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# Parameter Optimization for FEM based modeling of singlet oxygen during PDT using COMSOL

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# Outline

- Introduction
- Theory for PDT dosimetry model
- Optimization results
- PDT dosimetry quantity prediction for prostate using COMSOL
- Conclusions

# Outline

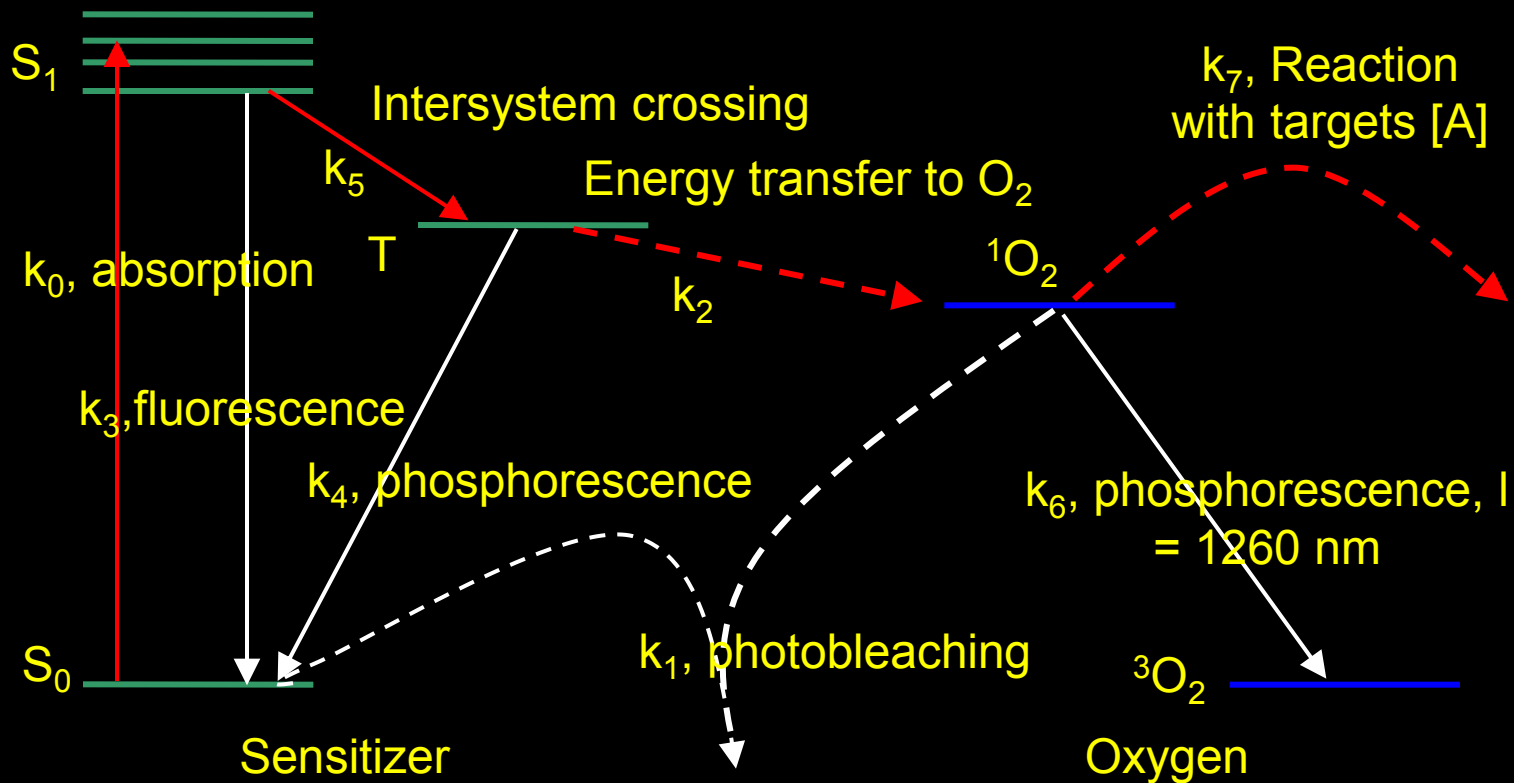
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# Introduction

- Photodynamic therapy (PDT) is an important treatment modality for cancer and other localized diseases.
- In PDT, **photosensitizers** excited by **light** react with ground state **oxygen**, which leads to generation of singlet oxygen - the major cytotoxic agent - to kill the surrounding tissues and cells.
- Compared with other treatment modalities, PDT has advantages including non-ionizing, localized photon delivery and better cosmetic outcome.

# Introduction

Jablonski Diagram for Type II PDT interaction



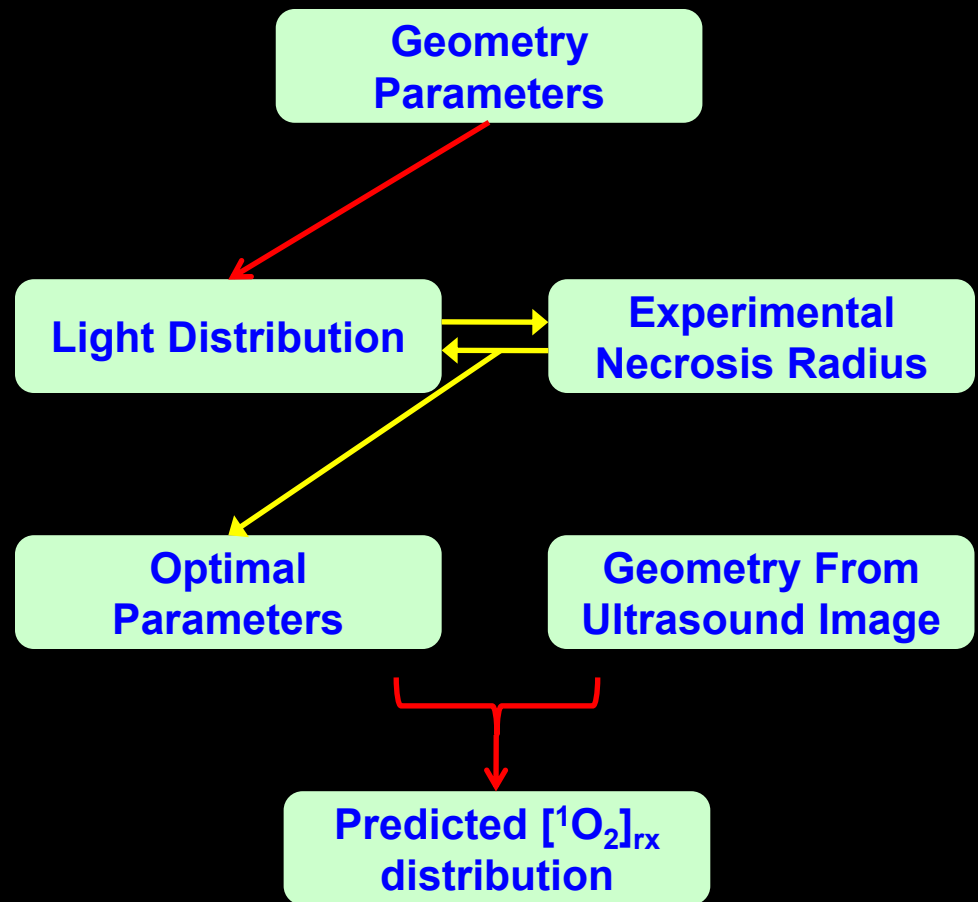
**Sensitizer (PS) + light + oxygen ( $^3O_2$ )  $\rightarrow$  singlet oxygen ( $^1O_2$ )**

# Introduction

Apparent reacted singlet oxygen  $[^1\text{O}_2]_{\text{rx}}$  was introduced as a PDT dosimetry quantity to better predict the PDT treatment outcome than PDT dose

→ By COMSOL

→ By COMSOL + MATLAB



Flow chart for PDT photophysiological parameter optimization and dosimetry prediction

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# Theory for PDT dosimetry model

Light  
diffusion  
equation

$$\mu_a \varphi - \nabla \cdot \left( \frac{1}{3\mu_s'} \nabla \varphi \right) = S$$



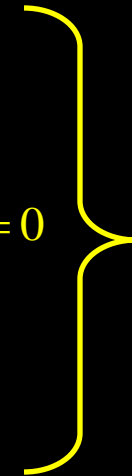
COMSOL

Photo  
chemical  
equations

$$\frac{d[S_0]}{dt} + \left( \xi \sigma \frac{\varphi([S_0] + \delta)[^3O_2]}{[^3O_2] + \beta} \right) [S_0] = 0$$

$$\frac{d[^3O_2]}{dt} + \left( \xi \frac{\varphi[S_0]}{[^3O_2] + \beta} \right) [^3O_2] - g \left( 1 - \frac{[^3O_2]}{[^3O_2](t=0)} \right) = 0$$

$$\frac{d[^1O_2]_{rx}}{dt} - \left( \xi \frac{\varphi[S_0][^3O_2]}{[^3O_2] + \beta} \right) = 0$$



MATLAB

$\varphi$  Light fluence rate

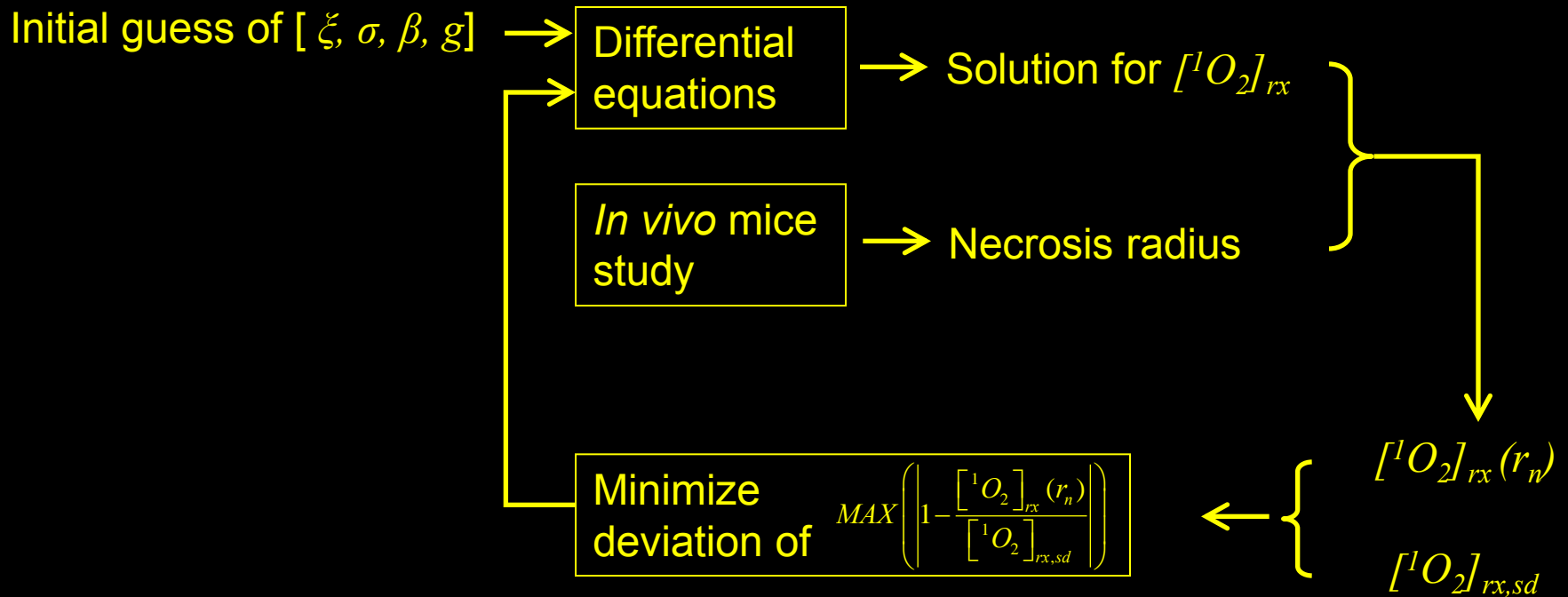
$[^3O_2]$  Ground triplet oxygen concentration

$[S_0]$  Ground sensitizer concentration

$[^1O_2]_{rx}$  Reacted singlet oxygen concentration



# Theory for optimization model

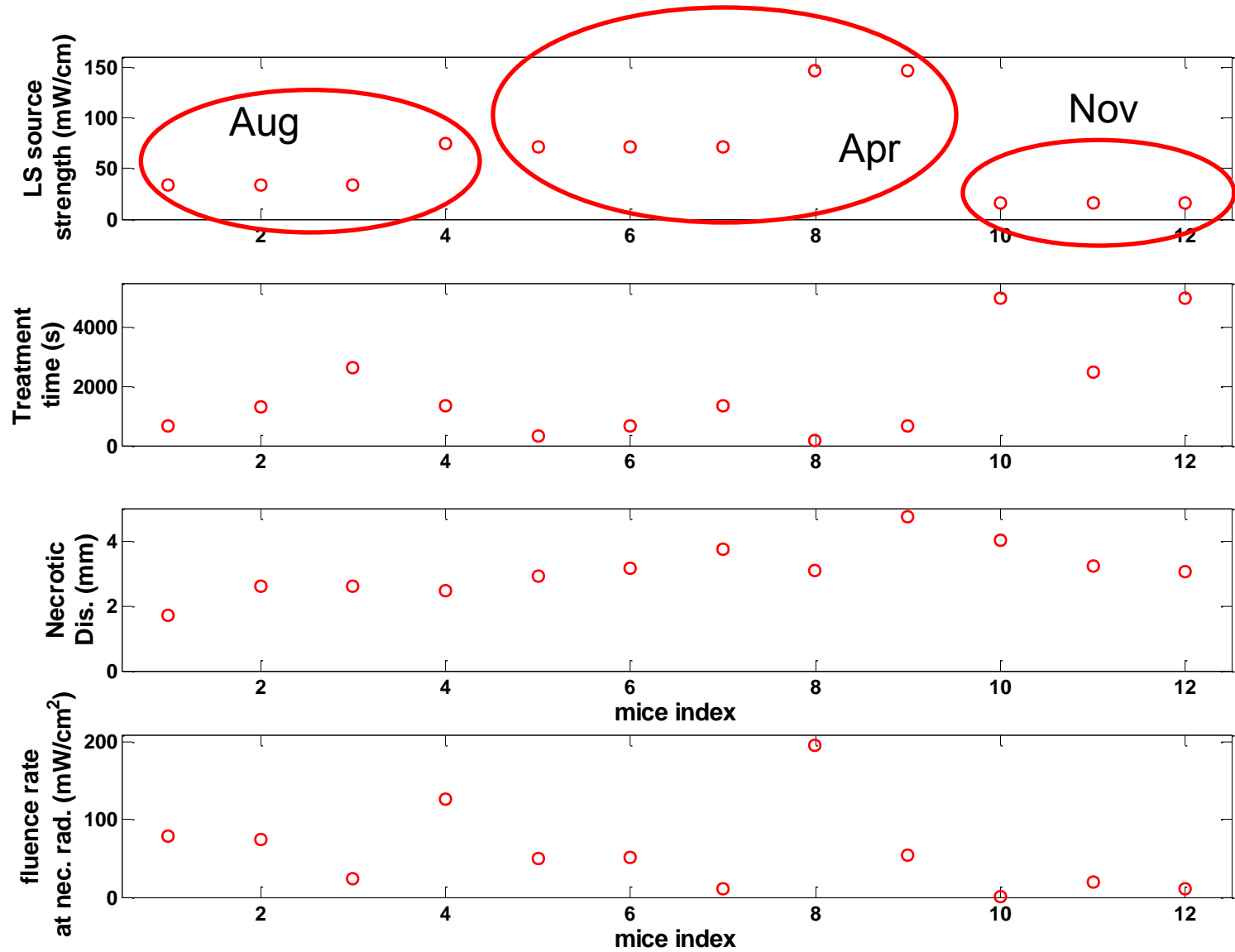


Fitting results  $[\xi, \sigma, \beta, g]$  and  $[^1O_2]_{rx}$

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# Optimization results



# Optimization results

Parameters	Final fit	Apr Fit	Apr and Aug Fit	Previous fit [1]	Published values
$\xi$ (cm <sup>2</sup> /s/mW)	$2.0 \times 10^{-3}$	$5.0 \times 10^{-3}$	$3.9 \times 10^{-3}$	$2.1 \times 10^{-3}$	$3.7 \times 10^{-3}$ [2]
$\sigma$ (1/ $\mu$ M)	$11.2 \times 10^{-5}$	$6.6 \times 10^{-5}$	$11.5 \times 10^{-5}$	$7.6 \times 10^{-3}$	$7.6 \times 10^{-3}$ [2]
$\beta$ ( $\mu$ M)	<b>11.9</b>	11.9	11.9	11.9	11.9 [3]
$g$ ( $\mu$ M/s)	<b>0.8</b>	0.62	0.56	0.69	—
$[^1O_2]_{rx,sh}$ (mM)	<b>0.41</b>	0.46	0.41	0.74	—

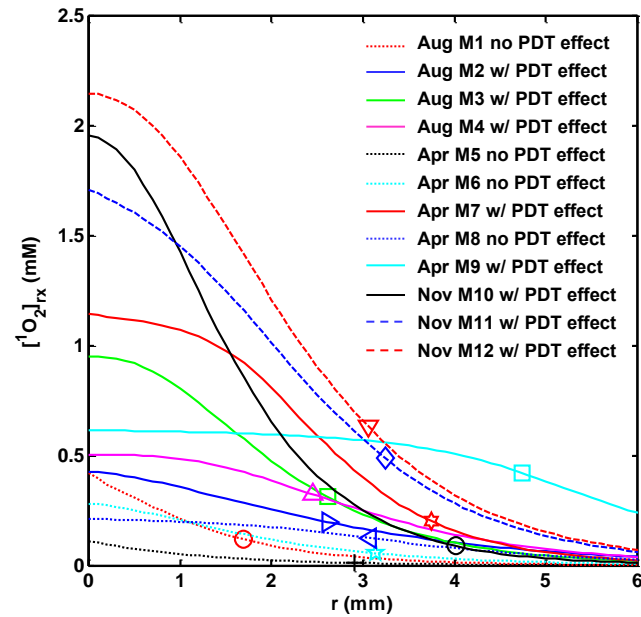
[1] Wang et al., J. Biophoton, 2010.

[2] Mitra et al., Photochem. Photobiol, 2005.

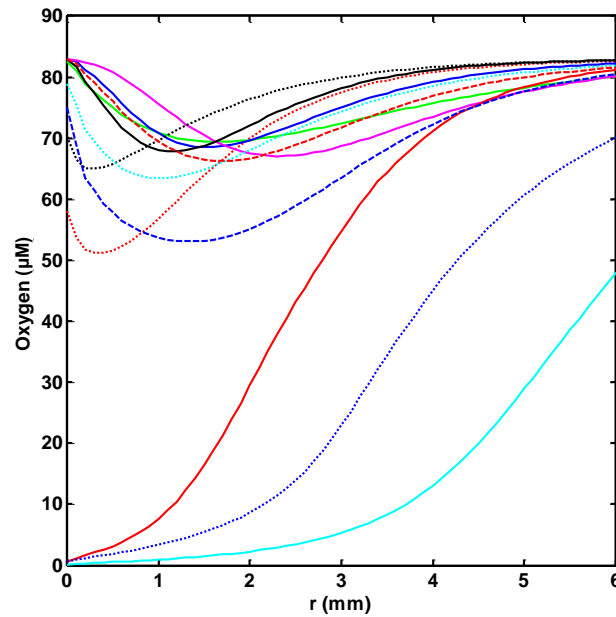
[3] Georgakoudi et al., Photochem. Photobiol. 1997.

# Optimization results

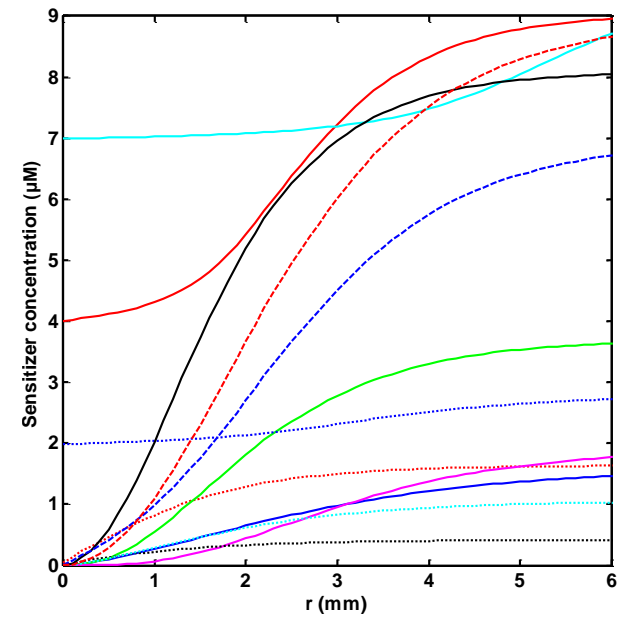
Final fitting results including experimental data from All data



Apparent reacted singlet oxygen concentration



Oxygen concentration



Photosensitizer concentration

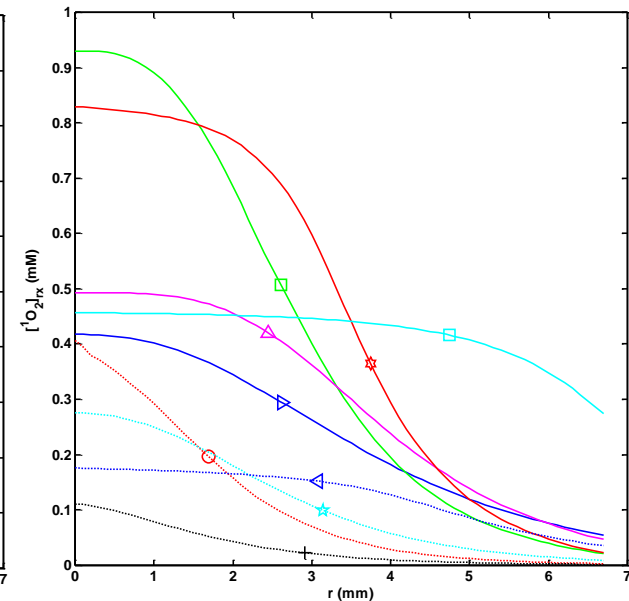
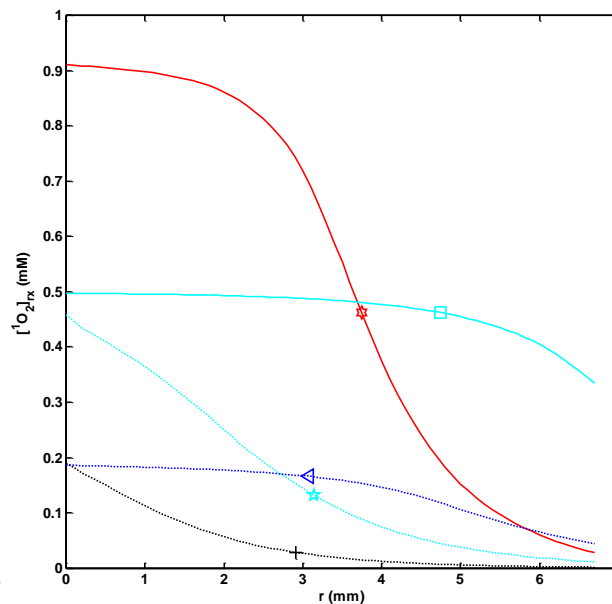
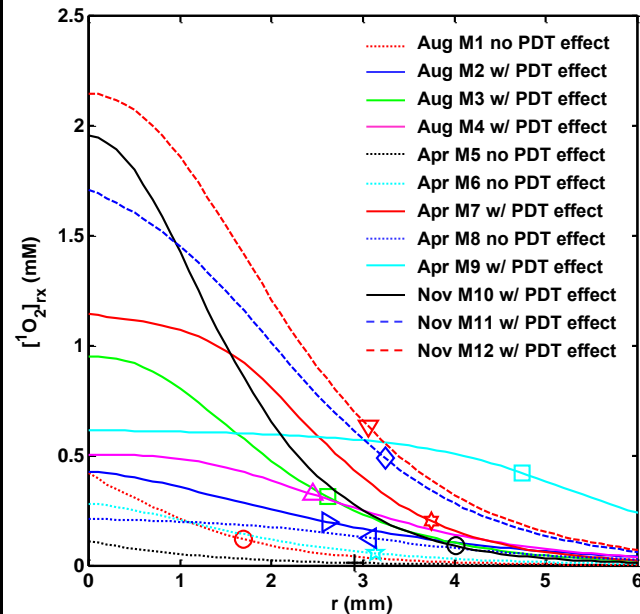
# Optimization results

Fitting results: apparent reacted singlet oxygen concentration

All data

April data

April and August data



$g = 0.8 \mu\text{M/s}$   
 $\xi = 2.0 \times 10^{-3} \text{ cm}^2\text{mW}^{-1}\text{s}^{-1}$   
 $\sigma = 11.2 \times 10^{-5} \mu\text{M}^{-1}$   
 $[^1\text{O}_2]_{\text{rx,sh}} = 0.41 \text{ mM}$

$g = 0.62 \mu\text{M/s}$   
 $\xi = 5.0 \times 10^{-3} \text{ cm}^2\text{mW}^{-1}\text{s}^{-1}$   
 $\sigma = 6.6 \times 10^{-5} \mu\text{M}^{-1}$   
 $[^1\text{O}_2]_{\text{rx,sh}} = 0.46 \text{ mM}$

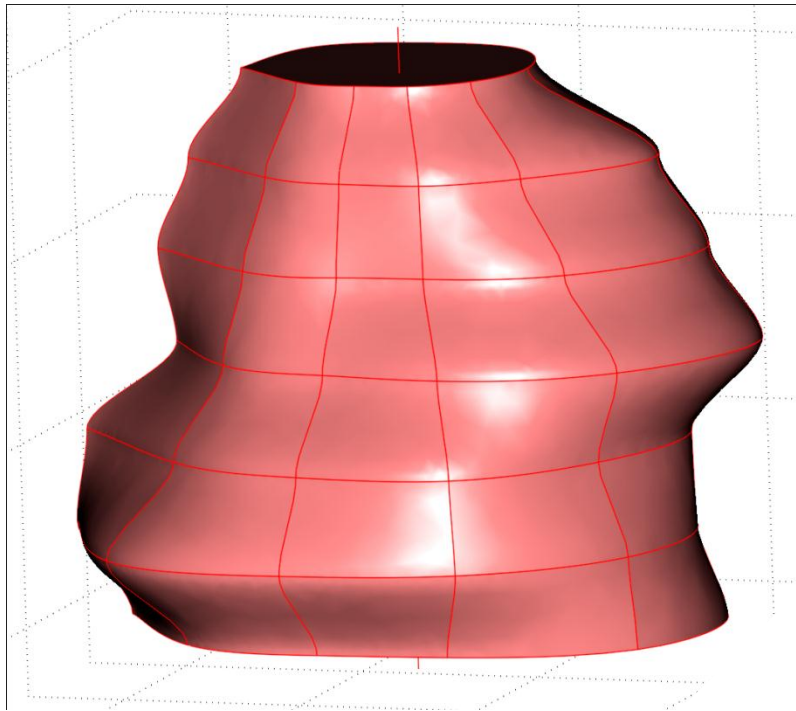
$g = 0.56 \mu\text{M/s}$   
 $\xi = 3.9 \times 10^{-3} \text{ cm}^2\text{mW}^{-1}\text{s}^{-1}$   
 $\sigma = 11.5 \times 10^{-5} \mu\text{M}^{-1}$   
 $[^1\text{O}_2]_{\text{rx,sh}} = 0.41 \text{ mM}$

# Outline

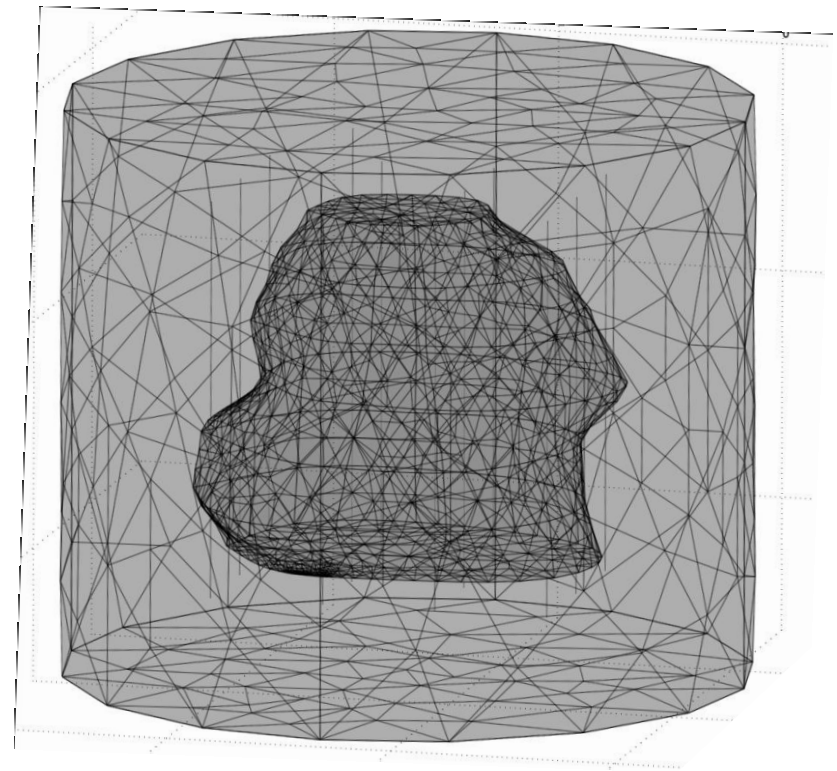
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# $[^1\text{O}_2]_{\text{rx}}$ prediction using COMSOL

Prostate geometry for prediction model



Geometry

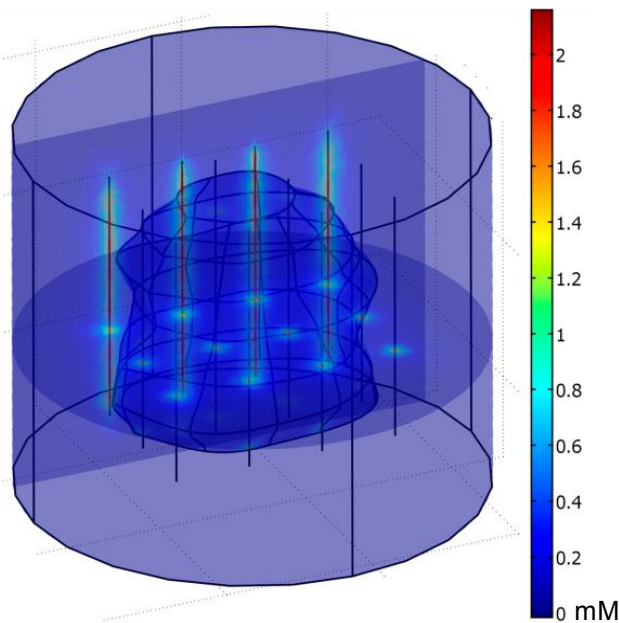


Meshing

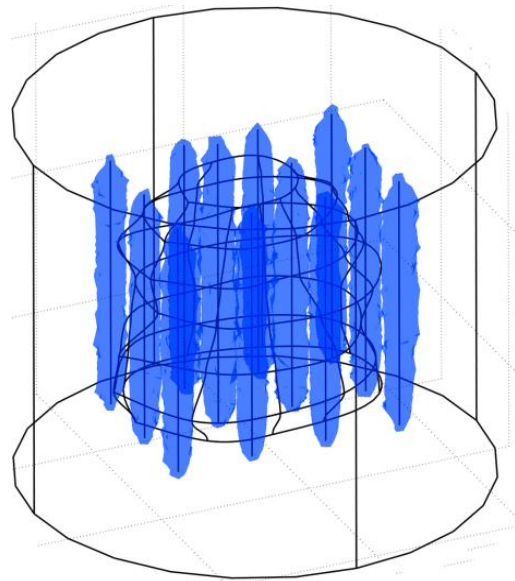


# $[^1\text{O}_2]_{\text{rx}}$ prediction using COMSOL

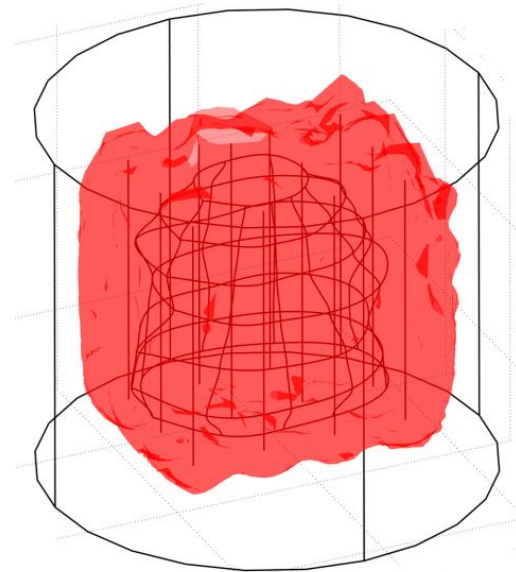
PDT dosimetry quantities for treatment up to 300 s in a homogeneous prostate



Slide view  
of  $[^1\text{O}_2]_{\text{rx}}$



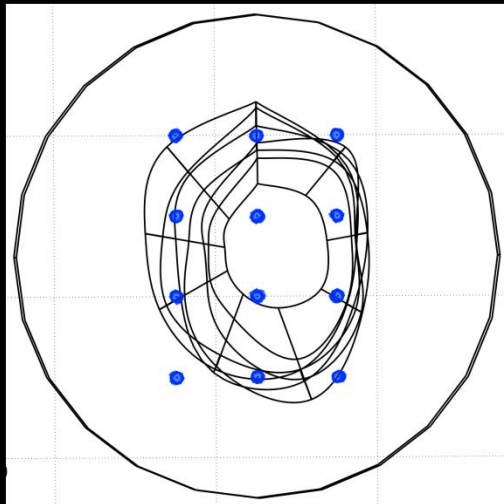
Isosurface of  
 $[^1\text{O}_2]_{\text{rx}}$  at 0.41 mM



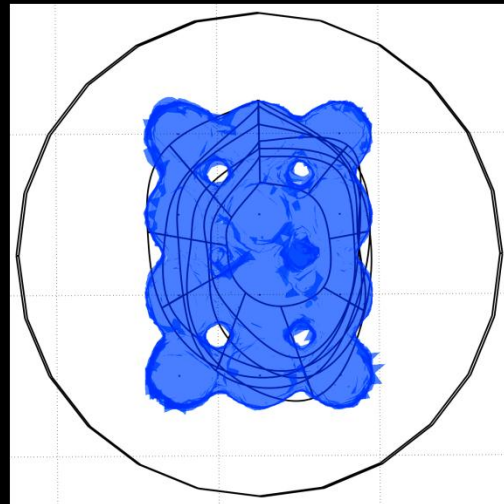
Isosurface of  
light fluence

# $[^1\text{O}_2]_{\text{rx,sh}}$ prediction using COMSOL

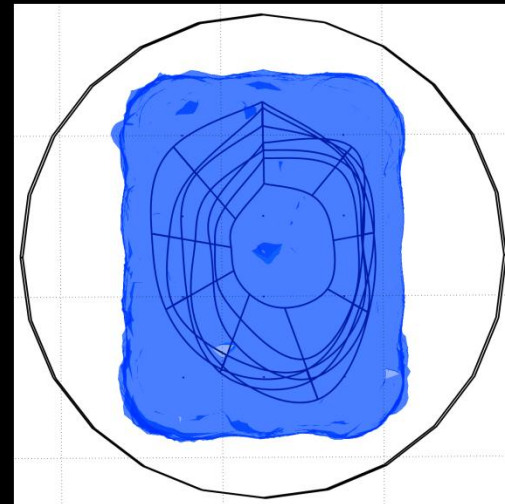
Top view of  
isosurface  
of  $[^1\text{O}_2]_{\text{rx}}$  at  
0.41 mM



200 s

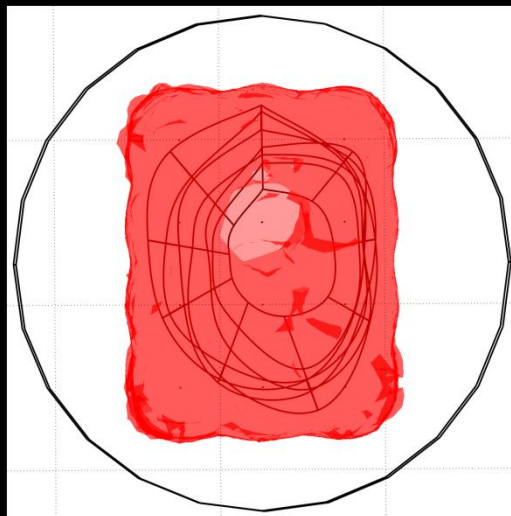


500 s

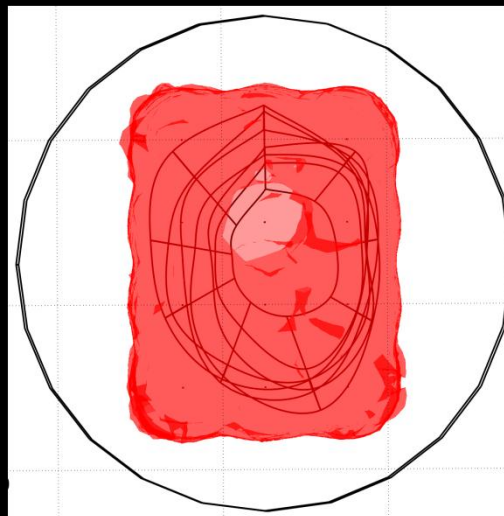


1000 s

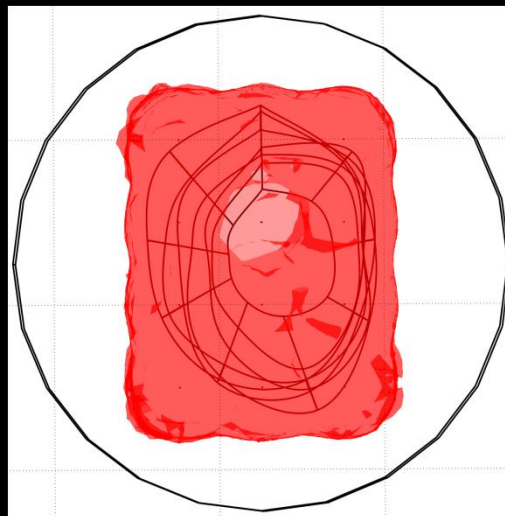
Top view of  
isosurface  
of light  
fluence



30 J/cm<sup>2</sup>



75 J/cm<sup>2</sup>



150 J/cm<sup>2</sup>

# Conclusions

- PDT model including light diffusion and PDT kinetics equations
- Optimized photo-chemical parameters in the PDT model
- PDT prostate model with homogeneous properties
- Prediction of PDT dosimetry quantities for treatment

**Thank you!**