EFFECT OF MASS ADSORPTION ON A RESONANT MEMS

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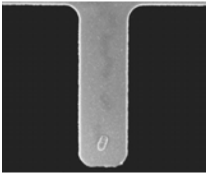
Outline

Introduction

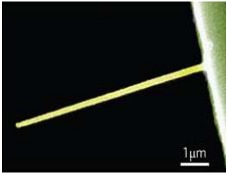
- In the last 15 years micro and nanoelectromechanical systems such as nanowires and cantilevers, have been developed and proposed for ultrasensitive mass detection.
- It has been reported that these systems are not just capable of detecting masses in the atto and zeptogram ranges, but they are devices to measure other properties of the adsorbate such as the Young's modulus.

Cantilevers



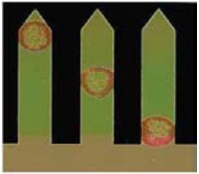
Ilic et al.: Single cell detection with micromechanical oscillators

Nanowires

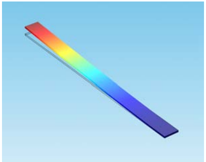


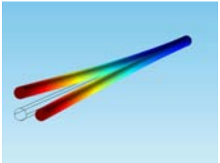
Gil-Santos et al.: Nature Nanotecnology 10.1038

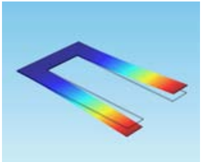
Arrays



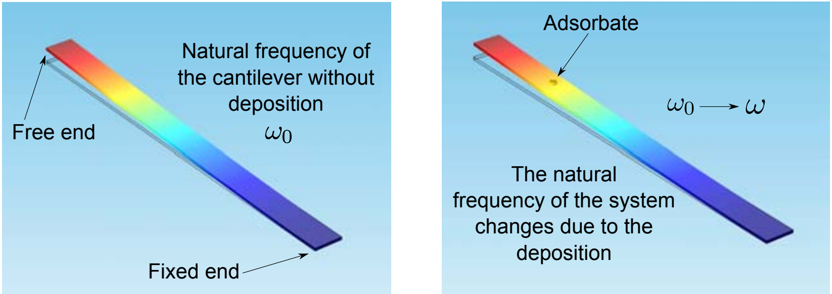
Harold Craighead.:Measuring more than mass





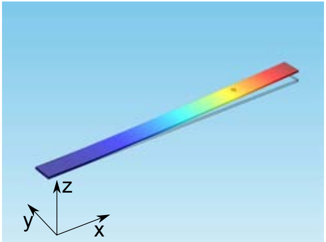


Basic principle



- We can measure frequency shifts for different modes of vibration.
- These frequency shifts give us information about the properties of the adsorbate.

- Before the adsorption, we calculate the natural frequency and mode shape ψ_n.
- We assume the adsorbate is small so the mode shape does not change after the deposition.
- Rayleigh-Ritz method is used to calculate the natural frequency of the system after the deposition.



Kinetic energy

$$K = \frac{1}{2}\int_0^L \rho A \left(\frac{\partial w}{\partial t}\right)^2 dx$$

Strain energy

$$U = \frac{1}{2}\int_0^L D \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx$$

where

$$w = w(x,t) = a\psi(x)e^{-i\omega t}$$

$$D = \int_A E z^2 dA$$

Particle

- If we can consider the adsorbate as a small particle with little contact with the cantilever surface, the frequency shift will be mostly due to the change in the total mass of the system.
- Aplying the Rayleigh-Ritz method we obtain the frequency shift.

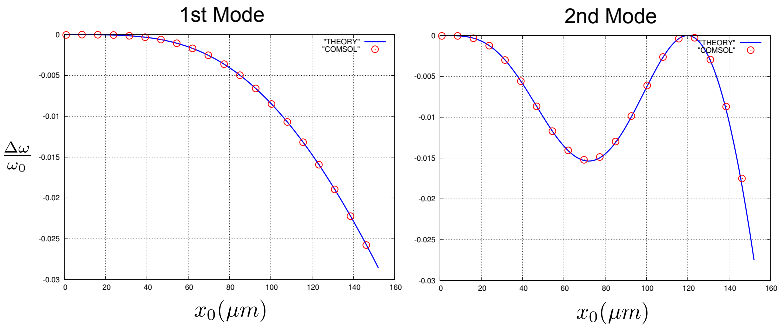
$$\frac{\Delta\omega}{\omega_0} \approx -\frac{1}{2}\frac{\Delta m}{m_0 + \Delta m}$$

where

$$\Delta m = m_{ad}\psi(x_0)^2$$

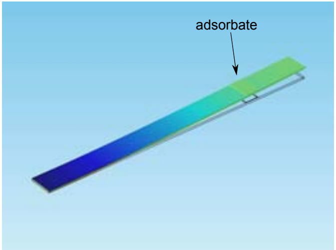
m_0 ≡cantilever mass, m_{ad} ≡adsorbate mass, x_0 ≡position of the adsorbate along the cantilever beam.

- We study the relative frequency shift as a function of the position of the adsorbate and then compare the theory with COMSOL simulations.
- The agreement between the simulation and the theory is very good.



Biological material with relevant surface of contact

- If the surface of contact is relevant, the adsorbate stiffness must be also taken into account.
- We consider a rectangular patch of a material with low young's modulus compared with that of the cantilever beam.



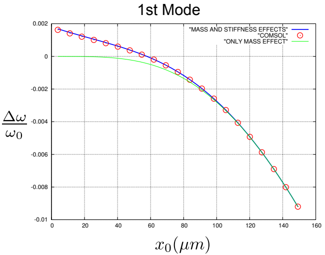
Effect of the adsorbate stiffness

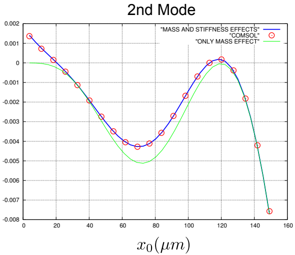
$$\frac{\Delta\omega}{\omega_0} \approx -\frac{\Delta m}{2(m_0+\Delta m)} + \frac{\Delta k}{2\omega_0^2(m_0+\Delta m)}$$

where

$$\Delta k \approx D_{ad} L_{ad} \left(\frac{d^2\psi(x_0)}{dx^2}\right)^2$$

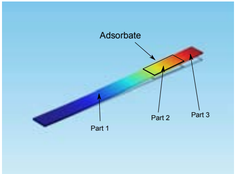
- Comparing the theory with the COMSOL simulation we observed again a good agreement.
- In the present case, the adsorbate has a Young's modulus which is 5% of the Young's modulus of the cantilever.
- If we had not considered the effect of the adsorbate stiffness there would be discrepancies.





Long adsorbates

- If the adsorbate is long enough, the Rayleigh-Ritz method is no longer valid as the mode shape changes.
- We must split the cantilever into three parts and each part will have its own differential equation.
- Then we apply continuity at all junctions and boundary conditions.



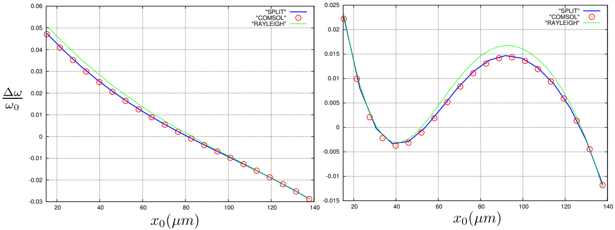
$$\frac{\partial^4 \psi_n}{\partial z^4} - \beta_n^4 \psi_n = 0 \qquad n = 1, 2, 3 \qquad \beta_n^4 = \omega^2 \frac{\rho_n A_n}{D_n}$$

$$\psi_n(z) = a_n sin(\beta_n z) + b_n cos(\beta_n z) + c_n sinh(\beta_n z) + d_n cosh(\beta_n z)$$

Boundary conditions and continuity

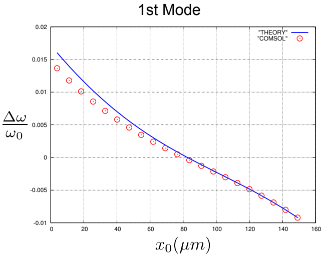
Fixed end	$\psi_1(0) = 0$	$\psi_1'(0) = 0$		
1st Discontinuity	$\psi_1(z_1) = \psi_2(z_1)$	$\psi_1'(z_1) = \psi_2'(z_1)$	$D_1\psi_1''(z_1) = D_2\psi_2''(z_1)$	$D_1\psi_1'''(z_1) = D_2\psi_2'''(z_1)$
2nd Discontinuity	$\psi_2(z_2) = \psi_3(z_2)$	$\psi_2'(z_2) = \psi_3'(z_2)$	$D_2\psi_2''(z_2) = D_3\psi_3''(z_2)$	$D_2\psi_2'''(z_2) = D_3\psi_3'''(z_2)$
Free end	$\psi_3''(L) = 0$	$\psi_3'''(L) = 0$		

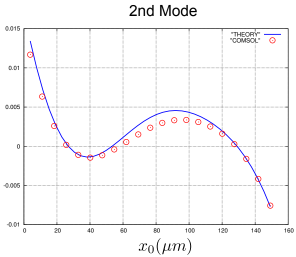
- In the present case, the adsorbate has a length which is 20% of the cantilever length.
- We can see the agreement between COMSOL and the theory that we expected.
- Although Rayleigh gets quite close, it is not very accurate for some zones.



Material with high Young's modulus

- As the Young's modulus of the patch is getting greater, we can observe certain discrepancies between the theory and the COMSOL simulation.
- In the present case, the adsorbate has a Young's modulus which is half the Young's modulus of the cantilever.





CONCLUSIONS

- Rayleigh is very accurate for particles and materials with low Young's modulus.
- If the adsorbate is long enough, Rayleigh gets close to the COMSOL simulation. However to accurately predict the frequency shift we must take into account the change in the mode shape.
- For materials with high Young's modulus the theory does not fit completely well the simulations.

References

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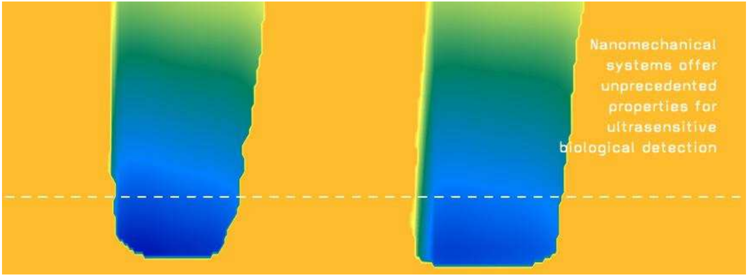

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