Using the Time Parameter as the Third Geometrical Dimension

Overview

- Background
- Theory
- Application in Comsol Multiphysics
- Results
- Conclusion

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Background

Problem description

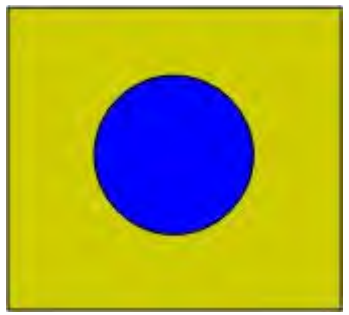
- Detailed Cross Section
- Complex Model (Multiphysical Model)

- Necessary to modify imported Geometry?
- Possible to save Calculation Memory?
- Possible to save Simulation Time?

Theory

$$Q = c \cdot m \cdot \Delta T$$

$$Q = c \cdot \rho \cdot V \cdot \Delta T = c \cdot \rho \cdot A \cdot \Delta z \cdot \Delta T$$



Introducing time parameter *t:*

$$P = \frac{dQ}{dt} = \frac{c \cdot \rho \cdot A \cdot \Delta z \cdot \Delta T}{dt}$$

$$P = c \cdot \rho \cdot A \cdot \Delta T \frac{\Delta z}{dt}$$

Distance *z* and time *t* are proportional:

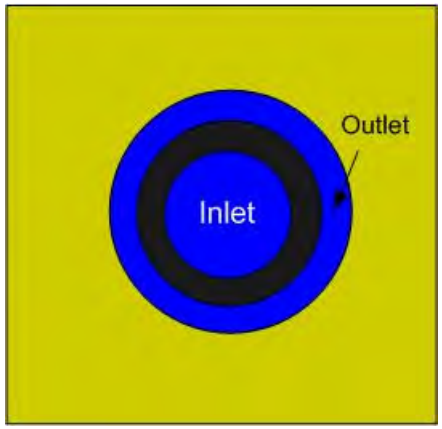
$$dt = \frac{1}{v} dz$$

2D 3D

An Application in Comsol:
A cooling water loop

Case 1: Loop

Application: Ground Source Heat Pump



Case 2: Pipe-in-pipe

Application : Bore Hole Heat Exchanger

An Application in Comsol

Thermal properties

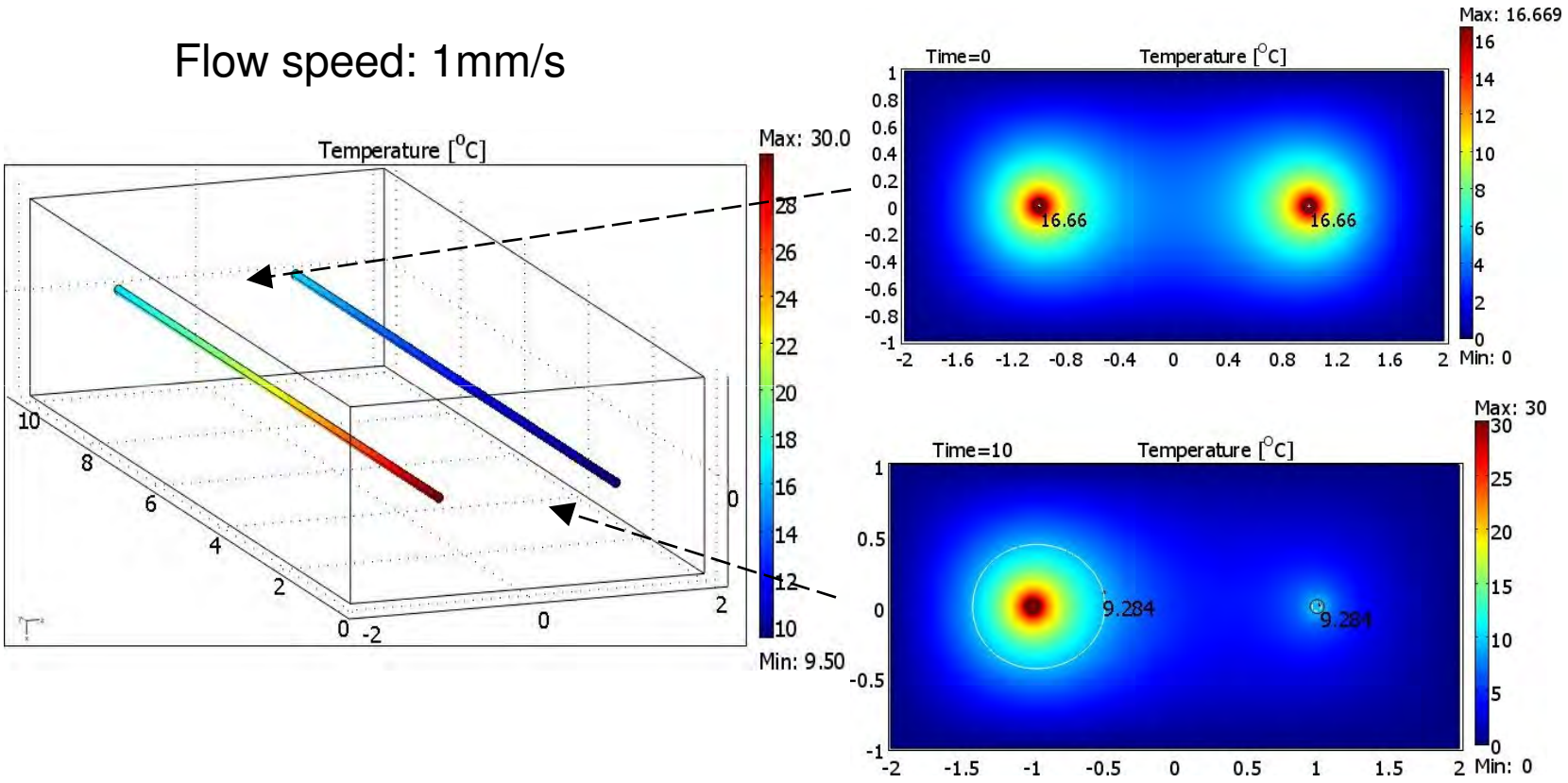
Part	k [W/K·m]	c_p [J/kg·K]	ρ [kg/m^3]	$T_{boundary}$ [°C]
Ground	1	1000	1000	0
Steel tube	50	1000	8000	N/A
Water	1000	4200	1000	30

Dimensions

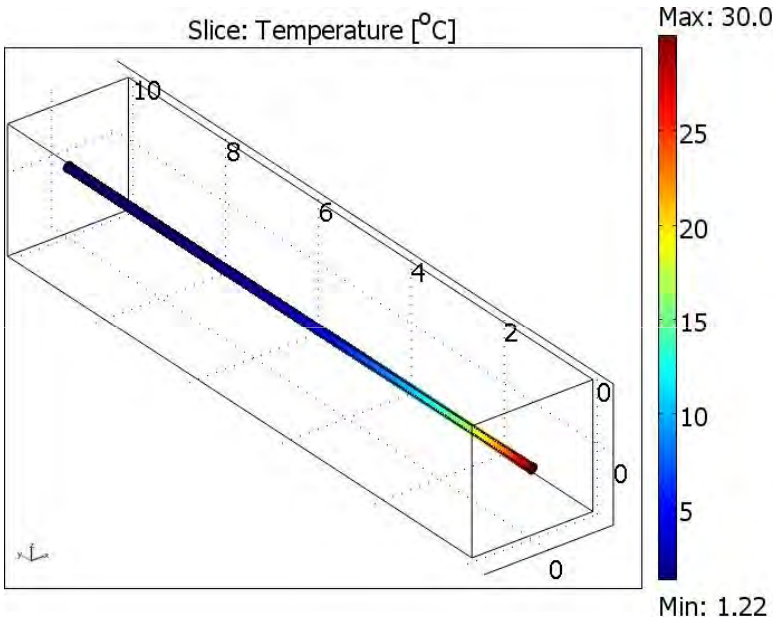
Dimension	Value
Depth under ground surface	1m
Length	10m
Inner diameter	10cm
Water cross section	ca. 80cm^2
For loop arrangement: Distance between tubes.	2m
For tube-in-tube arrangement: Inner tube material thickness.	1cm

Simulation Results: Loop

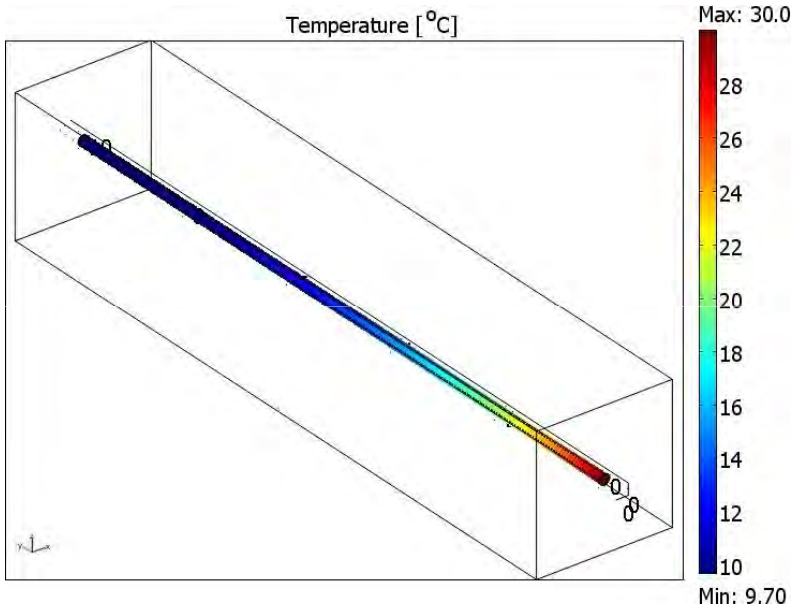
Flow speed: 1mm/s



Simulation Results: Pipe-in-pipe



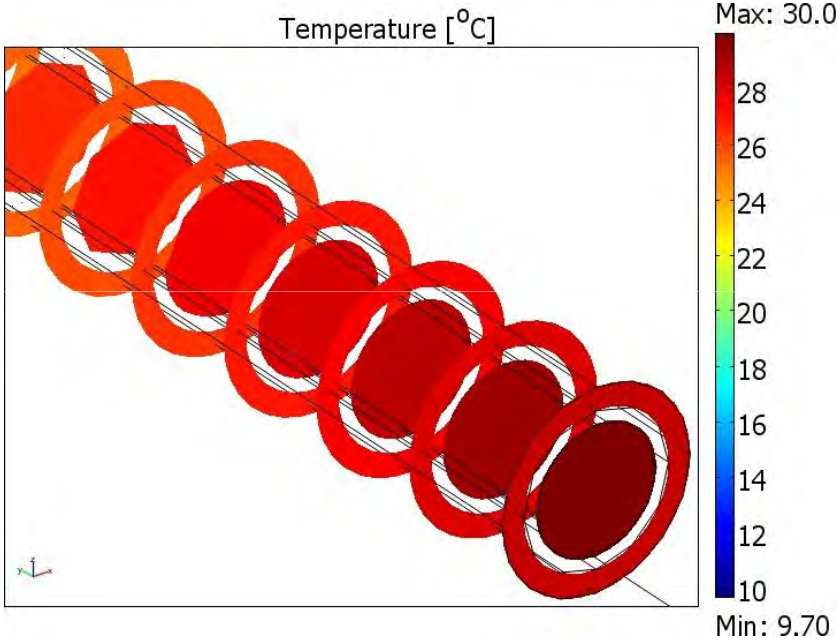
1mm/s

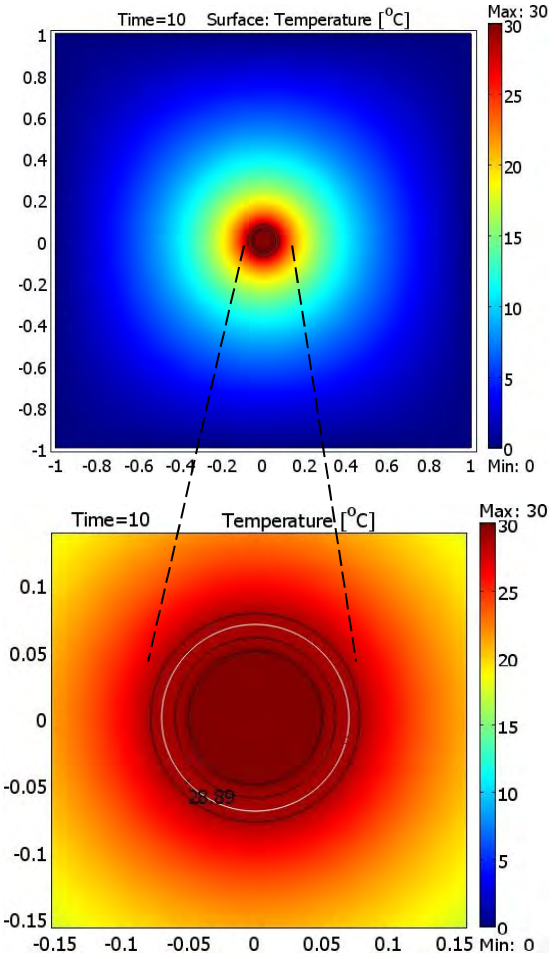


10mm/s

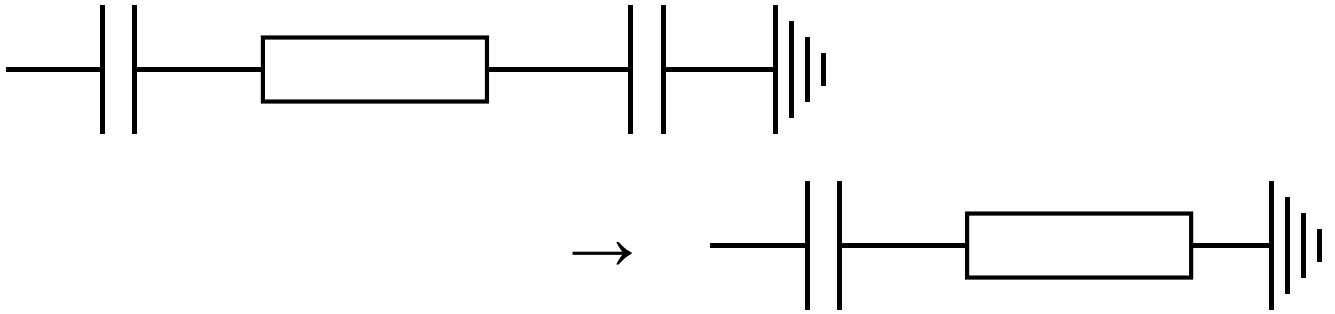
Simulation Results: Pipe-in-pipe

10mm/s





Simulation Results: Pipe-in-pipe



$$R = \frac{1}{\lambda} \cdot \frac{l}{A} = \frac{1}{50 \frac{W}{K \cdot m}} \cdot \frac{1cm}{2\pi \cdot 5.5cm \cdot 1m}$$

C/2

$$C = \frac{c_p}{2} \cdot \rho \cdot V = 2.1 \frac{J}{g \cdot K} \cdot 10^3 \frac{kg}{m^3} \cdot 2\pi \cdot (5cm)^2 \cdot 1m$$

$$\rightarrow \quad \tau = R \cdot C = 6.9 \mu \frac{W}{m^2} \cdot 16 \frac{kJ}{m} = 0.11s$$

$$T = T_0 \cdot e^{-\frac{\tau}{R \cdot C}}$$

Simulation Results: Overview

Loop

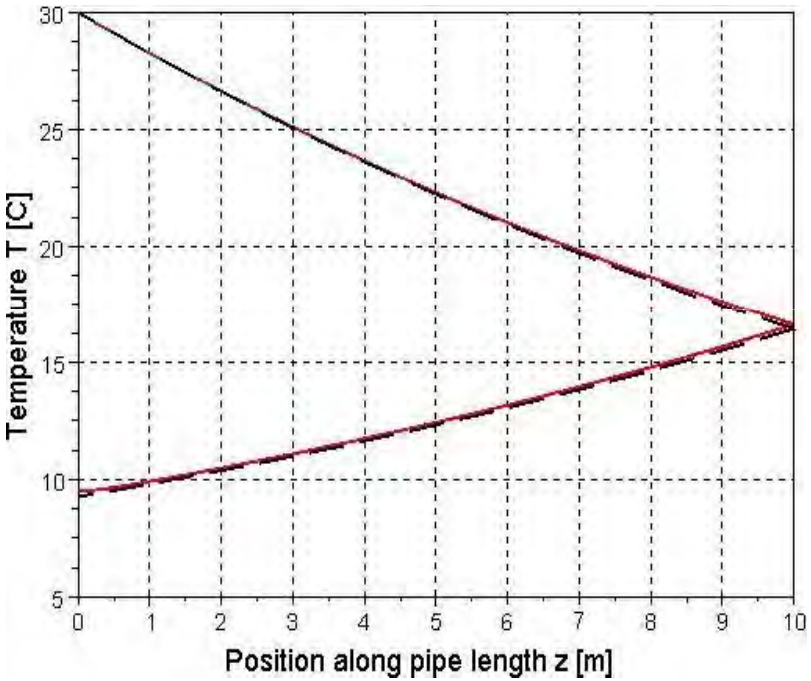
Water speed	Model	$T_{\text{outlet water}}$ [°C]	P by ΔT [W]	P by heat flux through surface [W]
1mm/s	3D	9.5	676	600
	2D	9.3	682	N/A
10mm/s	3D	26.6	1120	1100
	2D	26.6	1120	N/A

Pipe-in-pipe

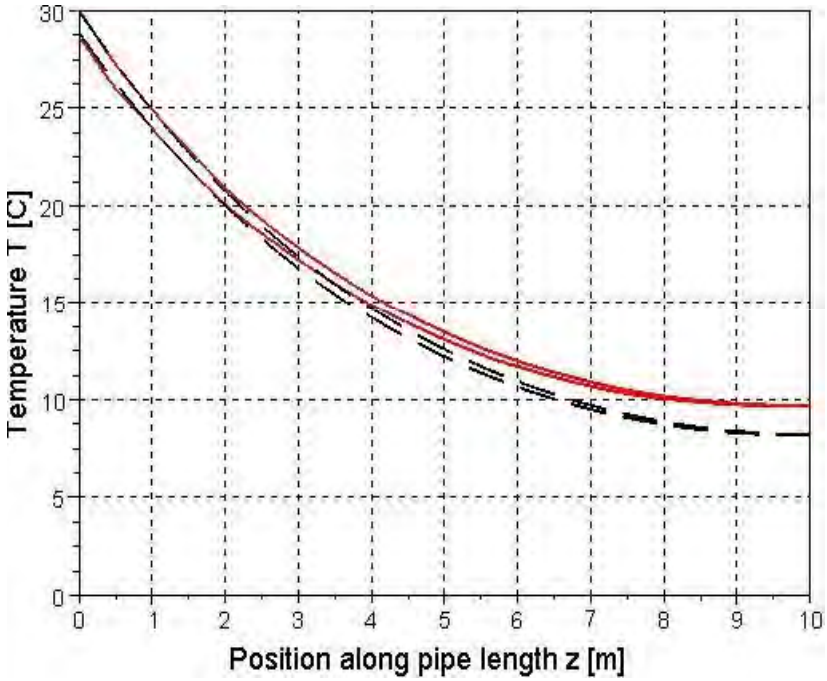
Water speed	Model	$T_{\text{outlet water}}$ [°C]	P by ΔT [W]	P by heat flux through surface [W]
1mm/s	3D	28.8	40	278
	2D	N/A	N/A	N/A
10mm/s	3D	28.62	455	426
	2D	28.9	363	N/A
100mm/s	3D	29.79	690	713
	2D	29.8	660	N/A

Simulation Results: Temperature profiles

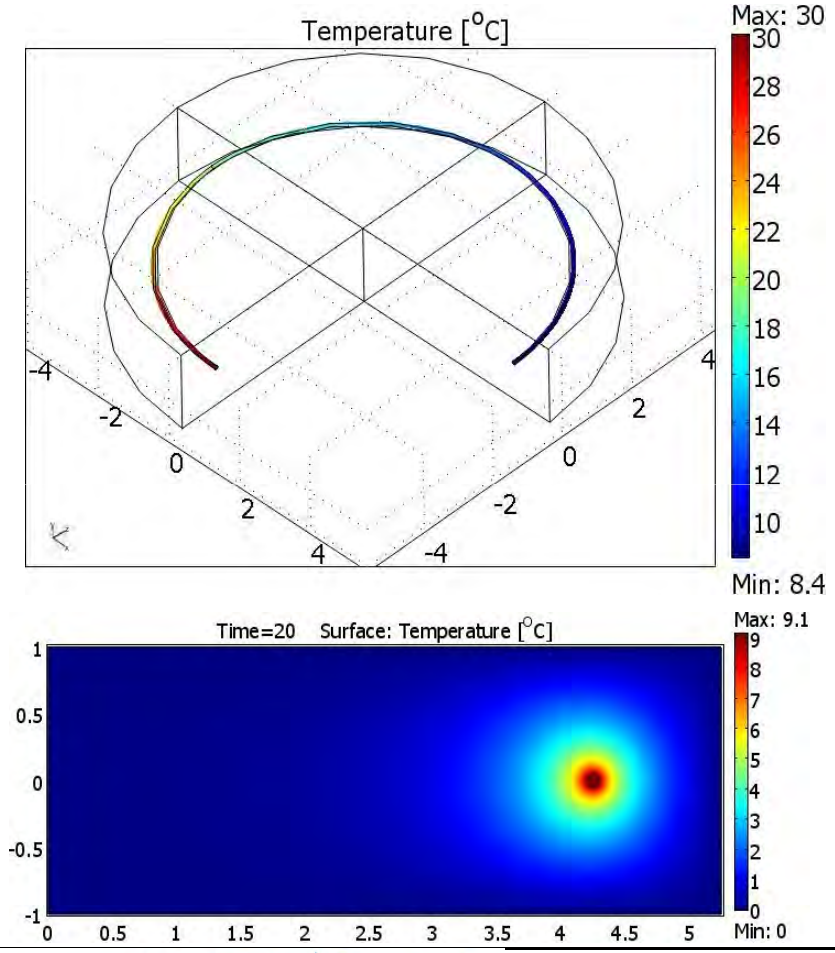
Loop



Pipe-in-pipe



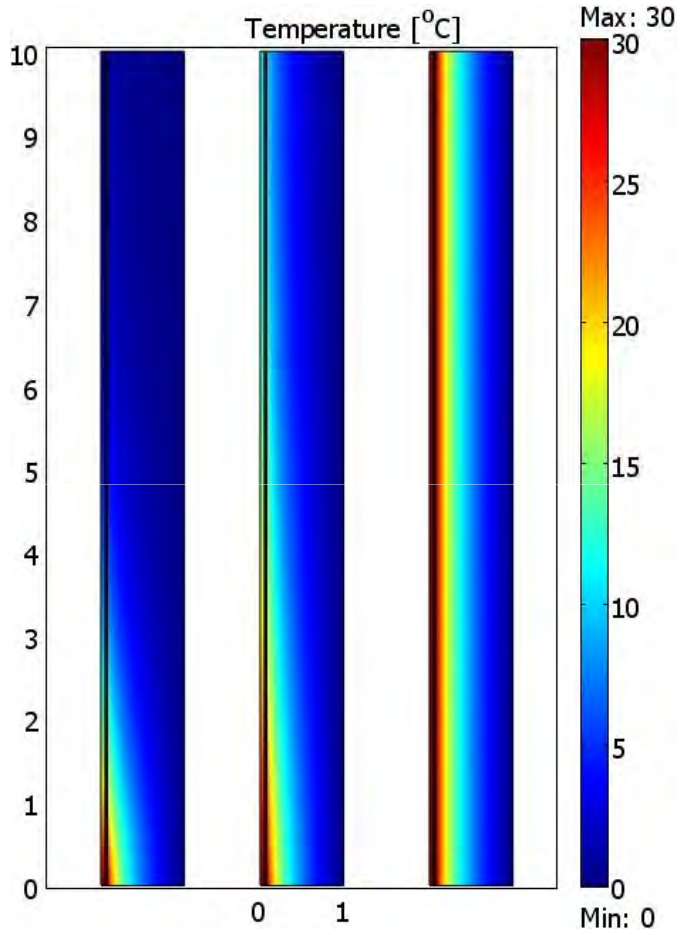
Simulation Results: Loop reviewed



Further verification by similar model in axial symmetrical coordinates

Flow speed: 1mm/s

Simulation Results: Pipe-in-pipe reviewed



Further verification by 2D axialsymmetrical model

1mm/s
10mm/s
100mm/s

Simulation performance

Model	Simulation time	Memory
2D Models	0.7s (8s)*	130MB (277MB)* UMFPACK
3D Models	54 s	687MB

* Worse values for pipe-in-pipe model due to more time steps

Conclusions

- New Model confirms the theory
- Model is feasible in Comsol Multiphysics
- Works for non-linear variations
- Better Convergence
- Faster (~10x)
- Less Memory Usage (~ 20%)

but:

- User has to know how to do (we know it)
- Model has to be tuned
- In special cases small time steps needed