Modelling thermal convection in the Earth's mantle using Comsol Multiphysics

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Abstract: A major tool for understanding thermal convection in the Earth's mantle is numerical modeling. To solve Boussinesq equations a finite element code has been applied. This is the first time this method has been used in Hungary, namely, modeling mantle convection on the Cartesian coordinate system. The simulations have been run in 2D Cartesian and cylindrical coordinate systems as well as in a "mantle-like" cylindrical-shell. The mantle dynamics are controlled by the Rayleigh number, which is the ratio of the buoyancy to viscous forces. The effect of *Ra* has been studied in the range of 1e4 to1e7. The significance of the cylindrical geometry is that at a given *rms* velocity the convection can carry the most heat to the surface and the results were close to the three dimensional case. This may imply that the upwelling part of the 3D mantle convection is cylindrical (mantle plume). In the cylindrical-shell domain an impressive approximate picture of the chaotic structure of the mantle convection has been shown. With the comparison of the three geometries it could be said that the cylindrical coordinate-system seems to be the most appropriate geometry to investigate the physical properties of an individual mantle plume.

Keywords: Mantle convection, geodynamics.

1. Introduction

A major tool for understanding thermal convection in the Earth's mantle is numerical modeling. The Boussinesq approximation has been used to formulate the partial differential equation system of the thermal convection. The governing equations (*Navier-Stokes and heat transfer*) were solved by a finite element method [Comsol Multiphysics 3.5, Zimmermann 2006]. The flexibility of the method allowed modeling of the thermal convection not only in rectangular domain, but in other geometries, as well. The results obtained in rectangular coordinate system were compared with the benchmark study of Blankenbach et al. [1989] and the agreement was within 1% error.

The simulations have been carried out in two dimensional rectangular, cylindrical, and in an "Earth's- mantle-like" cylindrical-shell domain. Using this method this was the first time in Hungary, when the thermal mantle convection could be modeled successfully out of Cartesian coordinate system.

The mantle dynamics is controlled by the Rayleigh number (*Ra*) which is the ratio of the buoyancy and viscous force that is the engine of the flow system. The effect of *Ra* has been studied in the range of 10^4 to10^7 (1e4-1e7). It was found that relationships between surface heat flow (*Nu*) and Ra ($Nu \sim Ra^{1/3}$), and root-mean-square velocity (v_{rms}) and *Ra* ($v_{rms} \sim Ra^{2/3}$), -originally derived from the thermal boundary layer theory for 2D rectangular domain-, are valid in the other studied geometries, as well. Obviously, for a given Rayleigh – number, *Nu*, v_{rms} and the mean temperature of the convection cell depend on the geometry: the highest values were obtained in case of rectangular model domain. The significance of the cylindrical geometry is that for a given *rms* velocity the surface heat flow is the highest. In that sense the most effective heat transport occurs in cylindrical shape convection systems. The dimensionless mean cell temperature was 0.5 in case of symmetric rectangular domain, and it was lower in cylindrical and cylindrical – shell domains. The lower mean cell temperature derives from the asymmetry of the flow regimes determined by these geometries. In case of cylindrical convection the surface of the hot upwelling plume in the centre is smaller than the surface of the cold downwelling flow at the rim of the cylinder. In case of cylindrical-shell geometry the outer cold surface of the domain is larger than the inner heated surface (like in case of the Earth) resulting in low cell temperature. Additionally in cylindrical geometry the results were close to the results of the three dimensional case, this has been alluded to that the upwelling part of the 3D mantle convection is cylindrical (mantle plume). Finally, in a sense with the comparison of the three geometries it could be said that the cylindrical coordinate-system seems to be the most appropriate geometry to investigate the physical properties and surface manifestations of an individual mantle plume.

1.1 Method and Theory, use of Comsol

These days the two dimensional numerical modelling has been widely still used [Čižkova, Matyska 2004; Mittelstaedt, Tackley 2006; Brunet, Yuen 2000], since this method uses lower memory therefore it runs faster and a lot of physical phenomena can be easier examined in this way. However it is obviously true that using only two dimensional simulations means a limitation for the real global mantle flow (which is spherical 3D), consequently the 2D physical parameters and results can be used only for studying the behaviour of a mantle-like fluid and they cannot explain the real global Earth Mantle circulation [Cserepes 1992, 2002.]. The main scientific target of this study is to identify the effect of the different geometries on the very high viscous "mantle-like" fluid flow system and to determine the impact of the Ra number. The following equations (Incompressible Navier-Stokes (1-2) and Heat transfer (3)) describe the Boussinesq approximation of the mantle convection [Chandrasekhar 1961]:

(1) $$\nabla \mathbf{u} = 0,$$

(2) $$\rho_0 \frac{d\mathbf{u}}{dt} = \rho_0 [1 - \alpha (T - T_0)] \mathbf{g} - \nabla p + \eta \Delta \mathbf{u})$$

(3) $$\rho_0 \, c_p \left[\frac{\partial T}{\partial t} + (\mathbf{u} \nabla) T \right] = K \, \Delta T \, .$$

where $\mathbf{u}$ is the velocity vector, ρ_0 is the density, α the thermal expansion, $\mathbf{g}$ is gravity, p is the pressure, η is viscosity, T is the temperature, c_p the specific heat and K thermal coefficient. Equations (1-3) were solved by a Comsol direct solver, UMFPACK [Zimmermann 2006], using the boundary conditions: mechanically slip/symmetry and thermically the lower (or inner) boundary had higher temperature than the upper (surface). The non-dimensional form of equation (2) contains the Ra number, which is the unique controlling parameter of the system. The Ra number (4), which is the ratio of the buoyancy and viscous force, is responsible for the dynamics of the mantle regime.

$$Ra = \frac{\rho_0 \cdot \alpha \ \cdot g \cdot \Delta T \cdot d^3}{\kappa \cdot \eta} = \frac{"bouyancy \ force"}{"viscous \ force"}, \quad (4)$$

Where d is the thickness of the mantle. Higher the Ra number is more vigorous the dynamics of the flow system. It is important to note that in the Earth Mantle the exact value of the Ra number is not known [Galsa 2008.], it is about 10^7 (1e7).
According to the linear stability theory [Schubert, Turcotte, Olson 2001] the mantle convection can exists only above a critical (minimum) Ra number (which is $\frac{27}{4}\pi^4$ ~ 657.5). So the Ra number of the Earth's mantle is probably more than four magnitudes higher than the critical value that suggests the convection should exists. A relevant factor is the impact of the Ra number on the mantle circulation therefore the 1e4-1e7 range has been used by modelling.

1.2 Results

To test the numerical accuracy of the Comsol Multiphysics, Blankenbach et al's 1989 study has been used and the results were very satisfactory (see Appendix, Table 1.), the agreement was within 1% error.

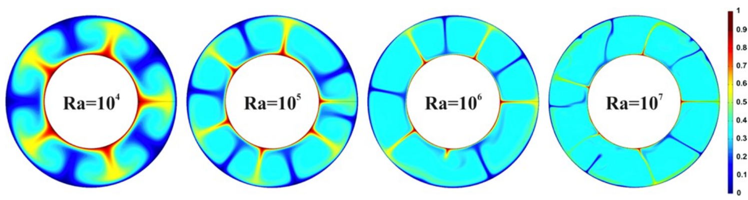

Figure 1. Cylindrical-shell, non-dimensional temperature field distribution, stationar. *Ra=10^4 and non-stationary solutions in case Ra=10^5-10^7.*

On Figure 1 the temperature distribution can be seen using cylindrical-shell domain. The higher the Ra number is the more chaotic the

structure of the flow system will be. The hot regions (red colour) are plumes (upwellings) and flowing to the surface, the cold regions (blue colour) are downwellings and streaming to the "Core-Mantle-boundary". Left to the right the Ra number is increasing from 1e4 to 1e7. It means that the first model is stationary but the others not those are time-dependent phenomena due to the chaotic attitude of mantle flow (see Figure 2.). It can be noticed that at higher Ra numbers
the up/down –wellings are getting thinner and the boundary layers (at the top and bottom) are also thinning because of the increscent velocity (see detailed Boundary Layer Theory, Schubert, Turcotte,
Olson 2001) . The above described phenomena are valid in Cylindrical and Cartesian geometry too [Herein et al. 2008].

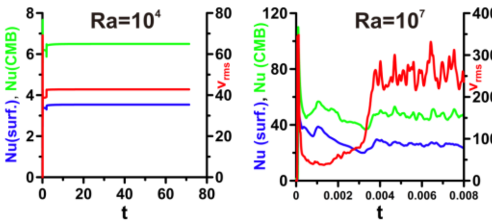

Figure 2. Non-dimensional parameters (Nu, vrms) versus non-dimensional time, Ra=1e4(stationary) , 1e7(time-dependent).

It was found that relationships between surface heat flow (Nu) and Ra (Nu ~ Ra $^{1/3}$), and root-mean-square velocity (vrms) and Ra (vrms ~ Ra $^{2/3}$), originally derived from the thermal boundary layer theory [Turcotte et al. 1967] for 2D rectangular domain, are valid in the other studied geometries, as well (Fig. 3, 7, 11). Obviously, for a given Rayleigh number, Nu, vrms and the mean temperature of the convection cell depend on the geometry: the highest values were obtained in case of rectangular model domain. The significance of the cylindrical geometry is that for a given rms velocity the surfaceheat flow is the highest (Fig. 3). In that sense the most effective heat transport occurs in cylindrical shape convection systems (Figure. 3.).

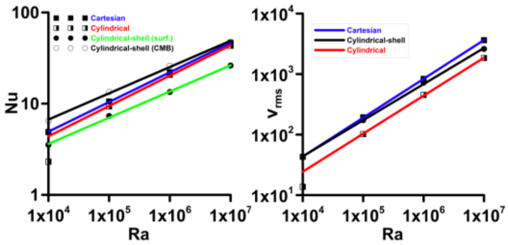

Figure 3. Relationship between Ra and the non-dimensional parameters (Nu, vrms), in Cartesian (blue), Cylindrical (red) and in Cylindrical-shell (black) geometry.

The dimensionless mean cell temperature was 0.5 in case of symmetric rectangular domain (Cartesian), and it was lower in cylindrical and cylindrical shell domains (Figure 4).

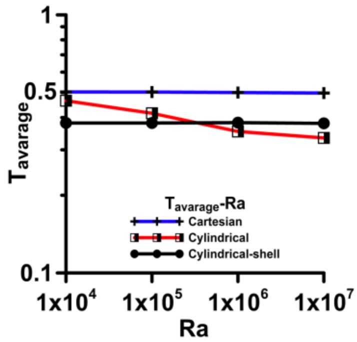

Figure 4. Relationship between Ra and the average temperature, in Cartesian (blue), Cylindrical (red), and Cylindrical-shell (black) geometry.

It is interesting that in cylindrical case the average temperature decreases with increasing Ra, but in the other two cases the average temperature is constant, there is no Ra dependence. The lower mean cell

temperature derives from the asymmetry of the flow regimes determined by these geometries. In case of cylindrical convection the surface of the hot upwelling plume in the centre is smaller than the surface of the cold downwelling flow at the rim of the cylinder. In case of cylindrical- shell geometry the outer cold surface of the domain is larger than the inner heated surface resulting in low cell temperature. Overall it could be said that the cylindrical-shell geometry provided the best way to analyze the chaotic structure of the thermal mantle convection.

2. Conclusions

Comsol Multiphysics proved as a good tool to model thermal mantle convection.
Using this method this was the first time in Hungary, when the thermal mantle convection could be modeled successfully out of Cartesian coordinate system. It had been testified that the geometry has an influence on the dynamics of the thermal convection. At high Ra number (1e7) the mantle flow is time-dependent and chaotic in all geometries. In total the cylindrical-shell geometry provided the best image of the chaotic structure of the thermal mantle convection.

3. References

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4. Acknowledgement

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5. Appendix

Table 1: Comparison between recent study and Blankenbach's 1989 study.

Parameter	*Ra*	This work	BLANKENBACH (1989)	Deviation [%]
Nu	10^4	4.88525	4.884409	0.0172
v_{rms}		42.864943	42.864947	0
Nu	10^5	10.567700	10.534095	0.319
v_{rms}		193.197400	193.21454	0.0088
Nu	10^6	22.061601	21.972465	0.4
v_{rms}		833.991497	833.98977	0.0002