

# Theory of Proportional Solenoids and Magnetic Force Calculation Using COMSOL Multiphysics

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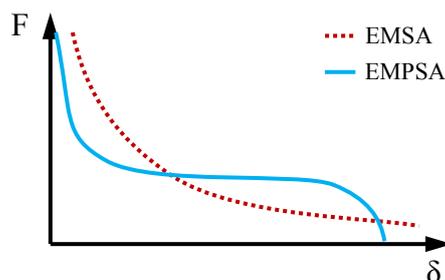
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**Abstract:** Proportional solenoids are well-known and used in a wide range of applications today. However, the theory behind the functional principle of proportional magnetic actuators is not very well represented in literature. This paper discusses methods of influencing the characteristic force-stroke-curves of magnetic actuators by means of different pole geometries. The conical design of the stator pole, which is primarily used to accomplish proportional solenoids, is analyzed by both a simple analytic reluctance model and a FEM model created with the AC/DC module of COMSOL Multiphysics. Furthermore, three different approaches of magnetic force calculation in COMSOL Multiphysics are introduced showing their advantages and disadvantages. The three methods are *Maxwell Stress Tensor* (COMSOL default method), *Maxwell Tensile Force* and *the method of virtual displacement*.

**Keywords:** Electromagnetic solenoid actuator (EMSA), electromagnetic proportional solenoid actuator (EMPSA), force calculation, magnetic energy, virtual displacement.

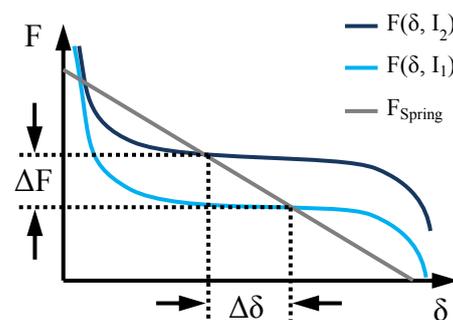
## 1. Introduction

There are various applications for EMSA today. Standard EMSA mostly have typical nonlinear force-stroke-curves exemplarily shown in figure 1. Due to the typical behavior of an electromagnetic actuator regarding the interrelationship of force and main air gap between stator pole and armature it is often



**Figure 1:** Typical force-stroke-curve of electromagnetic actuators

necessary to find methods for modifying force-stroke-curves of an actuator to fit requirements of applications. One application can be a proportional valve. This kind of valve must be able to vary its cross section area in proportion to an electric control signal. Therefore an electromagnetic proportional solenoid actuator (EMPSA) is needed. An EMPSA should have a force-stroke-curve showing an interval of preferably constant force along stroke as shown in figure 1. Figure 2 shows the interaction of an EMPSA and a linear spring. It becomes clear that with increasing current, stroke increases proportionally. There are mainly two principles to achieve this kind of behavior. One possibility is to implement electronic regulation, which is costly and also can have functional failure. In addition, the nonlinear behavior makes it quite difficult to regulate force properly along stroke, especially in range of a small main air gap. Thus the only chance of modifying the behavior of EMSA is to find pole geometries of stator and armature allowing different characteristics.



**Figure 2:** Functional principle of an EMPSA

## 2. Analytic Approach

This section is an analytic view of the topic. Reluctance models are used to model magnetic circuits.

### 2.1 Definition of Reluctances

Magnetic circuits usually contain at least one magnetic excitation (coil, permanent magnet) in line with a network of linear and nonlinear

magnetic resistors, so called reluctances. Linear reluctances show a linear inter-relationship between magnetic field H and magnetic flux B. In other words, they have a constant permeability. Linear materials are for instance gases (air) or plastics, and can be designated as nonmagnetic materials. Nonlinear reluctances do have a nonlinear interrelationship between magnetic field H and magnetic flux B. So the permeability of these reluctances is not constant at all (Fig. 3). Most common nonlinear materials are soft iron and low-alloy steel. Besides the material property, reluctances analog to electric resistors depend on their geometric dimensions as shown in figure 4. Linear and nonlinear reluctances can be calculated according to the following formulas

$$R_{\text{linear}} = \frac{l}{\mu_0 A}$$

$$R_{\text{nonlinear}} = \frac{H l}{B A}$$

As nonlinear reluctances do not have a constant permeability, it needs to be replaced by a H(B)- or a B(H)-curve. Instead a constant permeability can be assumed within approximately linear sections of B(H)-curve.

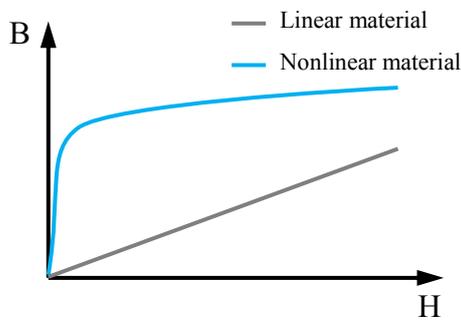


Figure 3: B(H)-curves of linear and nonlinear materials

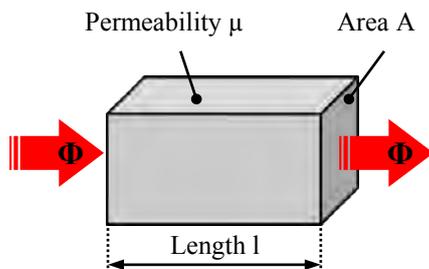


Figure 4: Reluctance dimensions

## 2.2 Series Connection of Reluctances

In an standard EMSA armature, the main air gap and stator pole can be assumed as a series connection of linear and nonlinear reluctances as shown in figure 5. Reluctance of stator pole is normally less than reluctance of the armature. Furthermore reluctance of the armature is much less than reluctance of the main air gap due to high permeability of the armature material. This is why in this examination reluctance of the armature and stator pole are negligible. So the total reluctance can be assumed as

$$R_{\text{tot}} = R_{\text{air}} = \frac{\delta}{\mu_0 A}$$

Magnetic flux passing the main air gap can be calculated by

$$\Phi = \frac{\Theta}{R_{\text{tot}}} = \frac{\Theta \mu_0 A}{\delta}$$

Magnetic force between armature and stator pole can be calculated by Maxwell Tensile Force (see also Chap.4.2)

$$F = \frac{\Phi^2}{2 \mu_0 A} = \frac{\Theta^2 \mu_0 A}{2 \delta^2}$$

Now it becomes clear that magnetic force of an EMSA has a characteristic  $F \sim 1 / \delta^2$  curve which cannot be manipulated in terms of linearization.

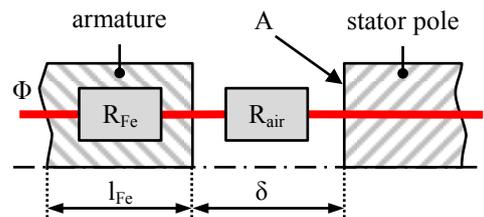


Figure 5: Series connection of reluctances within an EMSA

## 2.3 Parallel Connection of Reluctances

Most EMPSA today have a conical shaped tube on the stator pole which armature dives into. This conical tube and the main air gap can be assumed as a parallel connection of a linear and a nonlinear reluctance (Fig. 6). Reluctance of the armature and massive stator pole in comparison to main air gap and bypass

are very low and therefore are neglected in this examination. Total magnetic flux  $\Phi_{tot}$  reaching the parallel connection splits up into two components  $\Phi_{air}$  and  $\Phi_{Fe}$ . So component  $\Phi_{Fe}$  always bypasses the main air gap not contributing to axial force generation. Now two cases must be considered.

In case of a maximum main air gap  $\delta = \delta_{max}$  reluctance of bypass due to its conical shape and nonlinear material is very high, so that its permeability can be assumed as  $\mu_0$

$$R_{Fe} \approx \frac{l_{Fe}}{\mu_0 A_{Fe}} = \frac{\delta_{max}}{\mu_0 A_{Fe_{min}}}$$

With  $\delta = \delta_{max}$  reluctance of the main air gap is also at its maximum

$$R_{air} = \frac{\delta_{max}}{\mu_0 A}$$

The ratio of both flux components is reciprocally proportional to reluctances

$$\frac{\Phi_{air}}{\Phi_{Fe}} = \frac{R_{Fe}}{R_{air}} = \frac{A}{A_{Fe_{min}}}$$

As stator pole area  $A \gg A_{Fe_{min}}$  most of magnetic flux passes main air gap generating axial force.

In case of a minimum main air gap  $\delta = \delta_{min}$  reluctance of bypass is minimal due to increased bypass area  $A_{Fe}$  and permeability which is assumed as  $\mu_{Fe} = 100 \mu_0$

$$R_{Fe} \approx \frac{l_{Fe}}{100 \mu_0 A_{Fe}} = \frac{\delta_{min}}{100 \mu_0 A_{Fe_{max}}}$$

With  $\delta = \delta_{min}$  reluctance of main air gap is also at its minimum

$$R_{air} = \frac{\delta_{min}}{\mu_0 A}$$

The ratio of both flux components again is reciprocally proportional to reluctances

$$\frac{\Phi_{air}}{\Phi_{Fe}} = \frac{R_{Fe}}{R_{air}} = \frac{A}{100 A_{Fe_{max}}}$$

Compared to maximum main air gap, the ratio of flux components has changed considerably so that large amounts of total flux now are now bypassing the main air gap. With systematic control of flux component  $\Phi_{air}$  via bypass,

force-stroke-characteristics of an EMPSA can be achieved.

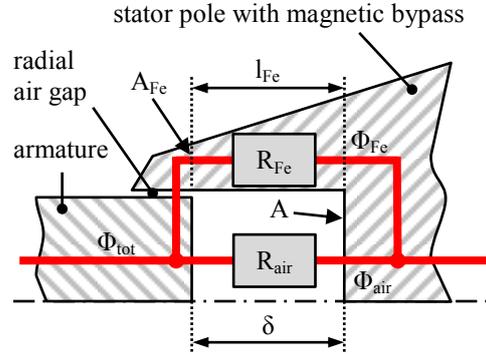


Figure 6: Parallel connection of reluctances within an EMPSA

### 3. Numeric Approach using COMSOL Multiphysics

This section deals with modeling and analyzing an exemplary EMPSA using the AC/DC module of COMSOL Multiphysics. Here an axisymmetric 2D-model is examined using magnetic field (mf) modus. First step of modeling always implies drawing models geometry (Fig. 7). Geometry of EMPSA consists of at least four domains: stator, armature, coil and surrounding air. The air domain is divided into several subdomains for meshing purposes. A ferromagnetic and therefore nonlinear material has been assigned to armature and stator. Material properties include  $H(B)$ -curve for field calculation and in addition an  $e_{mag\_co}(B)$ -curve used for energy calculation. Note that in order to activate nonlinear behavior of materials another ‘‘Ampère’s Law’’ the node needs to be added to ‘‘Magnetic Fields (mf)’’ node. Within this node

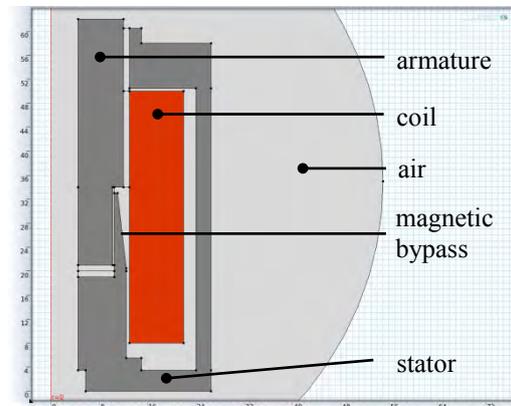


Figure 7: Geometry of EMPSA

nonlinear materials have been added to selection and relationship between magnetic field  $H$  and flux density  $B$  have been set to “HB-curve”. All other domains have been assigned to linear material (air) with permeability  $\mu_0$ . Furthermore a “Multiturn Coil Domain” has been added for implementing current and number of windings (280) to designated coil domain and a “Force Calculation” node for calculating forces on the armature. Additional variables and integration operations for energy calculation have also been added (see also Chap. 4.3). Geometries mesh (Fig. 8) consists of triangular elements in general. However in order to prevent calculated force-stroke-curves from being edgy domains of radial air gaps and axial main air gap consist of rectangular elements generated with a “Mapped Mesh” feature. COMSOL offers “Parameter Sweep” feature in solver node which can be used for calculating force-stroke-curves. Here LiveLink for MATLAB has been used for calculating and post processing force-stroke-curves (Fig. 9) instead. The curves clearly show the characteristic shape of an EMPSA until current exceeds magnitude of 5 A. As mentioned above, magnetic co-energy along stroke has also been calculated (Fig. 9). Note that in intervals of constant force along stroke magnetic co-energy increases linearly. Thus conformity to force calculation via virtual displacement method (Chap. 4.3) is easily understandable. In figure 11 proportional interrelationship between current and force becomes apparent. Fig. 12 to fig. 14 show a detailed surface plot of flux density and flux lines within the pole region at different armature positions. EMPSA with a maximum main air gap is shown in figure 12. Here the region near the tip of magnetic bypass is highly saturated due to magnetic flux

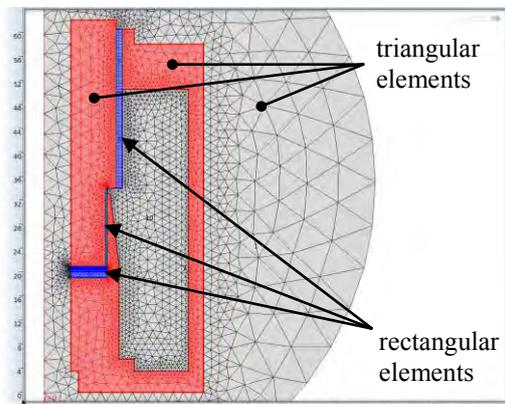


Figure 8: Meshed geometry of EMPSA

running through. This highly saturated area has permeability close to surrounding air ( $\mu_0$ ). Therefore a significant amount of magnetic flux disperses around that region heading towards less saturated region. Dispersed flux has an axial component so that it will contribute to axial force generation. At a medium main air gap (Fig. 13) travelling of highly saturated region with the front end of

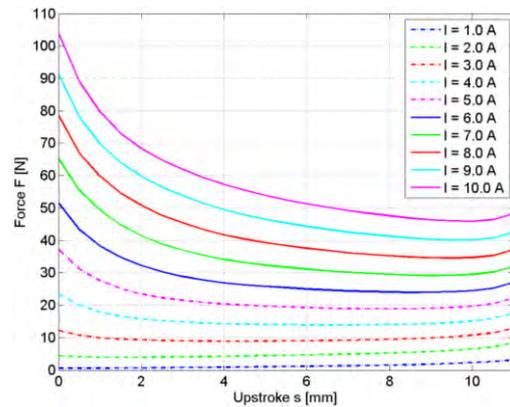


Figure 9: Force-stroke-curves of EMPSA

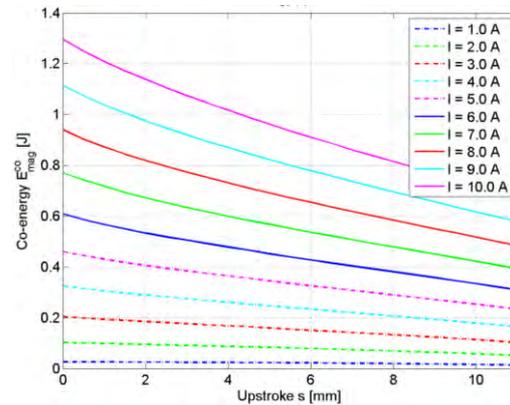


Figure 10: Co-energy-stroke-curves of EMPSA

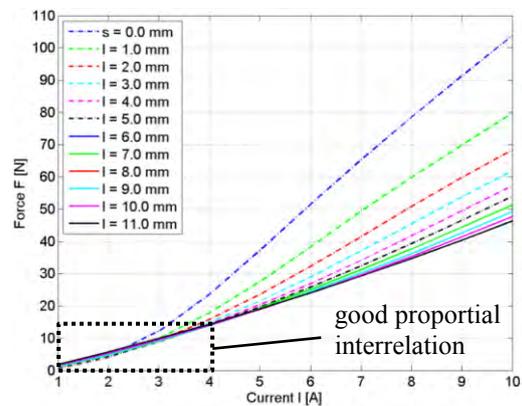


Figure 11: Force-current-curve of EMPSA

the armature can be observed. Here dispersed flux near the bypass as well as direct flux between poles exist, both contributing to axial force generation. Additionally, bypass flux has increased counteracting typical nonlinear ascent of force. Figure 14 shows armature at minimum main air gap position. Here straight flux lines and almost homogeneous flux density between poles can be observed. Even though being highly saturated, bypass reached

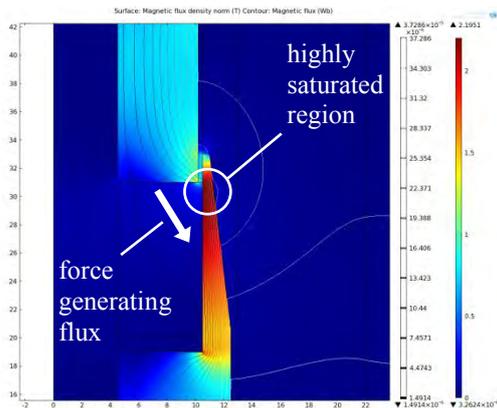


Figure 12: Armature at maximum air gap position

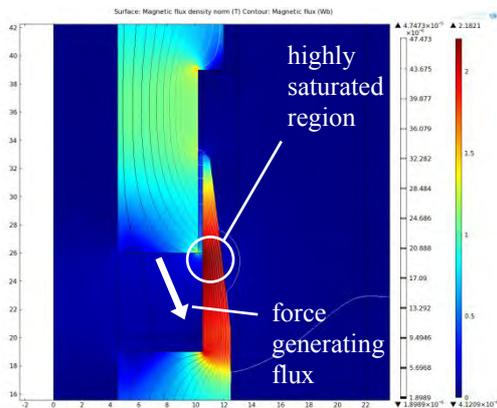


Figure 13: Armature at medium air gap position

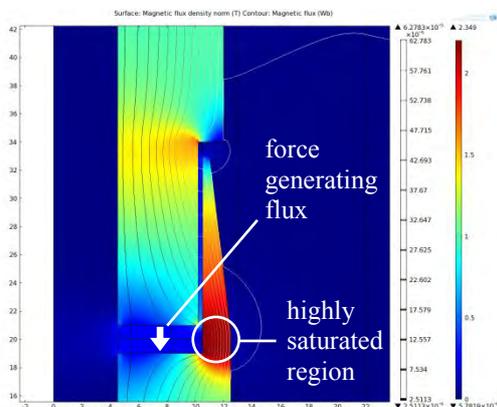


Figure 14: Armature at minimum air gap position

its minimum reluctance, dissipating maximum possible flux from main air gap.

#### 4. Methods of Force Calculation

This section is about three different approaches of electromagnetic force calculation. All three methods have been applied and tested in COMSOL Multiphysics.

##### 4.1 Maxwell Stress Tensor

The Maxwell Stress Tensor is mathematically an integral along a closed path (2D) or closed surface area  $A$  (3D) around a domain of interest (Fig. 15). When surface area  $A$  is located in a linear material ( $\mu_r = 1$ ) the following formula can be applied [1]

$$F = \frac{1}{\mu_0} \oint_A \left( B(B \cdot n) - \frac{1}{2} B^2 n \right) dA.$$

Note that  $n$  is a normal vector on surface area  $A$ . Force calculation using the Maxwell Stress Tensor is the force calculation method of COMSOL Multiphysics (AC/DC module) by default. To use this method you simply have to add a “Force Calculation” node to your COMSOL model. The next step is to select and add a domain (armature or stator) to the node selection on which calculation should be applied. After solving the model forces can be evaluated with global evaluation node. Force calculation results are very accurate with this method. Accuracy can be increased by increasing the mesh density in the location of surface area  $A$ . However this method also has its limits. For example retention force of a magnetic clamp cannot be calculated this way because of missing air gap between armature and stator pole. Not having an air gap will result in much too low force due to

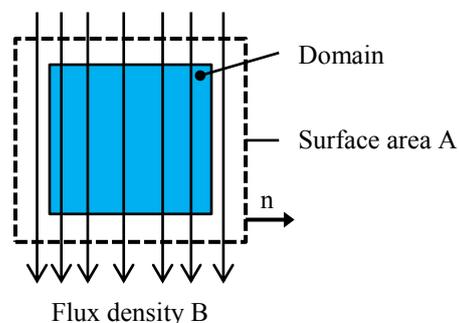


Figure 15: Principle of force calculation using Maxwell Stress Tensor (2D view)

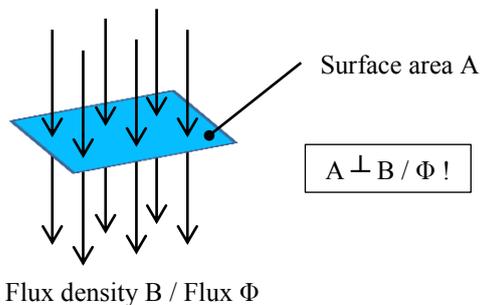
permeability of material being higher than  $\mu_0$ . In this case the only chance for proper force calculation is maintaining a very small air gap ( $\delta < 0.1$  mm) between poles. Alternatively, the following force calculation methods can be applied.

#### 4.2 Maxwell Tensile Force

This method of force calculation is common in analytic simulation tools where models are based on reluctance networks. It uses a very good mathematical approximation and can be applied in case of homogeneously distributed magnetic flux density over boundary surfaces of magnetic poles (Fig. 16). This method of force calculation accounts magnetic flux  $\Phi$  passing through a perpendicular area  $A$  according to the following formula [1]

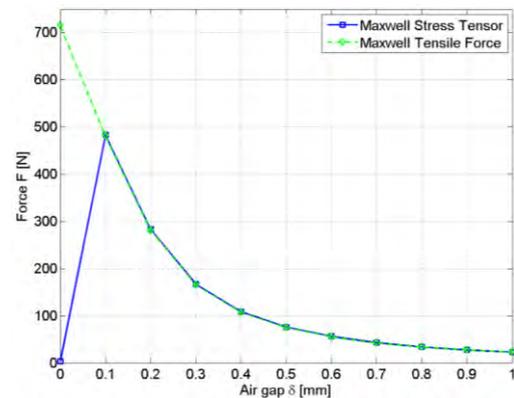
$$F = \frac{B^2 A}{2\mu_0} = \frac{\Phi^2}{2\mu_0 A}$$

Of course it is also possible to implement this method in COMSOL Multiphysics. Therefore it is necessary to add integration operations to the COMSOL model. The integration operation can be found by clicking the right mouse button on *Model 1 \ Definitions*. A context menu will open where *Model Couplings \ Integration* needs to be selected. Add the stator or armature pole area (3D) or edge (2D) to the integration node selection. In the next step a variables node needs to be added to the model. Within the variables node, the pole area and magnetic flux can be calculated using the integral operation. This method was tested in a COMSOL model of a magnetic clamp. A magnetic clamp is an EMSA with a simple plate for the armature. The range of usable stroke is very low. But when the armature is placed on the stator without any air gap between, retention force



**Figure 16:** Principle of force calculation using Maxwell Tensile Force

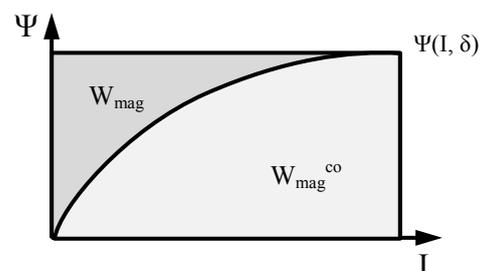
is immense. Figure 17 shows a comparison of force-stroke-curves calculated with COMSOL Multiphysics using Maxwell Stress Tensor and Maxwell Tensile Force. In fact, Maxwell Tensile Force gives very good results when magnetic poles have plane and parallel surfaces. It is not suitable for more complex pole geometries e.g. those of an EMPSA. The main advantages of this method are its simplicity and the fact that it still works even in the case of non-existing air gap between poles.



**Figure 17:** Comparison of Maxwell Stress Tensor and Maxwell Tensile Force calculation results

#### 4.3 Virtual Displacement

This method utilizes the energy stored in a magnetic circuit for force calculation. Note that besides magnetic energy, magnetic co-energy can also be used for this method (fig 18). In general the stored (co-)energy of a magnetic circuit will increase with decreasing main air gap [2]. In order to calculate the forces of an EMSA at a specific armature position, it is necessary to calculate the stored magnetic (co-)energy at this position. In a second step, the armature needs to be slightly displaced from its reference position as shown in fig. 19. The direction of displacement is



**Figure 18:** Magnetic energy and co-energy within a  $\Psi(I)$ -diagram at fixed armature position

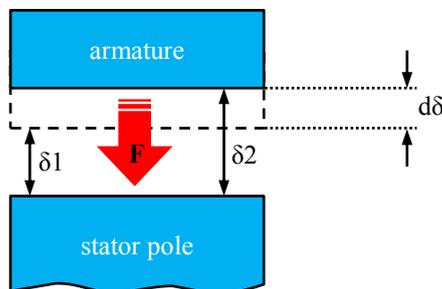
generally arbitrary. Magnetic energy needs to be calculated in the displaced position as well. The difference of magnetic (co-)energy equates to potential mechanic work. So force can be calculated according to the following formulas [1]

$$dW_{\text{mech}} = dW_{\text{mag}}$$

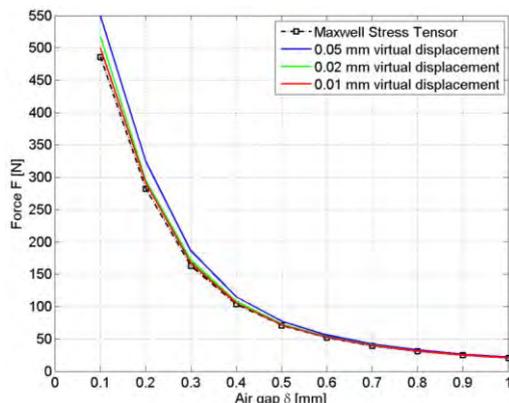
$$dW_{\text{mech}} = F dx = -F d\delta$$

$$F = -\frac{W_{\text{mag},1} - W_{\text{mag},2}}{d\delta} = -\frac{dW_{\text{mag}}}{d\delta}.$$

Therefore force  $F$  can be assumed as a boundary value for  $d\delta$ -values close to zero. So accuracy of this force calculation method basically depends on the grade of virtually applied displacement  $d\delta$ . It is also possible to perform a symmetric displacement in relation to reference armature position which increases accuracy and offers good results even with quite large degree of virtual displacement. For implementing this method of force calculation in COMSOL Multiphysics some things have to be considered. COMSOL offers a predefined variable for energy calculation. However just a



**Figure 19:** Principle of force calculation using a method of virtual displacement



**Figure 20:** Comparison of Maxwell Stress Tensor and virtual displacement calculation results

linear approach of materials is assumed. In case of having a nonlinear material, an  $H(B)$ -curve needs to be defined. Furthermore, a curve for magnetic (co-)energy density needs to be calculated by integrating areas of the  $H(B)$ -curve. This (co-)energy density curve needs to be stored in the model as an interpolation table. In order to calculate magnetic (co-)energy of the entire model, an integral operation can be used. This method has also been tested on a COMSOL model of a magnetic clamp. A comparison of calculation results is shown in figure 20. Note the converging results by decreasing the grade of virtual displacement. This method can be applied to any type of electromagnetic actuator not depending on pole geometry or remaining air gap between armature and stator pole.

## 7. Conclusions

Both analytic and numeric approaches have their advantages. Analytic models are usually most suitable to show influences of specific parameters to calculation results. However they can easily become very complicated even considering quite simple geometries. Therefore it is necessary to introduce simplifications wherever possible. Due to simplifications analytic models often are not that accurate. Within a FEM model very complex geometries can easily be examined. Resolution of critical regions or regions of special interest can be increased on demand. A very big advantage of FEM models are the various post processing capabilities of applied FEM-software for displaying e.g. flux density, tracing flux paths or calculating forces. The simultaneous application of both approaches can generally lead to a better and deeper understanding.

## 8. References

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