from the bottom conductor.

The results shown in this work correspond to a full domain 2D simulation and no symmetry simplifications were used to reduce the computing time. Although all the calculations were performed in DTU's blade server as described in Section 3, a simple test calculation of AC losses in commercially available coated conductors was carried out in less than two minutes using a laptop.

5. Conclusions

Use of mapped meshes and specifically, use of large aspect ratio elements, provides a considerable increase in the computing speed for calculation of AC losses in superconductors. Numerical simulations were performed showing a decrease up to 3 orders of magnitude in the computing time when compared with other 2D simulation were no mapped meshes are used.

Furthermore, although it could be argued that the computer used in the present work is different than the one used in the reference, the proposed approach offers a method that can be used just like any other Multiphysics module in COMSOL, allowing to couple as many conductors as required in one single module. In this way, the time spent modeling every single application while using large aspect ratio elements does not depend heavily in the number of conductors and offers similar computing time than the 1D formulation. Therefore, the work presented here offers a faster time to solution strategy for calculating AC losses.

Finally, the ease to set a problem using the proposed formulation makes it possible to think of further applications such as superconducting coils, and superconducting induction machinery, among others in the near future.

6. References

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