The effect of different geometries on the thermal mantle convection

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Outline

I. Introduction

II. Mathematical background, numerical method

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I. Introduction

I.1. Scientific target

- Modelling in 2D geometry: Cartesian, cylindrical, and cylindrical-shell
- Comparison between results of Cartesian-system and other study [BLANKENBACH ET AL. 1989.], Test of Comsol Multiphysics.

I.2. Thermal convection

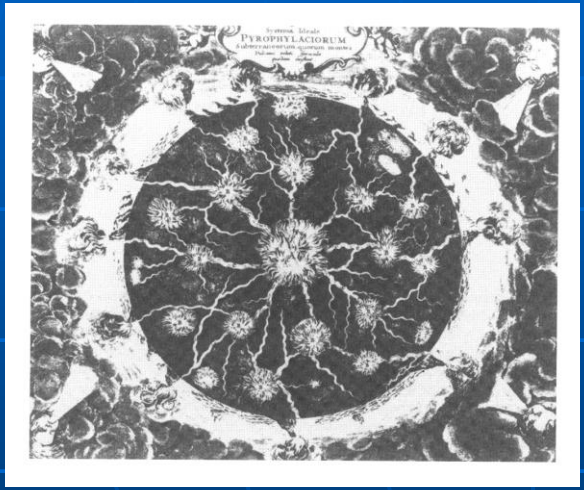


Figure1. The prevailing idea about the Earth interior, till the 19th century.

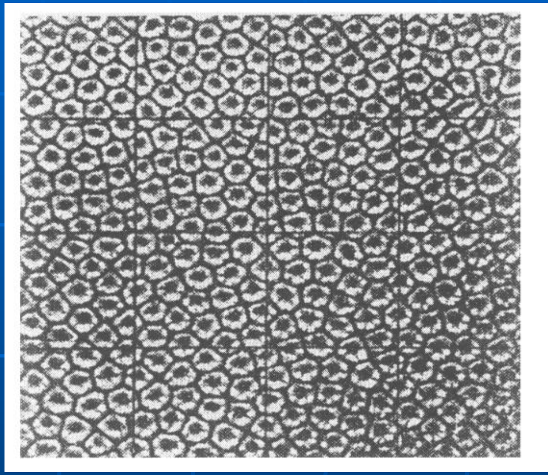
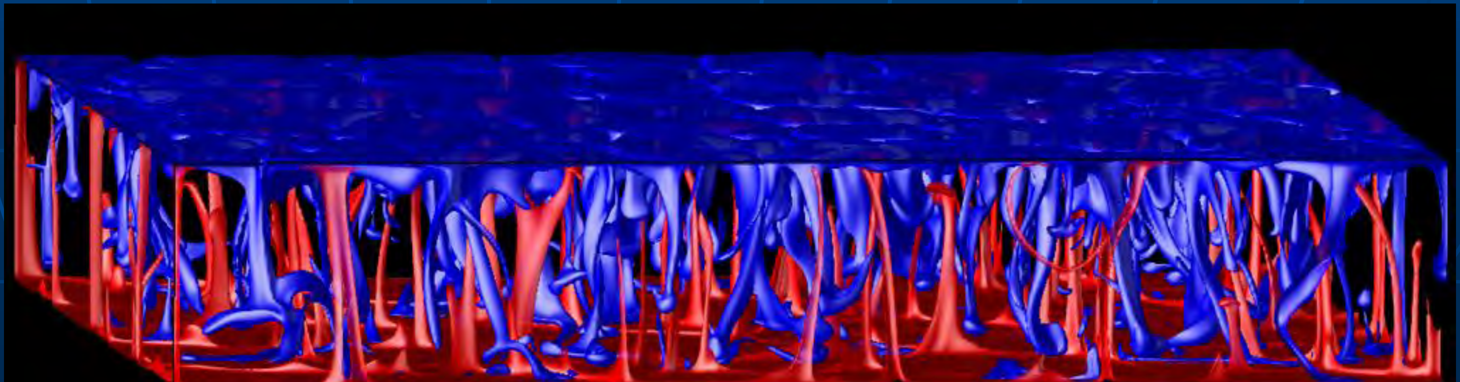


Figure 2. First photo of convection cells (Bénard 1901.)



3D mantle convection (Herein et al. 2010.)

II. Mathematical background, numerical method

II.1. Basic equations

(1) $$\frac{\partial u_i}{\partial x_i} = 0$$ Mass (continuity)

(2) $$\rho \cdot \frac{du_i}{dt} = -2 \cdot \rho \cdot \varepsilon_{ijk} \cdot (\Omega_j \cdot u_k) + \rho \cdot g \cdot e_i - \frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j}$$ Navier-Stokes (Momentum)

(3) $$\rho \cdot c_p (\frac{\partial T}{\partial t} + (u_j \cdot \frac{\partial}{\partial x_j})T) - \alpha \cdot T \cdot \frac{dp}{dt} = \frac{\partial}{\partial x_i}(K \frac{\partial T}{\partial x_i}) + \rho \cdot H + \Phi$$ Energy (heat transport)

+initial and boundary conditions+geometry!!

- Coupled partial differential equation-system (5 equations)
- Unknown: u, T, p, ρ (6 unknowns)$\rightarrow$+ 1 equation of state: $\rho = \rho_0 \cdot [1 - \alpha (T - T_0)]$
- Solution: analytical, laboratory experiments, numerical: **Comsol Multiphyiscs**

Direct solver (UMFPACK)

II.2. Non-dimensional parameters

Rayleigh number: $$Ra = \frac{\rho_0 \cdot \alpha_0 \cdot g \cdot \Delta T \cdot D^3}{\kappa_0 \cdot \eta_0} = \frac{\text{buoyancy force}}{\text{viscosus force}}$$ *Control parameter*

Important !! Convection exists only if Ra > 10^3

Sphere	**Rayleigh number**
Atmosphere	**~10^{17}**
Mantle	**~10^{7}**

Table 1: values of Ra

Nusselt number: $$Nu = \frac{q_T}{q_c} = \frac{K\int_0^d \frac{\partial T(x, y=d)}{\partial y} dx}{K\int_0^d \frac{\Delta T}{d} dx} = \frac{1}{\Delta T}\int_0^d \frac{\partial T(x,y=d)}{\partial y} dx$$ Dimensionless surface heat flow

Root mean square velocity: $$v_{rms} = \frac{d}{\kappa} \cdot \left\{ \frac{1}{d^2} \cdot \int_0^d \int_0^d (u^2 + v^2) dxdy \right\}^{\frac{1}{2}}$$

II.4. Applied model

- 2D domain in Cartesian and in cylindrical geometry
- Mechanical boundary conditions: slip and symmetry.
- Thermal boundary conditions: vertical insulating walls, horizontal isothermal boundaries, T_1 at the CMB, T_0 on the surface, $T_1 > T_0$.
- In the models the Rayleigh number ranged between 10^4-10^7.
- The variation of Ra is determined by the change of temperature, in the case of mantle: $\Delta T = 0.918$, 9.18, 91.8, 918 K.

Physical quantity	Mantle	
ρ [kg/m^3]	4500	density
η [Pas]	$2 \cdot 10^{21}$	viscosity
C_p [J/kgK]	1200	heat capacity
K [W/mK]	5.4	thermal conductivity
α [1/K]	$2 \cdot 10^{-5}$	thermal expansion coefficient
g [m/s^2]	9.92	Gravitational acceleration
d [m]	$2.9 \cdot 10^6$	cell size

Table 2: Mantle's physical parameters

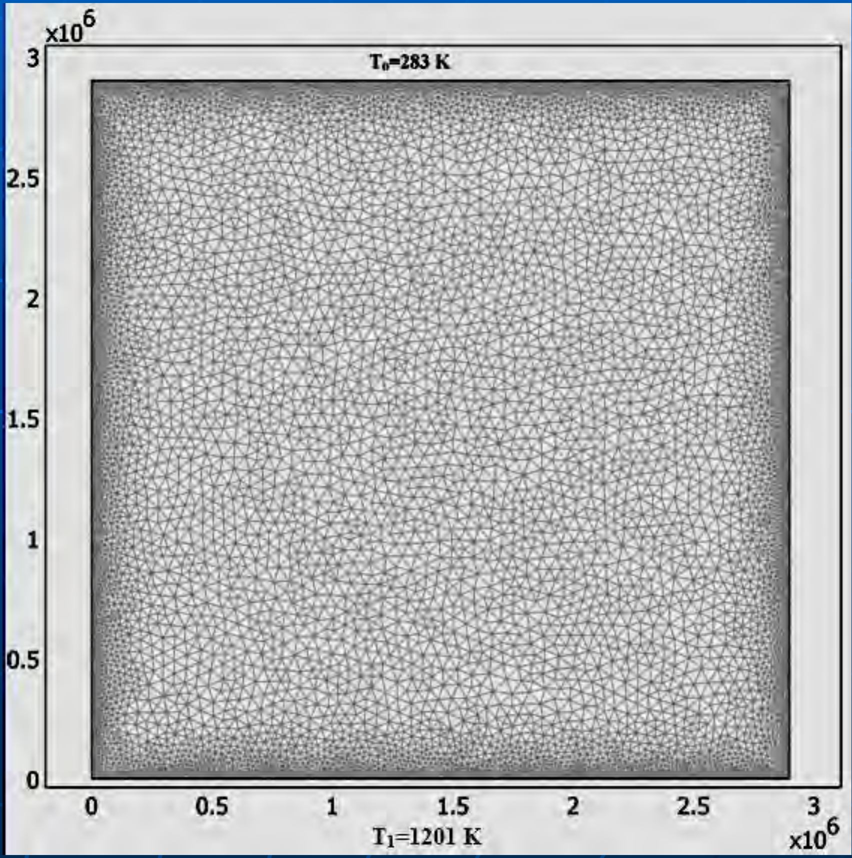


Figure 3. Finite element model, Ra=10^7

III. Results

III.1. Mantle convection in Cartesian geometry

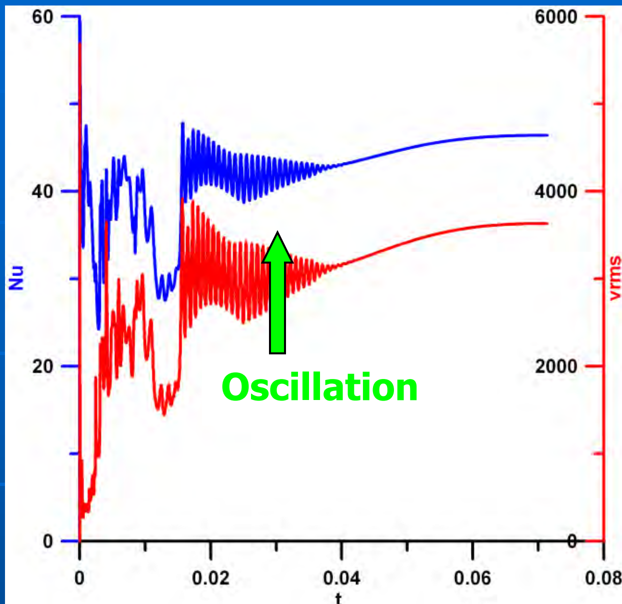


Figure 4. Nu (blue) and vrms (red) versus non-dimensional time, Ra=10^7 oscillating solution

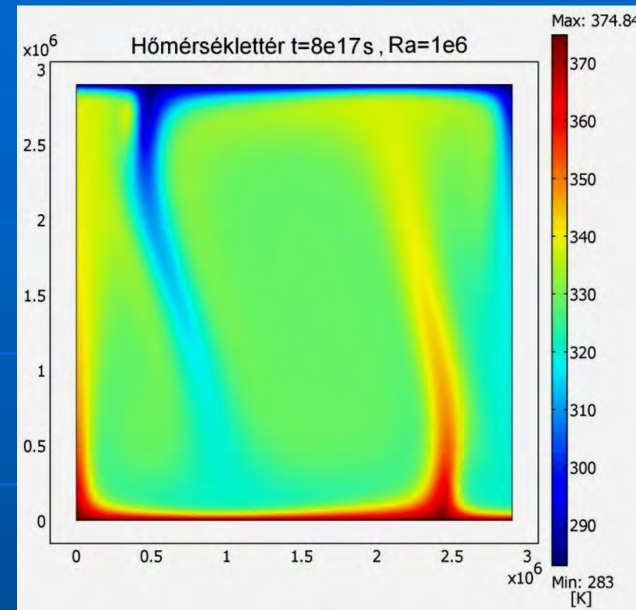
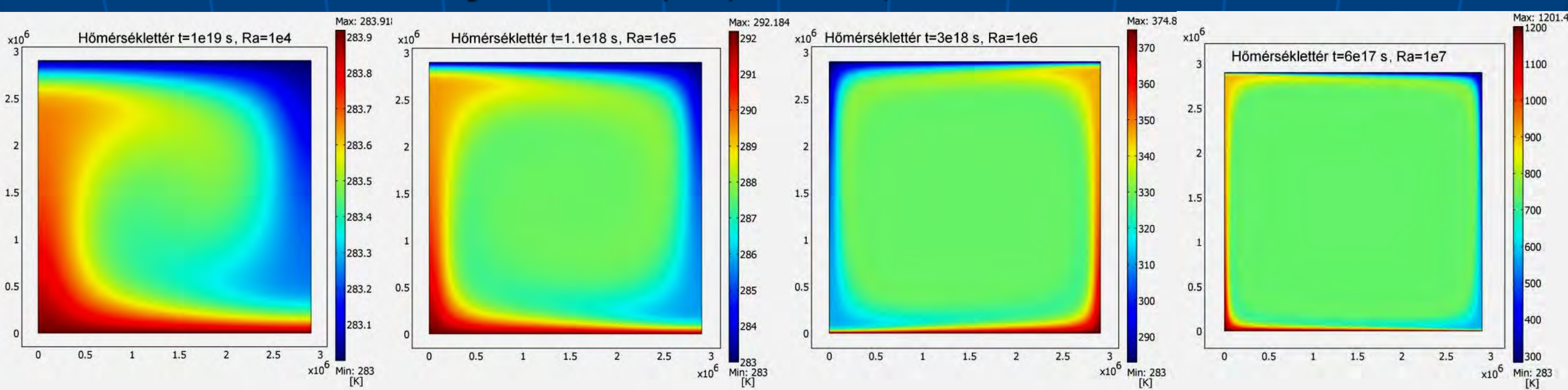


Figure 5. Oscillating solution, Ra=10^7

Figure 6. Stationary temperature field, Ra=10^4-10^7



III.2. Mantle convection in cylindrical geometry

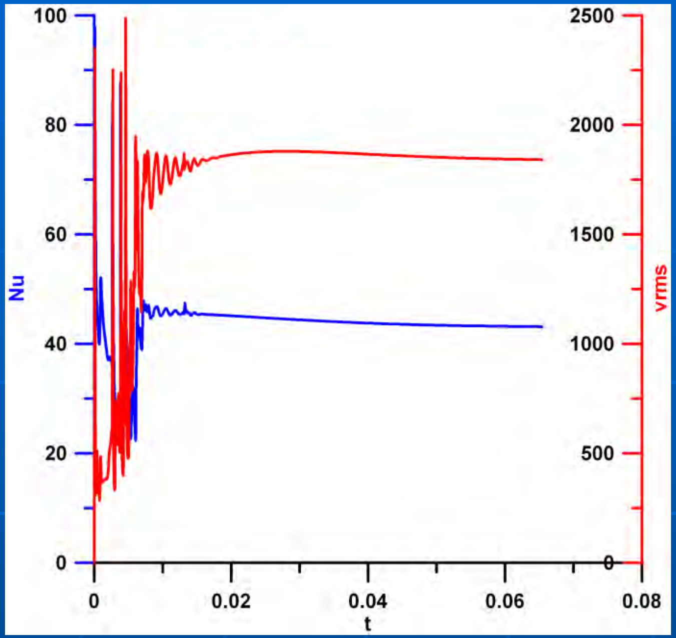
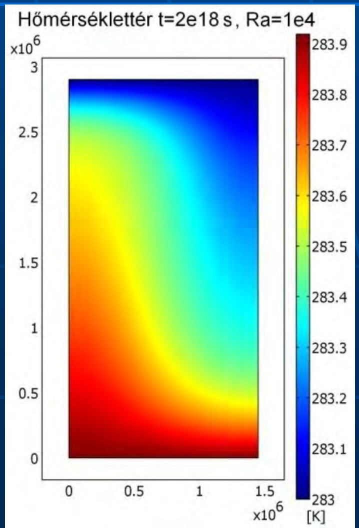
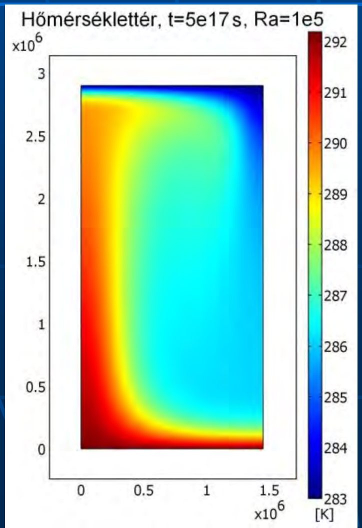
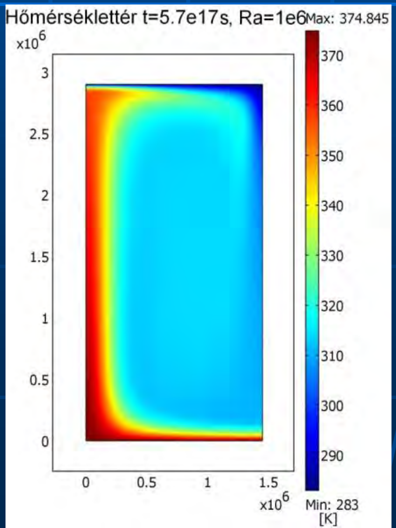


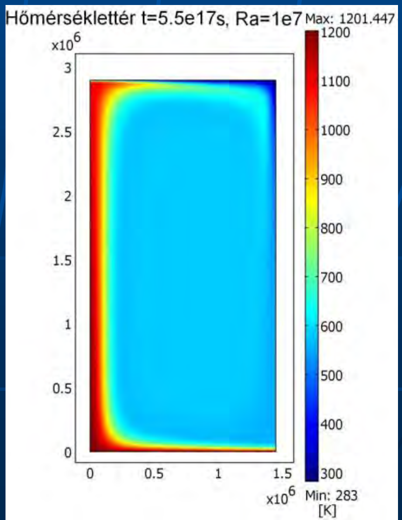
Figure 7. Nu (blue) and (red) versus non-dimensional time $Ra=10^7$

Figure 8. stationary temperature field, $Ra=10^4$-10^7









III.3. Mantle convection in cylindrical-shell geometry

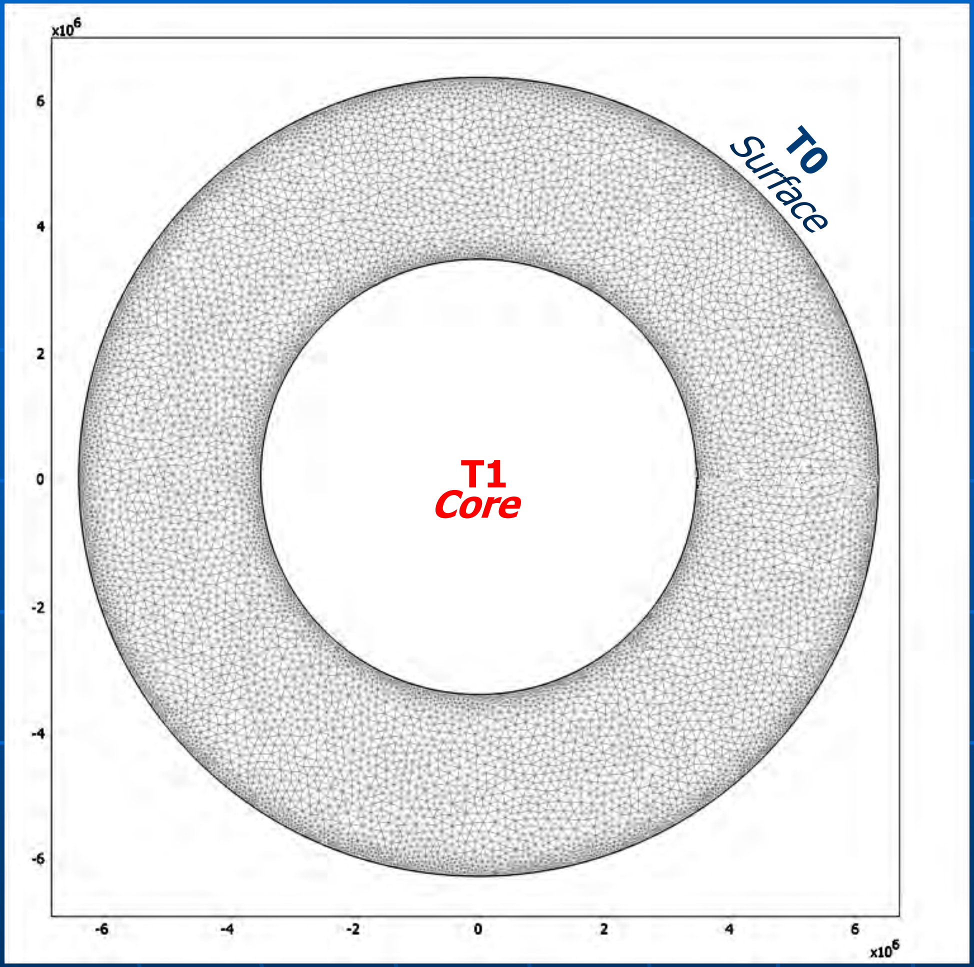


Figure 9. Discretization of a cylindrical-shell domain

- Mechanical boundary conditions: slip and symmetry
- Thermal conditions: at the CMB T_1 on the surface T_0 ($T_1 > T_0$)
- $Ra = 10^4 - 10^7$

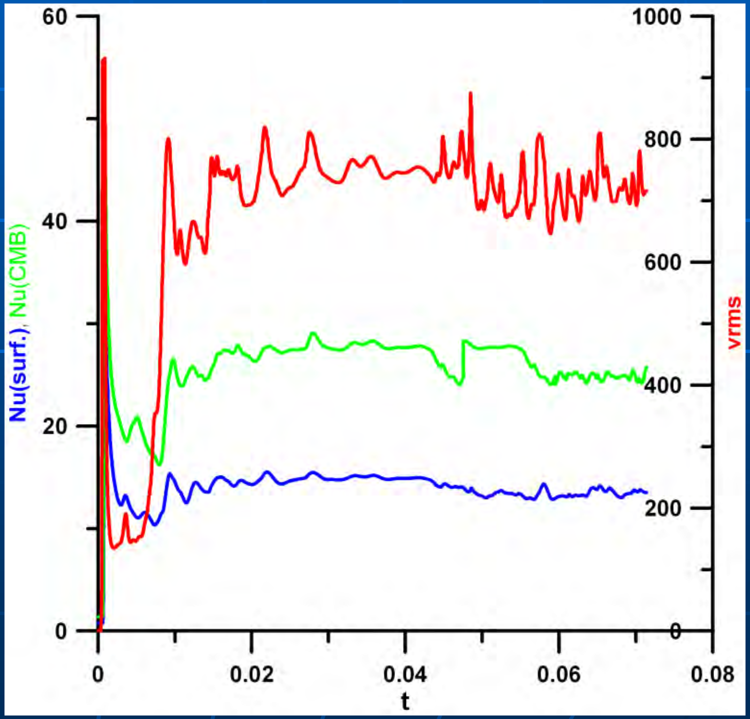


Figure 10. Nu (blue,green) and vrms(red) versus non-dimensional time, Ra=10^6 non-stationary solution

Stationary solutions: Ra=10^4-10^5

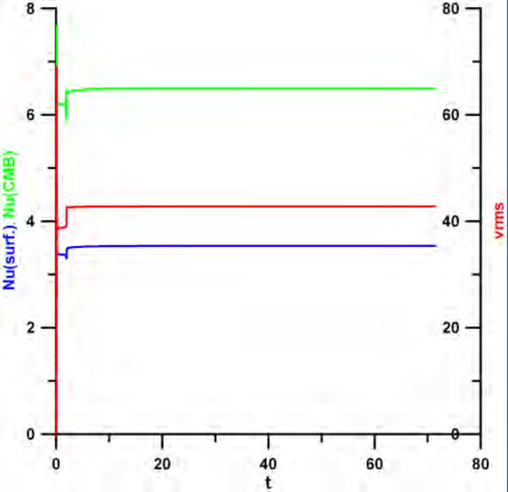


Figure 11. Nu (blue, green) and vrms(red) versus non-dimensional time Ra=10^4 stationary solution

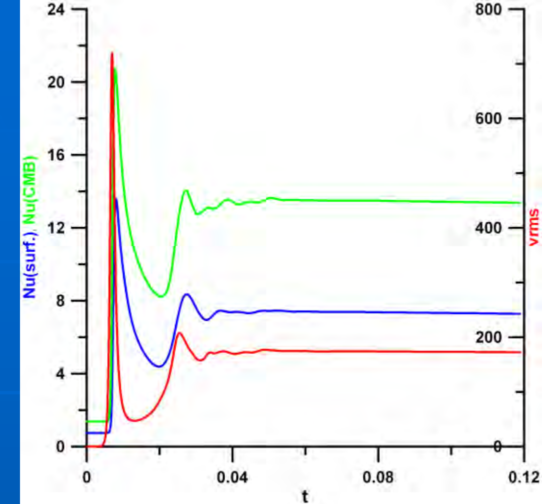


Figure 12. Nu (blue,green) and vrms (red) versus non-dimensional time Ra=10^5, stationary solution

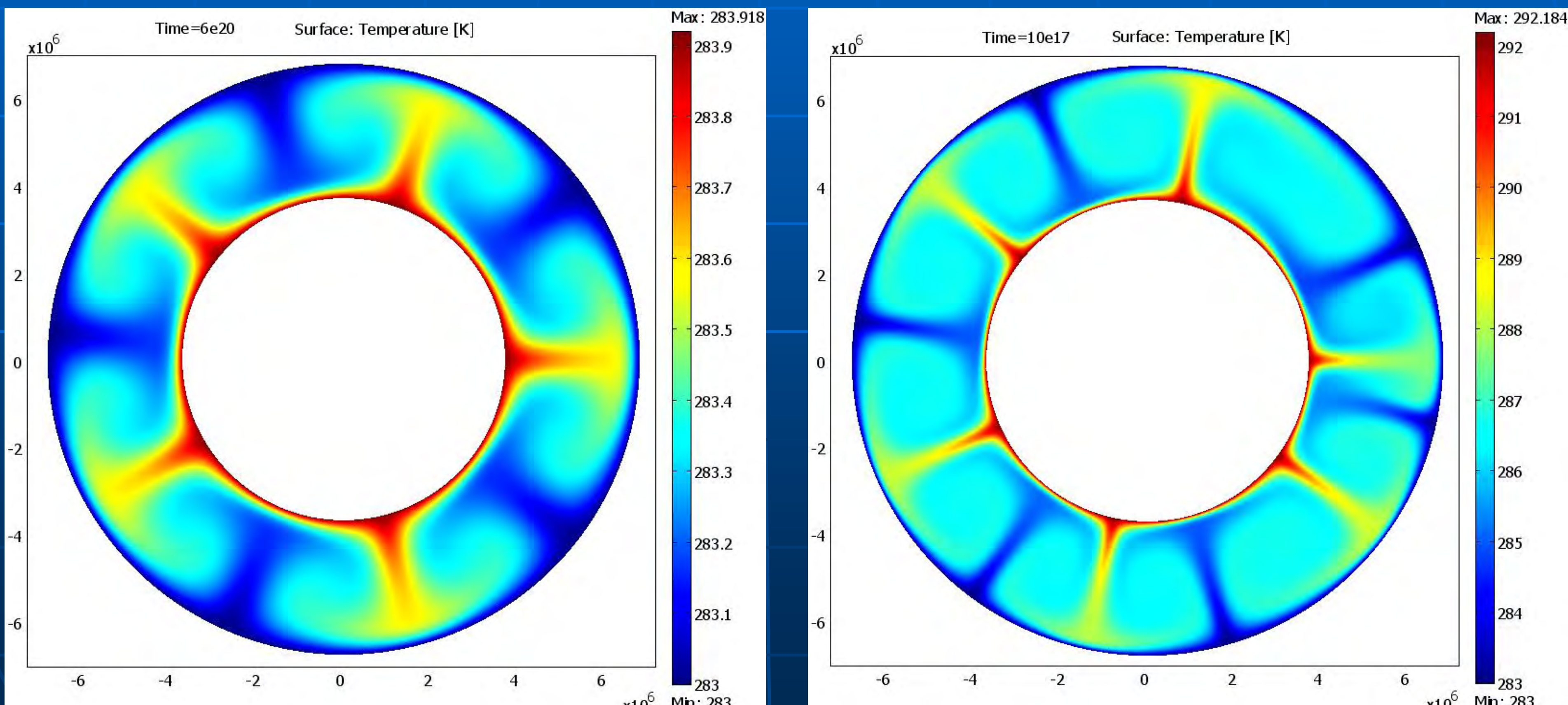


Figure 13. Stationary temperature field, Ra=10^4-10^5

Non-stationary solutions, Chaotic behaviour: $Ra=10^6$-10^7

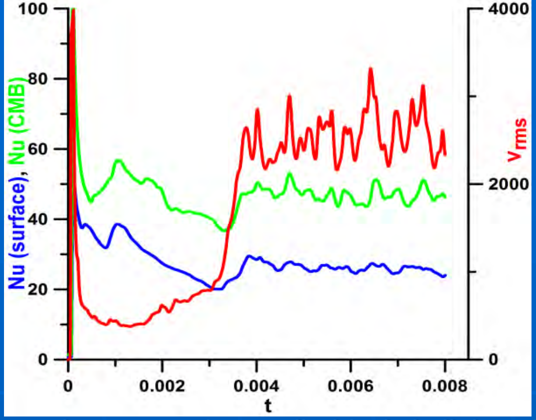


Figure 15. Nu and vrms vs. Time, cylindrical-shell domain, $Ra=10^7$

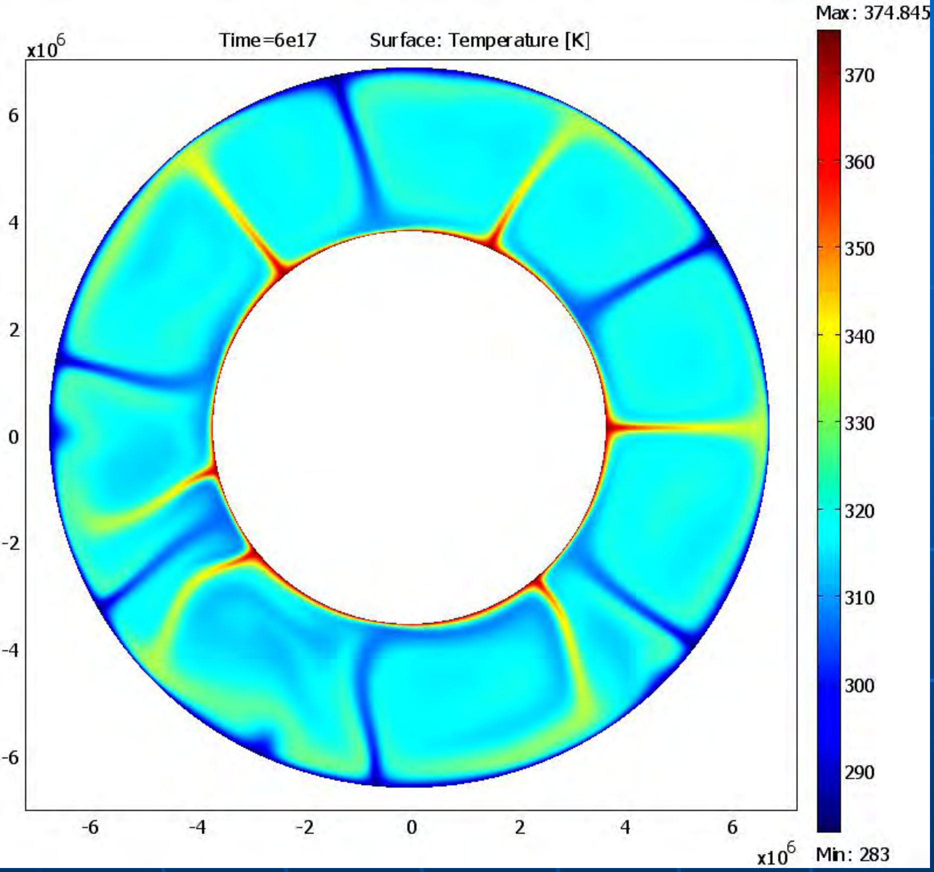


Figure 14. non-stationary temperature field, $Ra=10^6$

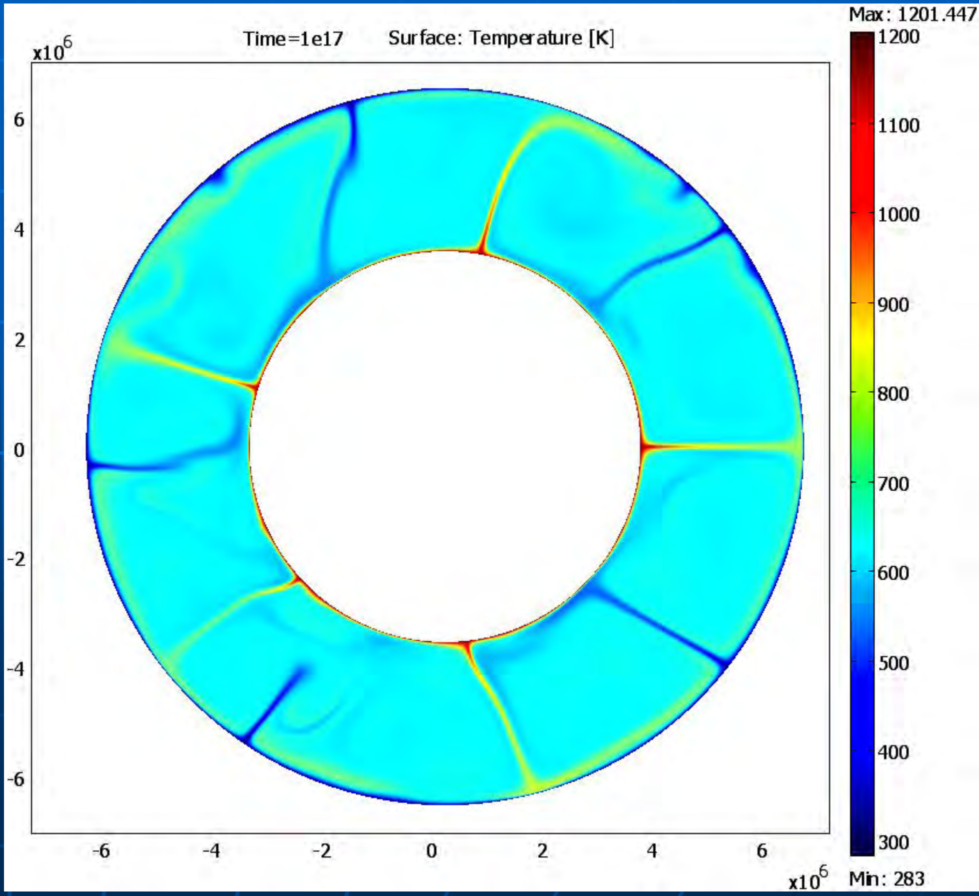
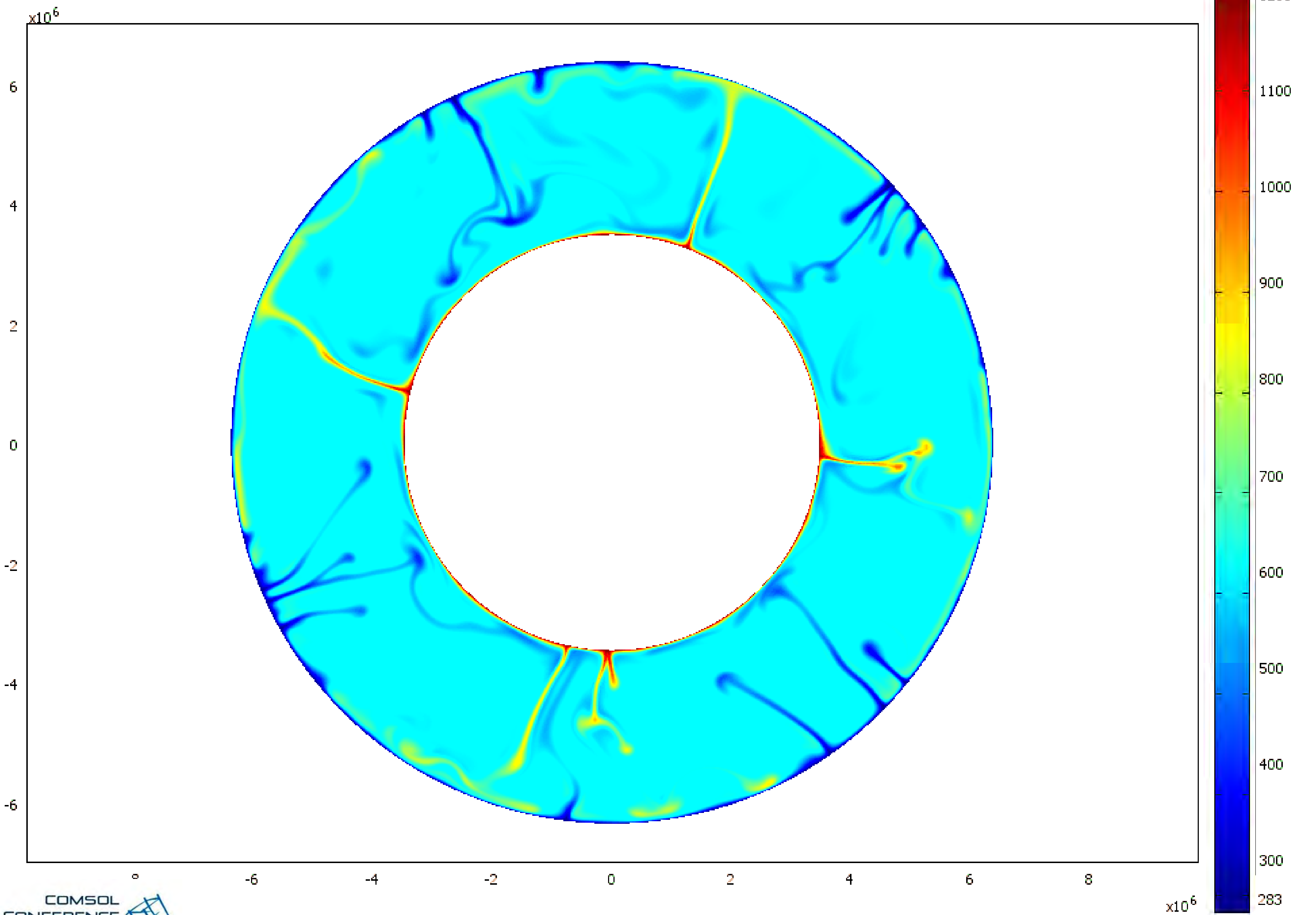


Figure 16. non-stationary temperature field, $Ra=10^7$

Ra=10^7 chaotic convection

V ~ 1 cm/year, q ~ 50 mW/m^2



IV. Interpretation

IV.1. Comparison

The final 2D Cartesian results were compared to BLANKENBACH et al's study.

	Ra	This work	BLANKENBACH ET AL. (1989)	Deviation [%]
Nu	10^4	4.88525	4.884409	0.0172
v_{rms}		42.864943	42.864947	0
Nu	10^5	10.567700	10.534095	0.319
v_{rms}		193.197400	193.21454	0.0088
Nu	10^6	22.061601	22.072465	0.0005
v_{rms}		833.991497	833.98977	0.0002

Table 3: Comparison between this study and Blankenbach's study

The deviation was within 0.5 % !

IV.2. Comparison between the Cartesian, cylindrical and cylindrical-shell geometry

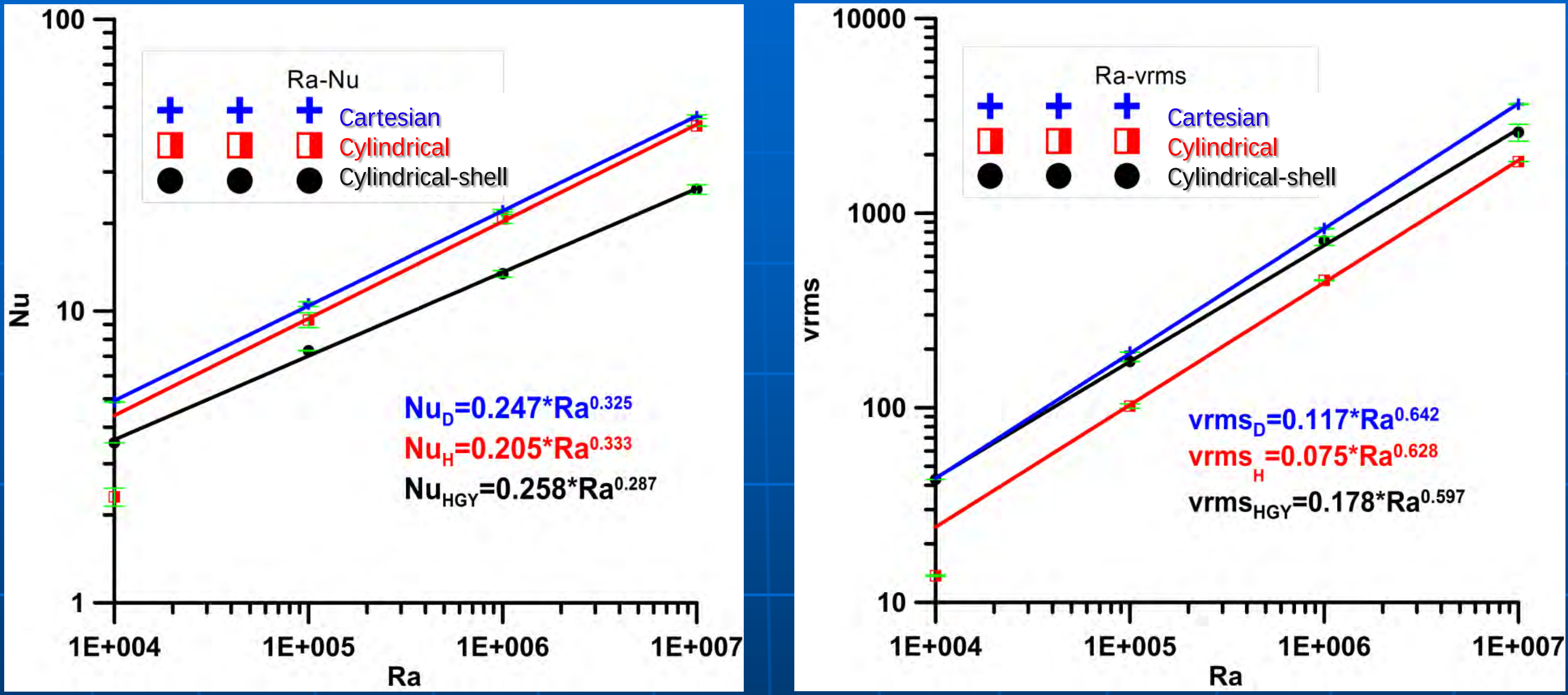


Figure 17. Relationship between the Rayleigh number and the non-dimensional parameters (Nu, vrms), in Cartesian (blue), cylindrical (red) and cylindrical-shell geometry (black)

Geometry's influence on the flow and the efficiency of flow → **Cylindrical geometry is the most effective!**

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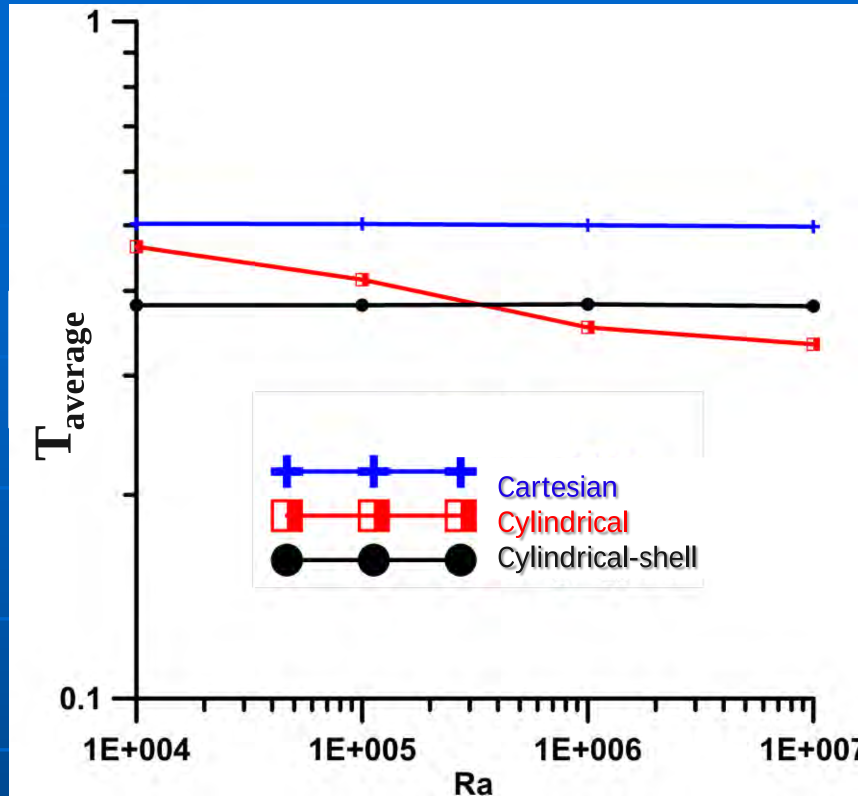

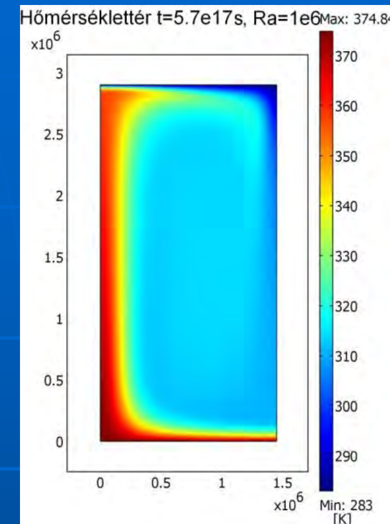

Asymmetrical convection

Figure 18. Relationship between the Rayleigh number and the average temperature, in Cartesian (blue), cylindrical (red) and cylindrical-shell geometry (black)

This shows that the average temperature is independent of the Rayleigh number in Cartesian and Cylindrical-shell geometries!!

In cylindrical geometry a permanent decrease in the average temperature can be observed.→ The average temperature of the cell is always determined by the stream flowing along the outer wall!!

V. Summary

- Comsol proved as a very good and flexible modelling tool.
- The results in 2D Cartesian geometry are practically identical with Blankenbach et al's study.
- Cylindrical geometry: The average temperature of the cell is always determined by the stream flowing along the outer wall!!
- Results in cylindrical geometry were close to the 3D results (in reality we imagine a plume in a cylindrical way ⇨ seismic tomography), very fast computation.
- Cylindrical system seems to be the most appropriate geometry to model individual plumes .
- In cylindrical-shell model we got an impressive picture of the chaotic structure of mantle convection, the mean velocity is very close to global Tectonics.

VI. Plans for the future

- Modelling in Cylindrical and in Cylindrical-shell domains to analyze the effect of viscosity and radioactive heat production.
- Thermochemical convection modelling. (Together with Dr. Attila Galsa).
- 3D modelling (Cartesian and Spherical geometry).
- Phase transition at 660 km, 2D, 3D studies, studying the impact of phase transition zone (Using ATLASZ Cluster at Eötvös Uni.). (In progress).

Acknowledgements

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Thank you very much for your attention!