Using Superposition Principle and Edge Current Model to Compute Impedance of Coil in Logging Tool

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Abstract: Coil antennas are widely used in the well-logging tools. Accurate design needs to know how to model the coil impedance in the presence of formation environment, and with the complex tool structure, e.g., the collar and the shield housing. This paper introduces how to accurately compute the impedance of the coil with very thin wires at high frequency with complex 3D configuration. To predict the resonant frequency, stray capacitance is also obtained in the impedance computation.

Keywords: Edge Current Model, Inductance, AC Resistance, Stray Capacitance, Input impedance

1. Introduction

Antenna input impedance is one of the main parameters in induction/propagation tool design for well-logging application. It allows the system to design the current, power input and the signal noise ratio for the antenna.

The impedance of the coil antenna can be simplified to RLC circuit network. Below resonant frequency, the impedance of the coil is simply a series resistance with inductance. The resistance comes from various losses of the structure: the conductor loss from the collar, shield, and dielectric loss from the insulator the coil is embedded. The inductance is from the coil, which is a magnetic energy storage component. The impedance Z can be expressed as $Z = R + j\omega L$ under time harmonic condition. The amplitude and phase of Z can be easily represented as amplitude and phase:

$$|Z| = \sqrt{R^2 + (\omega L)^2} \text{ , and } \tan^{-1} Z = \frac{\omega L}{R}.$$

There are many published literature on analytical formulas for resistance and inductance [1-6].

In theory, the general formula to calculate low frequency self inductance is Neumann formula:

$$L = \frac{\mu}{4\pi} \int_C \int_{C'} dS \cdot dS' / |R|,$$

where C and C' are curves spanned by wires, dS and dS' are the line segment, and R is distance between two points. In Figure 1, for a helix coil, the two line segment dS and dS', and the distance between them

$|R|$ are shown. When the frequency is high, the skin effect causes the electric current to flow mainly
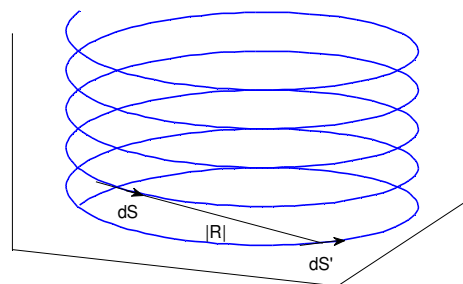

Figure 1. The helix coil, the line segment and distance between them are shown.

at the "skin" of the conductor, thus a zero R will cause error. For example, for an AWG 18 wire, the diameter is 1mm, and the skin depth is 0.1mm and 0.046mm at 400 kHz and 2 MHz, for a copper conductor. It is obvious that current density is the greatest at the surface rather than in the center of the conductor. The distance R between two points is smaller than 0.1mm. Even with some correction terms introduced, skin effect makes numerical result difficult to obtain. COMSOL is a FEM-based software, and the computation depends on the meshing of the region of interest, and there is a limit on the minimum mesh to get an accurate numerical value.

In well-logging environment, as shown in Figure 2, a metal 3D housing structure surrounds the coil for protection, and a big conducting cylinder is located in the center of the coil. Meanwhile, the coil sits on top of insulating material between the conducting collar and the housing. The complex environment makes the analytical method hard to obtain.

There are many manuals and papers dedicated in computing the inductance of the coil from COMSOL database. In COMSOL version 4.0 above, inductance can be computed and output in postprocessor directly. Helix coil can be drawn directly and surface current density can be assigned to excite the coil to get the inductance. However, the meshing of the wire in high frequency is difficult, and the setup in COMSOL needs to extend the feed and output to the outer

boundaries, which is hard with the presence of the outside housing structure in our study.

In addition, at high frequencies, the effect of the capacitance starts to show up. With the introduction of $1/j\omega C$, where C represents the capacitance and the capability of storing electric energy, the impedance expression is $Z = 1/\left(1/\left(R + j\omega L\right) + j\omega C\right)$. With the shield and collar present, the parasitic capacitance comes from two parts, the internal one from turn-to-turn capacitance, and the external one from the turn to the grounded housing and collar. An iterative method [7] is used to compute the overall capacitance from the distributed capacitance.

In this paper, edge current model is used in COMSOL to compute the inductance and resistance, and a correction of AC resistance is then applied. Capacitance is obtained from electrostatics application module. Numerical results are presented and compared to measured results.
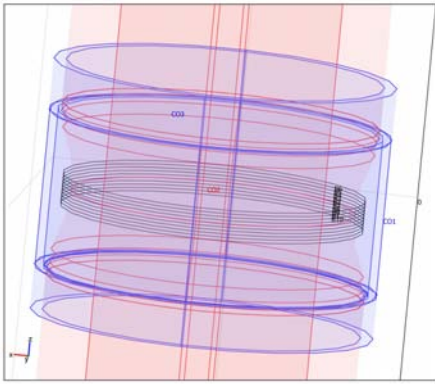

Figure 2. The 3D structure surrounding the coil, the blue part represents the housing, and the orange part is the collar.

2. Use of COMSOL Multiphysics

The accurate numerical modeling of the input impedance can be separated into three parts: the modeling for the inductance and resistance, and the modeling for the parasitic capacitance.

2.1. Modeling of inductance:
For L and R modeling, COMSOL uses 3D RF Module with electromagnetic waves harmonic propagation application. The reason that a 3D model is required comes from the complex 3D housing configuration, which is also the reason for the application of numerical method. Because the current concentrates on the surface at the frequencies of interest, an alternative form for the wires of the coil can be used: edge current. Instead of trying to mesh the finite dimension of the wire in the order of the

skin depth, the thin wire is reduced to infinitesimally thin edge. To avoid a blown up value from small R, alternate arrangement of transmitter and receiver within the coil is used to make sure R between the source and field is not zero. Figure 3. shows the alternate assignment of transmitters and receivers turns for an example of a 10-turn coil. The red and blue colors in the figure are used to differentiate the transmitters and receivers.

The weights applied on currents on 5 transmitters T1-T5 are 0.125, 0.25, 0.25, 0.25 and 0.125. The distribution gives higher weight on the center turns of the coil, and lower weight on the edge currents. The overall sum of weight is 1. We set the 5 transmitters to have a total of I=1[A]. The induced voltages on the 5 receivers R1-R5 have the same weight distribution, and the overall sum of weight is 1. The induced voltage of the receiver is
Vtotal=0.125*(V1+V5)+0.25*(V2+V3+V4),
where V1-V5 are the induced voltage in receivers R1-R5 respectively. The voltage is obtained in "Edge Integration Variables" by selecting the receiver coils, then defining the variable as
V=(tEx*t1x*signz+tEy*t1y*signz+tEz*t1z*signz),
where tEx,y,z are the tangential E-field, t1x,y,z are the tangential vectors, and
signz=sign((x-x0)*t1y-(y-y0)*t1x),
is the sign extraction of the normal vector, the cross product of the radial vector and the tangential vector. The setup helps to get the correct sign for the edge current to ensure right-hand rule. The variable is then integrated to get the voltage. The impedance Z then is computed using Z=N^2*(Vtotal/I), where N is the number of turns. Vtotal/I computes one turn only because the weight sum up to 1. The imaginary part of the Z gives the inductance.
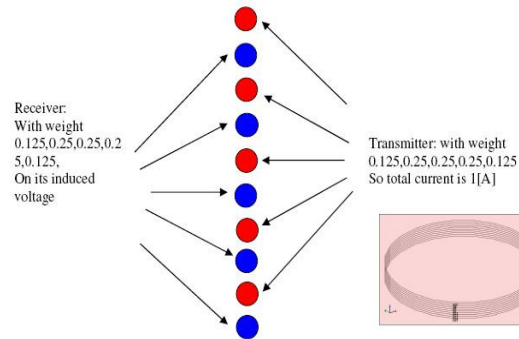

 Figure 3. The alternating Transmitter and Receiver assignment for 10-turn coil.

2.2. Correction of AC resistance

The real part of the Z from section 2.1 is the resistance. Since the wire is modeled as an edge, the AC resistance will not be accurate. To correct the inaccuracy from the simplification of the model, a superposition method is applied to model the impedance more accurately. The first step is to model the impedance of the coil in air only using COMSOL, and it is called $Zair\_num$. Then we use the same coil, but with collar, housing and insulator material, and the numerical value is called $Zcollar$. Analytical term $Zair\_anal$ of the coil in air can be found from papers [3-5]. It is found that the Rac (AC resistance) increases with frequency, and Lac is almost constant. Numerical result for $Zair$ confirms that the imaginary part divided by angular frequency ω is constant, indicating that L value only changes with frequency slightly and the change can be ignored. The corrected impedance is: $Zcollar - Zair\_num + Zair\_anal$ should be the accurate value. Lac_anal=30.76μH, is the analytical inductance in air without the 3D collar and shield, and is obtained by using equation in ref[8-9]. The Lac_num=30.14 μH, is the numerical inductance in air.

2.3. Stray Capacitance

There are turn-to-turn capacitances and turn-to-ground capacitances. In COMSOL electrostatics study, each turn is defined as a port, and has an input voltage. The housing and collar are connected as ground. COMSOL can obtain the capacitance between each port, defined as C11, C21, C31……C101. C11 is the self capacitance or the one versus the ground, and C21 is the adjacent one from the top turn to the second turn, and C31 is the capacitance of the third turn versus the first turn. With the increase of distance between turns, the non-adjacent capacitance drops in the order of 10. To simplify the final capacitance calculation, only the adjacent capacitances are included in the model. Figure 4 shows the simplified diagram for capacitance only. C11, C22, … C1010 are simplied to Cg in the diagram, and C12=C21, and the mutual capacitance terms are equal, and indicated as Ct (turn-to-turn) in Figure 4.

The COMSOL 2D axial-symmetric model is shown in Figure 5. The 3D housing structure is simplified to a 2D strucuture, and together with the collar, they are defined as ground in the model.
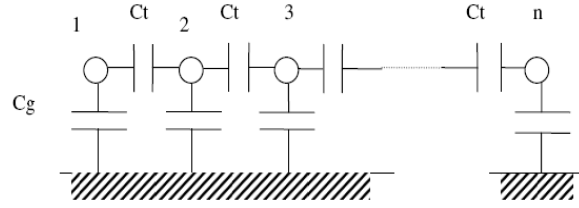

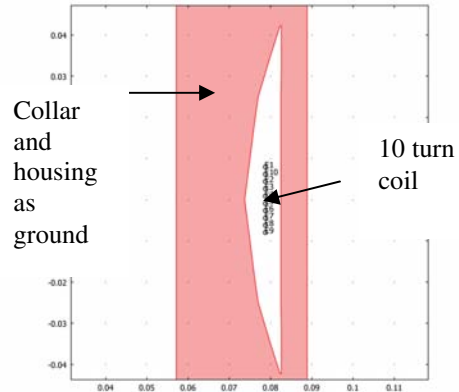**Figure 4**. The simplified equivalent circuit for a single-layer air-core inductor with a shield.

Figure 5. The 2D axial-symmetric geometry setup in COMSOL for capacitance computation.

3. Numerical Results

The field plot from COMSOL model of the coil antenna with the collar and the housing is shown in Figure 6. Table 1 shows the COMSOL impedance of the coil with the collar. The detailed values: the analytical value of the AC resistance Rac, the COMSOL modeled $Zcollar$, and the corrected impedance of coil with collar $Zcollar*$, as described in section 2 are compared with the measured results $Zmeas$. The difference between numerical and measured results are listed in the table as well. It shows the impedance value at two chosen frequencies: 400kHz and 2MHz. It shows that the error for magnitude are less than 6% at 2MHz, and less than 1% at 400kHz. The impedance frequency response is shown in Figure 7.
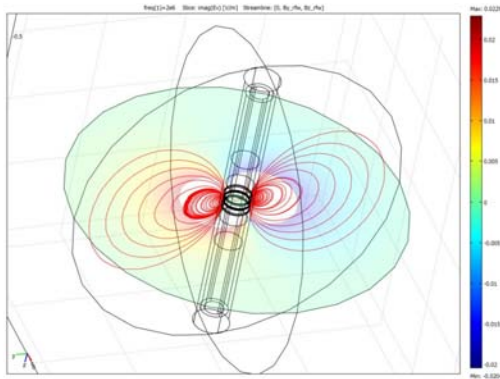
Figure 6. The electric and magnetic field plot for the antenna from COMSOL.

Table 1. The comparison of modeling and experiment result of input impedance.

| Freq. | COMSOL Model | | Analytical |
| | Z_{COLLAR} | Z_{AC} (air) | Z_{AC} (air) |
(MHz)	(Ω)	(Ω)	(Ω)
0.4	1.54 + i22.29	0.0032+i76.03	0.49+i77.28
2.0	3.56 + i107.30	0.08+i378.78	1.03+i386.42

| Freq. | Final | Measurement | Z_{collar}– Z_{MEAS} | |
| | Z_{TOTAL}* | Z_{MEAS} | Mag. | Phase |
(MHz)	(Ω)	(Ω)	(Ω)	(°)
0.4	2.03 + i23.54 (23.63∠ 85.08°)	22.43 ∠85.54°	0.09	0.51
2.0	4.51 + i114.94 (115.03 ∠87.75°)	101.4∠ 88.02°	5.96	0.08

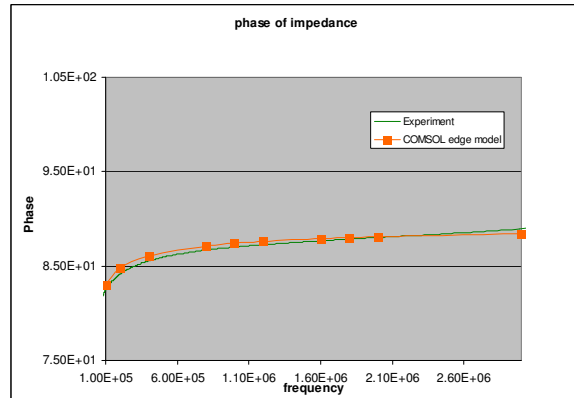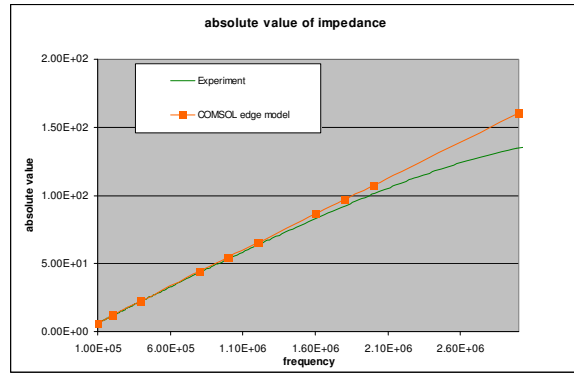*Z_{TOTAL}=Z_{COLLAR}-Z_{AIR_COMSOL}+$Z_{AIR-ANALYTIC}$

Figure 7. Impedance versus frequency from modeling and measured result for coil antenna.

It is observed that the edge current model with alternating TX and RX configuration provides impedance close to the measurement result. The further correction using addition of analytical coil R and L calculation in air result, and subtraction of coil in air of numerical result can potentially improve the accuracy, but it is highly depended on the accuracy of both the numerical result and analytical formula prediction. Our result shows that the AC resistance correction is not critical, while the L difference between the analytical and numerical will be enlarged in higher frequency.

The potential field plot for the internal turn-to-turn stray capacitance is shown in Figure 8. The one turn wire over ground plane potential field is shown in Figure 9.
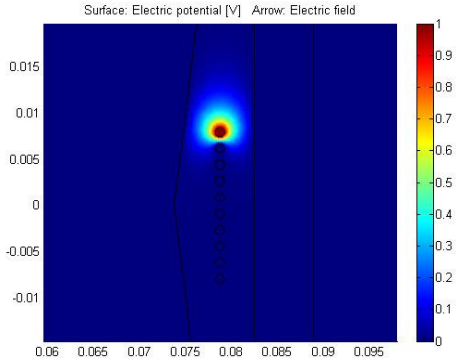
Figure 8. Electric potential field when the top turn is excited as the source port.
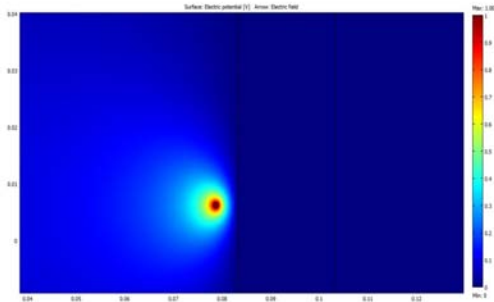

Figure 9. One turn wire over ground plane potential field plot.

The whole matrix for every turn of the 10-turn coil is shown below, with non-adjacent terms ignored. The first and last row of the term have only two none-zero values, one turn-to-ground C and one turn-to-turn C. For the intermediate turns, there are 2 turn-to-turn Cs and 1 turn-to-ground C. The matrix first row includes C11, C12, and the second row includes C21, C22, anc C23, etc. The matrix capacitance can be related to Figure 4.

C=[0.0159   0.0075   0   0   0
      0   0   0   0   0;
    0.0075   0.0198   0.0073   0   0
      0   0   0   0   0;
      0   0.0073   0.0198   0.0073   0
      0   0   0   0   0;
      0   0   0.0073   0.0198   0.0073
      0   0   0   0   0;
      0   0   0   0.0073   0.0198
    0.0073   0   0   0   0;
      0   0   0   0   0.0073
    0.0198   0.0073   0   0   0;
      0   0   0   0   0
    0.0073   0.0198   0.0073   0   0;
      0   0   0   0   0

0   0.0073   0.0199   0.0073   0;
      0   0   0   0   0
      0   0   0.0073   0.0199   0.0075;
      0   0   0   0   0
      0   0   0   0.0075   0.0161] nF;

The overall capacitance from the two end turns is obtained from the method introduced in Appendix[7], and the intermediate capacitance versus number of nodes are plotted below in Figure 10. It converged to 10.78pF after 5-6 turns.
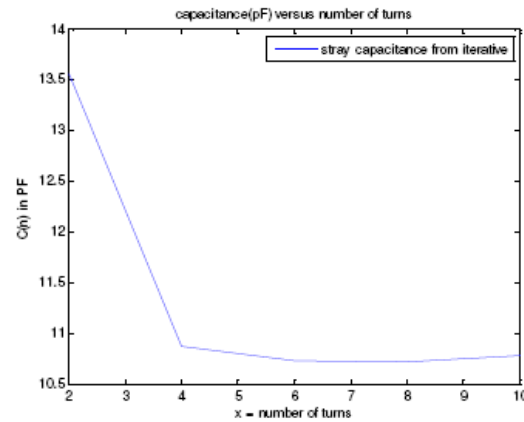

Figure 10. The overall capacitance in pF versus number of turns.

4. Application

The fiber glass is the insulator in between the collar and the housing. When water permeates the region, the original insulator electrical properties will change, depending to the porosity and how much water will occupy the pores. For simplicity, the case that water totally occupying the region is modeled. We assume AC resistance change with frequency, and inductance and capacitance stay relatively constant: $L = 8\mu H$, and $C=10.897pF$. Meanwhile the water invasion changes the permittivity of the insulator. Suppose ε_r (relative permittivity) is changed from air (=1) to water-filled (=80). The insulator has a dielectric constant in the range of 3 to 10. The impedance will be modified as

$$Z = 1/\left(1/\left(R + j\omega L\right) + j\omega \varepsilon_r C\right),$$

where C is the stray capacitance in air.

Below in Figure 11 shows the change of impedance with different ε_r (=1, 33, 67 and 100). The resonant frequency drops from 17MHz, to 2.9M, 2.08M and 1.705MHz. With water invasion, the

permittivity increases, thus the stray capacitance increases, and the resonant frequency decreases. From around 500kHz, with water invaded, the impedance starts to show dramatic change.
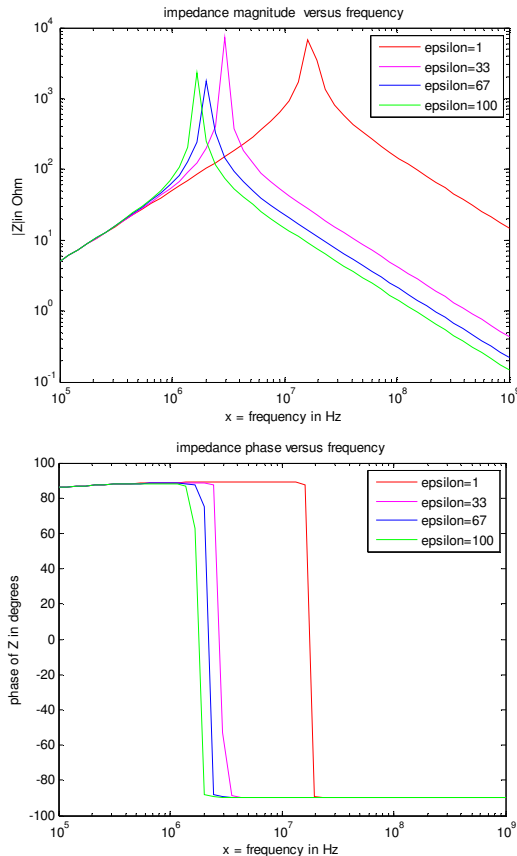

Figure 11. The magnitude and phase of the coil self impedance, with different dielectric constant for the insulator inside the housing.

5. Conclusions

This paper shows how to compute the input impedance accurately for a coil antenna with complex 3D structures surrounding it at high frequency. The inductance can be predicted accurately from the edge current model by alternating the source and receiver in the coil. The AC resistance is corrected by superposition principle. The stray capacitance is obtained from 2D electrostatic module, and an iterative method is used to get the overall capacitance. The model is used to investigate a water-invasion scenario in logging environment. Comparison with experiment result verifies the modeling setup and accuracy.

6. References

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7. Appendix

The following shows how to calculate overall stray capacitance:

(a) For odd number of turns (even number of nodes)

Figure 12 shows how to compute C starting from the middle two nodes. The center one-turn equivalent capacitance is $Ct(2)=Ct0+Cg/2$, where Ct0 is the capacitance of turn-to-turn, and it is shown in both Figure 4 as Ct, and in C matrix as C12, C21 etc as the mutual capacitance term; and Cg is the turn-to-ground capacitance. After adding two more turns, $Ct(n)=1/(1/Ct0+1/Ct0+1/C(n-2))+Cg/2$.

Normalized Cnt(n)=Ct(n)/Ct0 is defined. Cnt(n) versus number of nodes across the turns shows that after 5 or 6 turns the C converges.
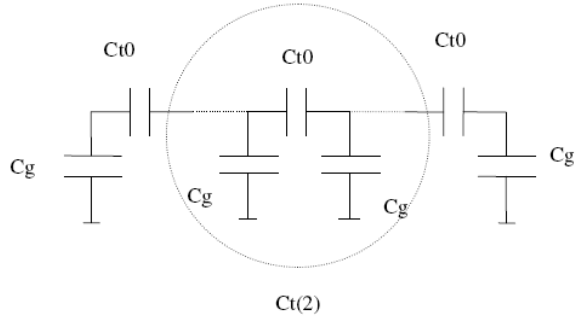

Figure 12. The capacitance network computation from center two nodes(one turn) demonstrating how to compute Ct(2) and Ct(4).

(b) For even number of turns (odd number of nodes)

To compute the capacitance for 10 turns, we can start with the middle two turns Ct(3), where the T network is transformed to Pi network first as shown in Figure 12:

C1=((1/(Ct*Ct)+1/(Ct*Cg)+1/(Ct*Cg))*Ct,
C2=((1/(Ct*Ct)+1/(Ct*Cg)+1/(Ct*Cg))*Cg,
C3=((1/(Ct*Ct)+1/(Ct*Cg)+1/(Ct*Cg))*Ct,

Then Ct(3) is obtained using similar iterative method to get 10 turns capacitance. Ct(3)=1/(1/(C1+Cg) + 1/(C3+Cg))+C2. Since C1=C3, Ct(3)=C2+(C1+Cg)/2. It indicates that method (a) and (b) are both applicable because both results converge after 5-6 turns.
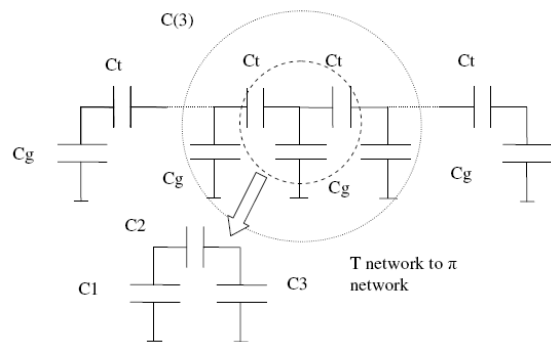

Figure 13. The capacitance network computation from center three nodes(two turns) on how to compute Ct(3) and Ct(5).