An Analysis of Spin Diffusion Dominated Ferrofluid Spin-up Flows in Uniform Rotating Magnetic Fields

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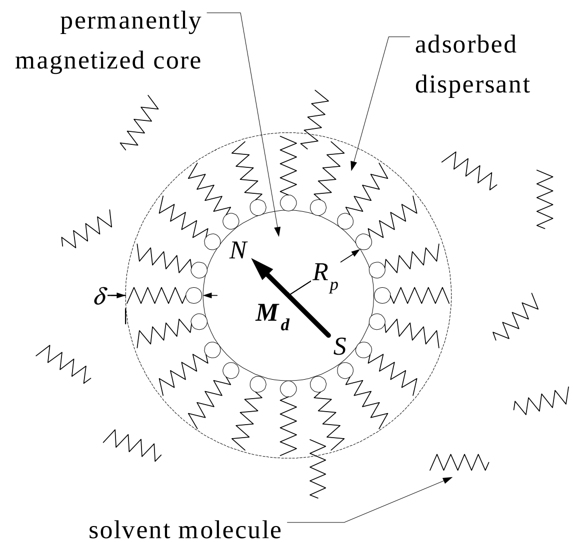
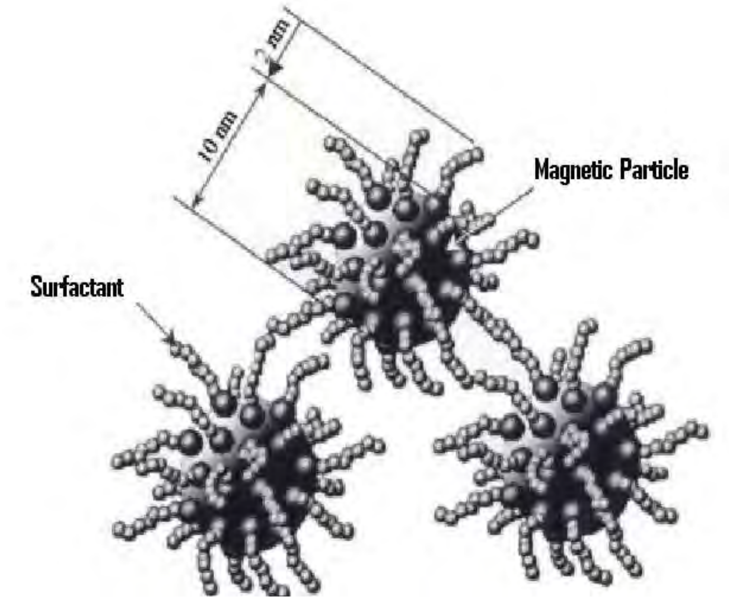
Ferrofluids

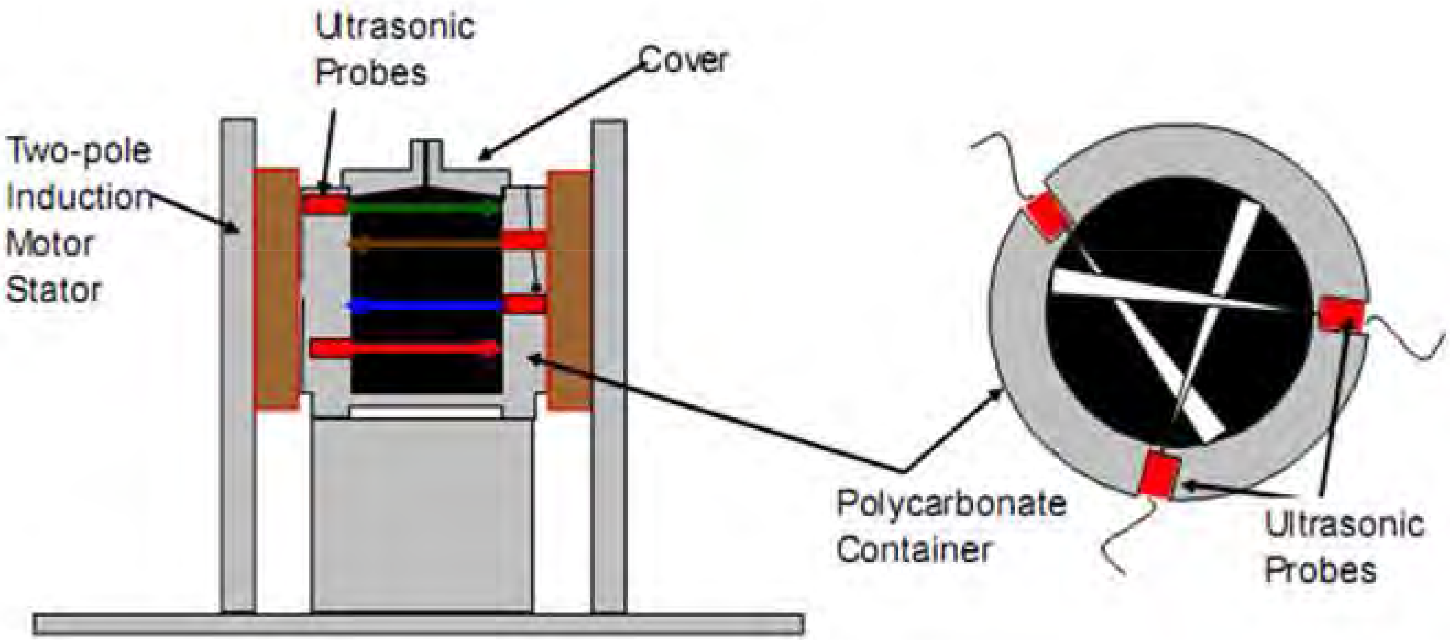
- Ferrofluids
  - Nanosized particles in carrier liquid (diameter~10nm)
  - Super-paramagnetic, single domain particles
  - Coated with a surfactant (~2nm) to prevent agglomeration
- Applications
  - Hermetic seals (hard drives)
  - Magnetic hyperthermia for cancer treatment





S. Odenbach, *Magnetoviscous Effects in Ferrofluids: Springer, 2002.*

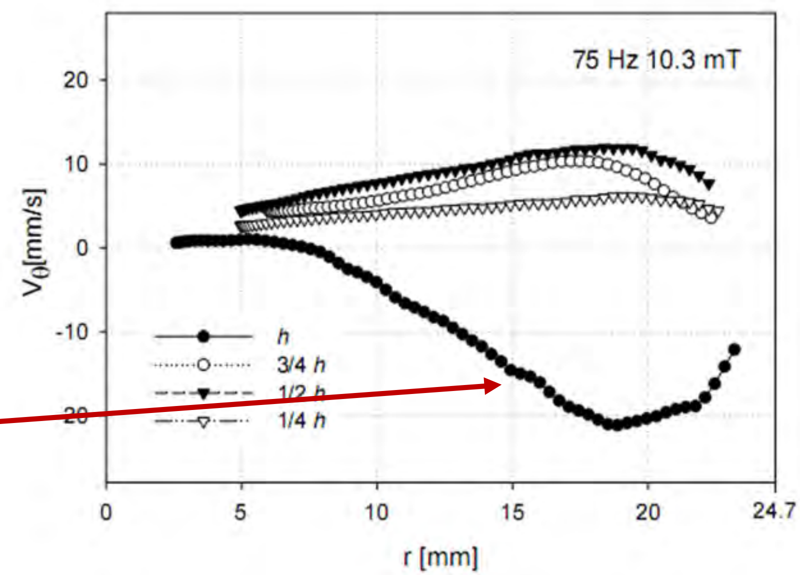
Bulk Spin-up flow experiments

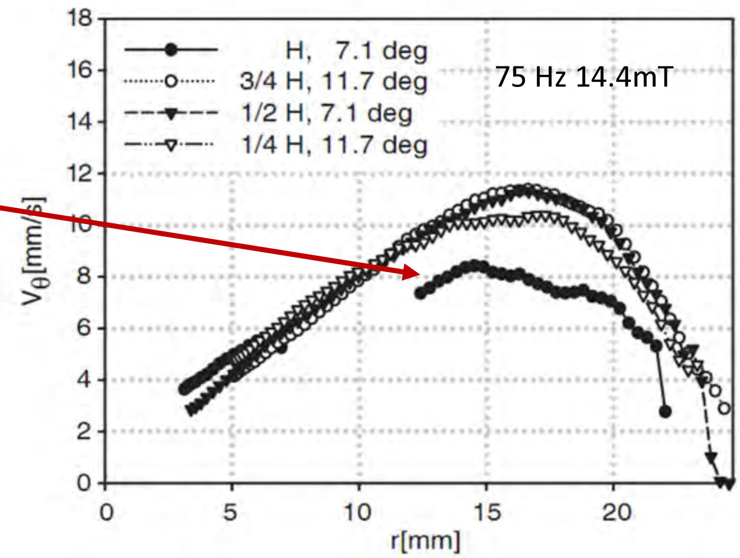


A. Chaves, C. Rinaldi, S. Elborai, X. He, and M. Zahn, *Bulk flow in ferrofluid in a uniform rotating magnetic field*, Physical Review Letters 96 (2006), no. 19, 194501-4.

Surface and Bulk driven flows

- Bulk flow velocity profiles co-rotate with the field
- If there is a free surface, there is *counter-rotation* at the surface (concave)
- If there is no free surface there is *co-rotation* near the surface





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Bulk Spin-up Flows

- Inhomogenous heating of fluid and spatial variation in magnetic susceptibility driving flow [1-4]
- Non-uniform magnetic field due to demagnetizing effects associated with shape of finite height cylinder [5-7]

1. Pshenichnikov*, et al.*, "On the rotational effect in nonuniform magnetic fluids," *Magnetohydrodynamics,* vol. 36, pp. 275-281, 2000.
2. A. V. Lebedev and A. F. Pshenichnikov, "Motion of a magnetic fluid in a rotating magnetic field," *Magnetohydrodynamics,* vol. 27, pp. 4-8, 1991.
3. M. I. Shliomis*, et al.*, "Ferrohydrodynamics: An essay on the progress of ideas," *Chem. Eng. Comm.,* vol. 67, pp. 275 - 290, 1988.
4. A. V. Lebedev and A. F. Pschenichnikov, "Rotational effect: The influence of free or solid moving boundaries," *Journal of Magnetism and Magnetic Materials,* vol. 122, pp. 227-230, 1993.
5. S. Khushrushahi, "Ferrofluid Spin-up Flows in Uniform and Non-uniform Rotating Magnetic Fields," PhD, Dept. of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, 2010.
6. S. Khushrushahi and M. Zahn, "Ultrasound velocimetry of ferrofluid spin-up flow measurements using a spherical coil assembly to impose a uniform rotating magnetic field," *Journal of Magnetism and Magnetic Materials,* vol. 323, pp. 1302-1308, 2011.
7. S. Khushrushahi and M. Zahn, "Understanding ferrofluid spin-up flows in rotating uniform magnetic fields," in *Proceedings of the COMSOL Conference*, Boston, 2010.

Spin Diffusion Model

- Neglects demagnetizing effects associated with shape of finite height cylinder
- Experimental fit values of spin viscosity are many orders of magnitude greater than theoretically derived values
- This work analyzes the Spin Diffusion model

1. S. Khushrushahi and M. Zahn, "Ultrasound velocimetry of ferrofluid spin-up flow measurements using a spherical coil assembly to impose a uniform rotating magnetic field," JMMM, vol. 323, pp. 1302-1308, 2011.
2. S. Khushrushahi and M. Zahn, "Understanding ferrofluid spin-up flows in rotating uniform magnetic fields," in *Proceedings of the COMSOL Conference*, Boston, 2010.
3. R. E. Rosensweig, *Ferrohydrodynamics*: Dover Publications, 1997.
4. K. R. Schumacher*, et al.*, "Experiment and simulation of laminar and turbulent ferrofluid pipe flow in an oscillating magnetic field," *Physical Review E,* vol. 67, p. 026308, 2003.
5. O. A. Glazov, "Role of higher harmonics in ferrosuspension motion in a rotating magnetic field," *Magnetohydrodynamics,* vol. 11, pp. 434-438, 1975.

Magnetic Field Equations

- Maxwell's equations for non-conducting fluid

$$\nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \cancel{\frac{d\mathbf{D}}{dt}} = 0$$

$$\mathbf{H} = -\nabla \psi$$

$$\mathbf{B} = \mu_0 \left(\mathbf{H} + \mathbf{M} \right)$$

$$\nabla^2 \psi = \nabla \bullet \mathbf{M}$$

- Magnetic Relaxation Equation

$$\frac{\partial \boldsymbol{M}}{\partial t} + \mathbf{v} \bullet \nabla \boldsymbol{M} - \boldsymbol{\omega} \times \boldsymbol{M} + \frac{1}{\tau_{eff}} (\boldsymbol{M} - \boldsymbol{M}_0) = 0$$

- Langevin Equation

$$\boldsymbol{M}_0 = \boldsymbol{M}_s [\coth(a) - \frac{1}{a}], a = \frac{\mu_0 H_0 M_d V_p}{kT}$$

$$\frac{1}{\tau_{eff}} = \frac{1}{\tau_B} + \frac{1}{\tau_N} \quad \tau_B = 3V_h \frac{\eta_0}{kT}, \tau_N = \frac{1}{f_0} exp\left(\frac{K_a V_p}{kT} \right)$$

$\mathbf{M_s}$ [Amps/m] represents the saturation magnetization of the material, $\mathbf{M_d}$ [Amps/m] is the domain magnetization (446kA/m for magnetite), V_h is the hydrodynamic volume of the particle, V_p is the magnetic core volume per particle, T is the absolute temperature in Kelvin, $k = 1.38 \times 10^{-23}$ [J/K] is Boltzmann's constant, f_0 [1/s] is the characteristic frequency of the material and K_a is the anisotropy constant of the magnetic domains

Spin-diffusion Governing Equations

- Extended Navier-Stokes Equation

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\bullet\nabla)\mathbf{v}\right]=-\nabla p+\mu_0(\mathbf{M}\bullet\nabla)\mathbf{H}+2\zeta\nabla\times\boldsymbol{\omega}+(\lambda+\eta-\zeta)\nabla(\nabla\bullet\mathbf{v})+(\zeta+\eta)\nabla^2\mathbf{v}$$

(=0, *Neglecting Inertia*; =0, *Incompressible flow*)

  - Boundary condition on **v,** $\mathbf{v}(r=R_{wall})=0$

- Conservation of internal angular momentum

$$J\left[\frac{\partial \boldsymbol{\omega}}{\partial t}+(\mathbf{v}\bullet\nabla)\boldsymbol{\omega}\right]=\mu_0(\boldsymbol{M}\times\boldsymbol{H})+2\zeta(\nabla\times\mathbf{v}-2\omega)+(\lambda'+\eta')\nabla(\nabla\bullet\omega)+\eta'\nabla^2\boldsymbol{\omega} \qquad \zeta=\frac{3}{2}\eta\phi$$

(=0, *Neglecting Inertia*; =0)

  - Boundary condition on **ω** unless η’=0, $\boldsymbol{\omega}(r=R_{wall})=0$

ρ [kg/m³] is the ferrofluid mass density, p [N/m²] is the fluid pressure, ζ [Ns/m²] is the vortex viscosity, η [Ns/m²] is the dynamic shear viscosity, λ [Ns/m²] is the bulk viscosity, ω [s⁻¹] is the spin velocity of the ferrofluid, v is the velocity of the ferrofluid, J [kg/m] is the moment of inertia density, η′ [Ns] is the shear coefficient of spin viscosity and λ′[Ns] is the bulk coefficient of spin viscosity, ϕ[%] is the magnetic particle volume fraction

Assumptions

- Applied field not strong enough to magnetically saturate the fluid

$$\mathbf{M}_{eq} = \chi \mathbf{H}_{fluid}$$

- Low Reynolds number flow – inertial effects set to 0
- Infinitely long cylinder – no demagnetizing effects

$$\boldsymbol{\omega} = \omega_z \mathbf{i_z}$$

Theoretical solution computed using Mathematica

$$\mathrm{v}_{\varphi}(r) = v_0 \left[\frac{r}{R} - \frac{I_1(\kappa r)}{I_1(\kappa R)} \right]$$

$$\omega_z(r) = \frac{\zeta + \eta}{\eta(R)} \left(\frac{\mu_0 |\mathrm{M}| |\mathrm{H}_{\mathrm{fluid}}|}{4\zeta} \sin\alpha \right) \left[1 - \frac{I_0(\kappa r)}{I_0(\kappa R)} \right]$$

$$\eta(R) = \eta + \zeta \left[1 - \frac{2 I_1(\kappa R)}{\kappa R I_0(\kappa R)} \right]$$

$$v_0 = \frac{1}{2\kappa\eta(R)} \left(\mu_0 |\mathrm{M}| |\mathrm{H}_{\mathrm{fluid}}| \sin\alpha \right) \frac{I_1(\kappa R)}{I_0(\kappa R)}$$

$$\kappa^2 = \frac{4\eta\zeta}{(\zeta + \eta)\eta'}$$

$$x^3 - (\Omega\tau_{\mathrm{eff}}) x^2 + (P+1) x - \Omega\tau_{\mathrm{eff}} = 0$$

$$P = \frac{\mu_0 \mathrm{M}_{\mathrm{eq}} \mathrm{H}_{\mathrm{fluid}} \tau_{\mathrm{eff}}}{4\zeta}$$

$$x = \tan\alpha$$

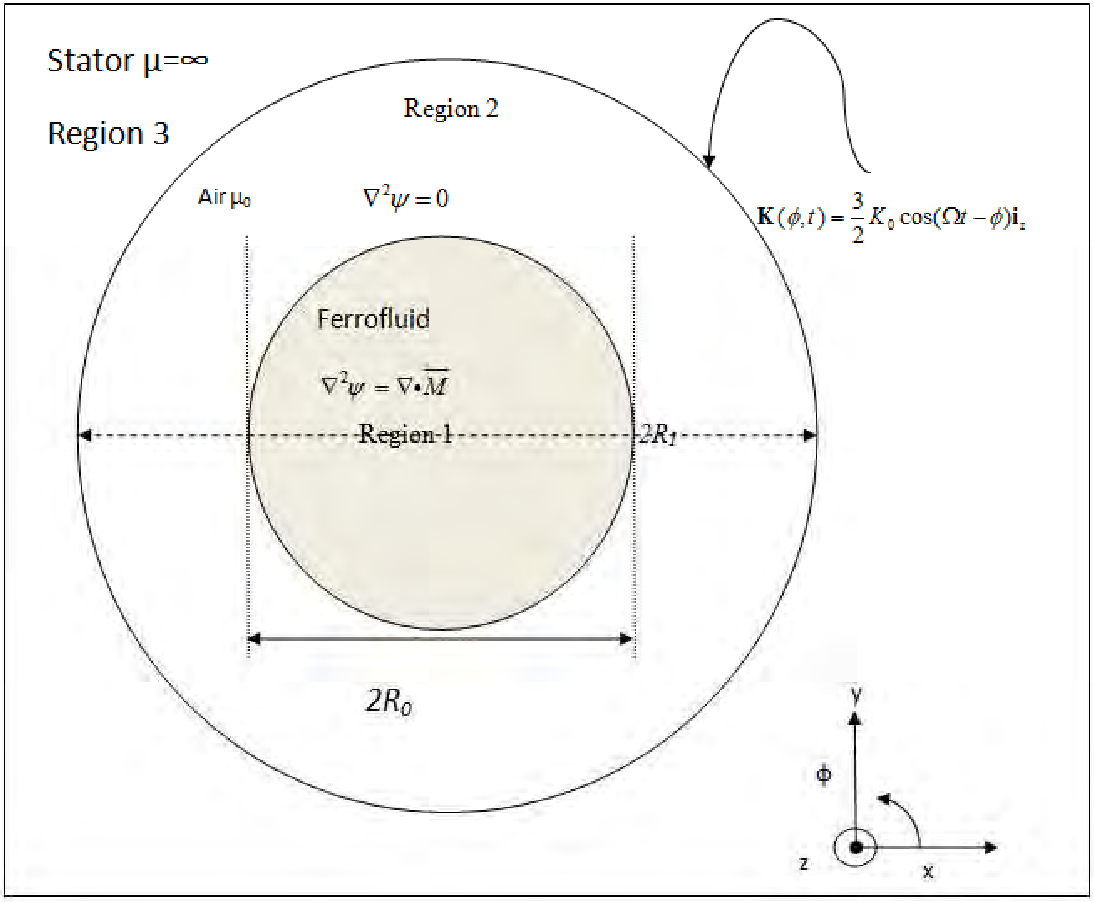
$$\mathrm{H}_{\mathrm{fluid}_x} = \mathrm{H}_{\mathrm{applied}_x} - \frac{1}{2}\mathrm{M}_x$$

$$\mathrm{H}_{\mathrm{fluid}_y} = \mathrm{H}_{\mathrm{applied}_y} - \frac{1}{2}\mathrm{M}_y$$

1. R. E. Rosensweig, *Ferrohydrodynamics*: Dover Publications, 1997.
2. V. M. Zaitsev and M. I. Shliomis, "Entrainment of ferromagnetic suspension by a rotating field," *Journal of Applied Mechanics and Technical Physics,* vol. 10, pp. 696-700, 1969.

Modeling the Magnetic Field

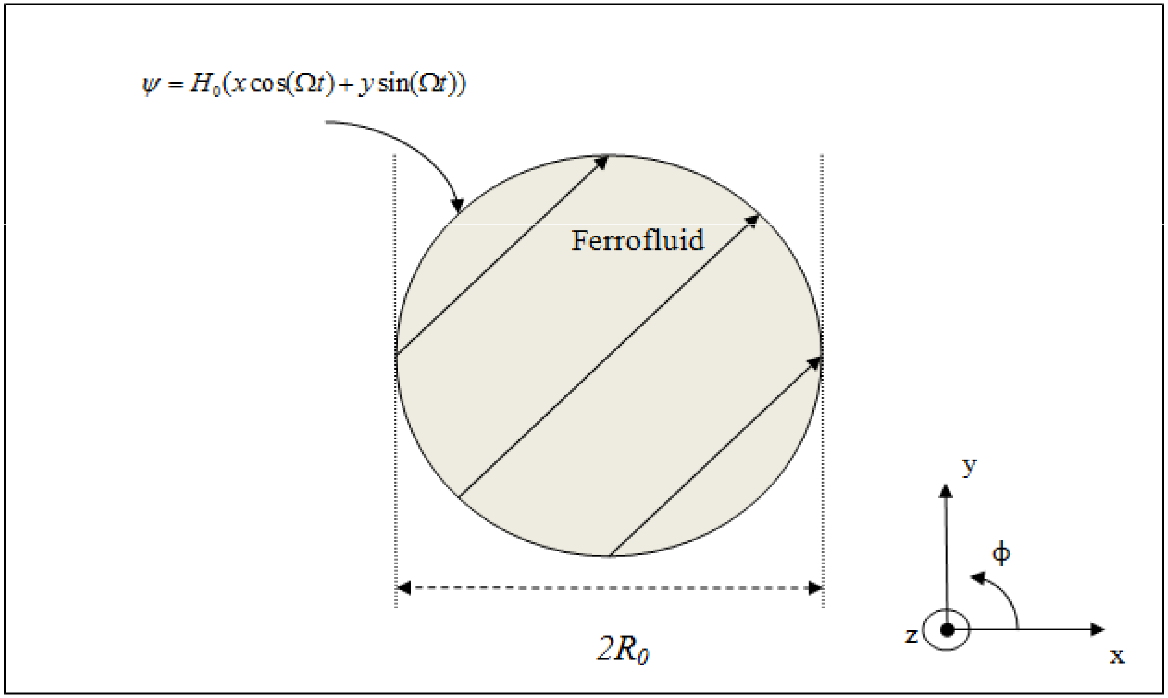
- 1) Surface Current Method



S. Khushrushahi, "Ferrofluid Spin-up Flows in Uniform and Non-uniform Rotating Magnetic Fields," PhD, Dept. of Electrical Engineering and Computer Science, MIT, Cambridge, 2010.

Modeling the Magnetic Field

- 2) Scalar Potential Method



S. Khushrushahi, "Ferrofluid Spin-up Flows in Uniform and Non-uniform Rotating Magnetic Fields," PhD, Dept. of Electrical Engineering and Computer Science, MIT, Cambridge, 2010.

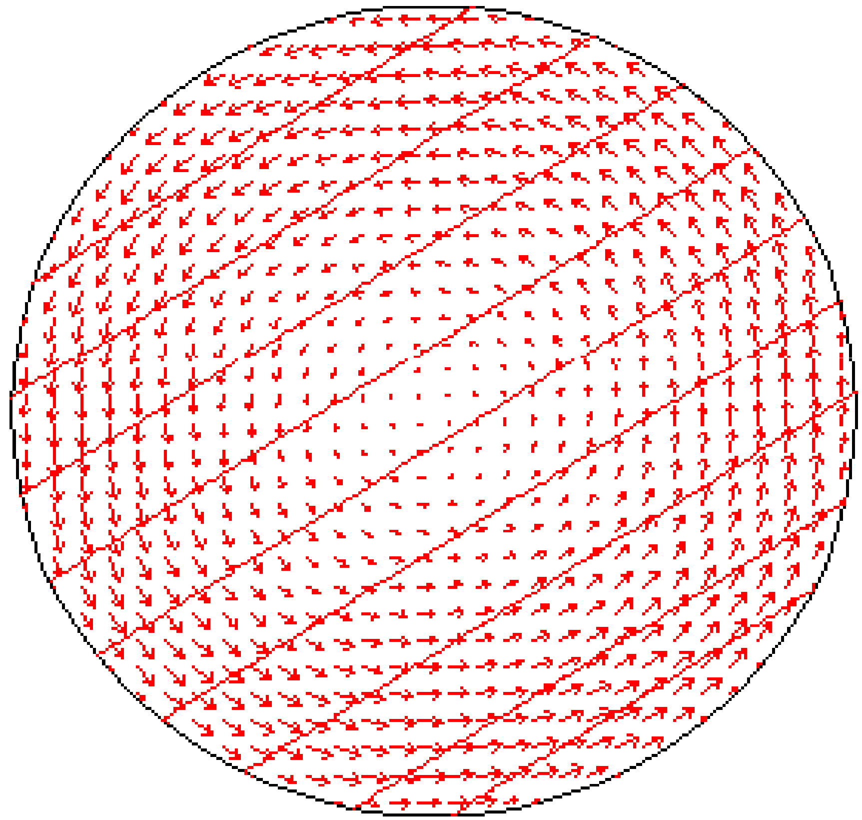
Model Setup and Parameters

- Magnetic field
  - Surface current method
    - AC/DC module, Perpendicular Induction Currents, Vector Potential
  - Scalar potential method
    - General PDE
- Linear Momentum Equation
  - Fluid Mechanics Module
  - No slip velocity boundary condition
- Angular Momentum Equation
  - Diffusion Module
  - $\omega_z=0$ (Boundary condtion for $\eta' \neq 0$)
- Magnetic Relaxation Equation
  - 2 convection and diffusion modules used (for x and y magnetization)
- All equations are non-dimensionalized and a Transient analysis was computed

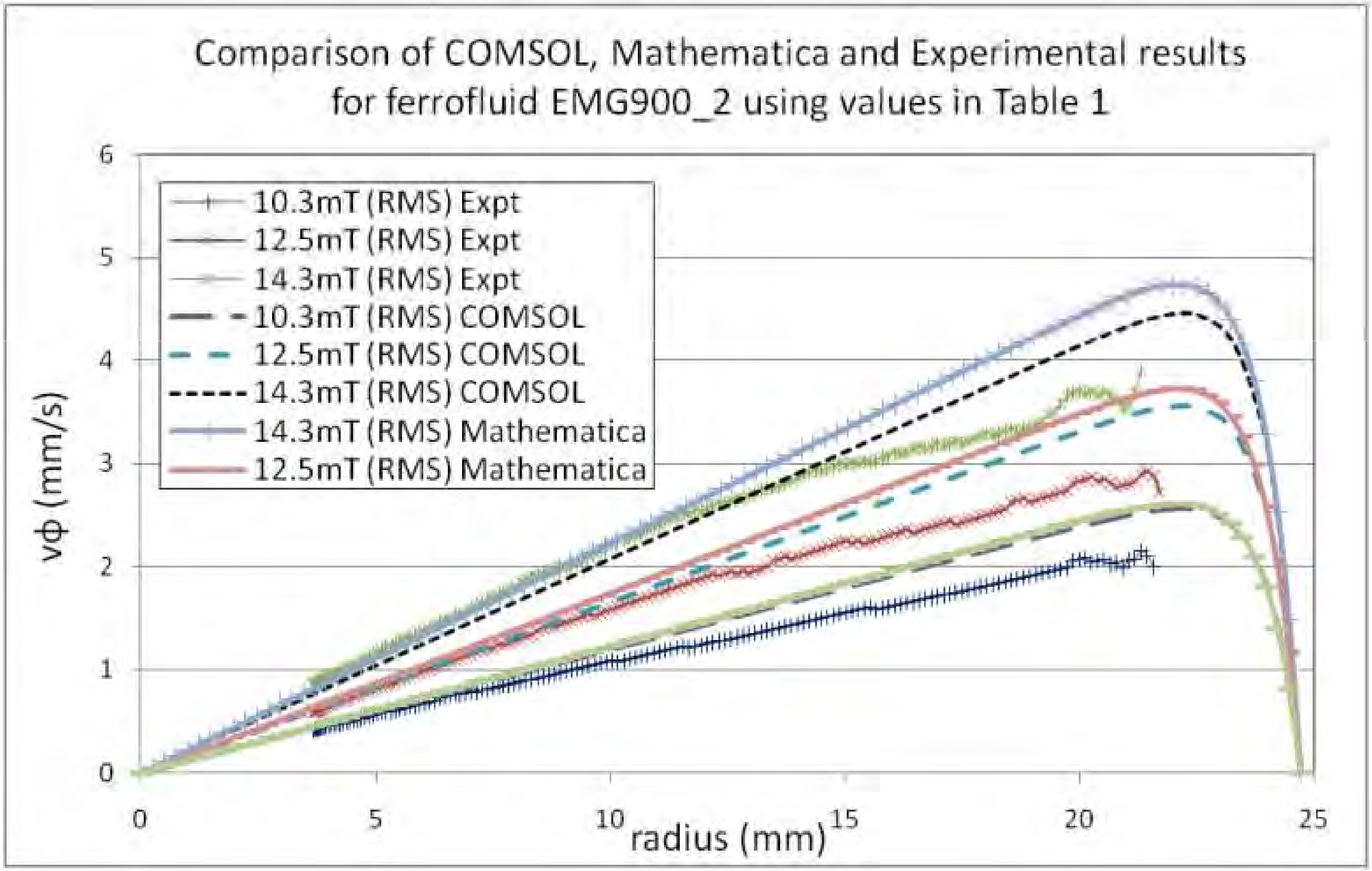
Parameter	Value
τ_{eff} (s)	$1\text{x}10^{-6}$
ρ (kg/m^3)	1030
η (Ns /m^2)	0.0045
$\mu_0 M_s$(mT)	23.9
ζ (Ns/m^2)	0.0003
Frequency (Hz)	85
Radius of cylindrical vessel (m)	0.0247
Radius of stator (m)	0.0318
Volume Fraction (%)	4.3
Magnetic Susceptibility χ	1.19
Ω (rad/s)	534.071
η' (kg m/s)	$6\text{x}10^{-10}$
B_0 (mT) *RMS*	10.3,12.5, 14.3
B_0 (mT) *amplitude*	14.57,17.68, 20.22

A. Chaves*, et al.*, "Spin-up flow of ferrofluids: Asymptotic theory and experimental measurements," *Physics of Fluids,* vol. 20, p. 053102, 2008.

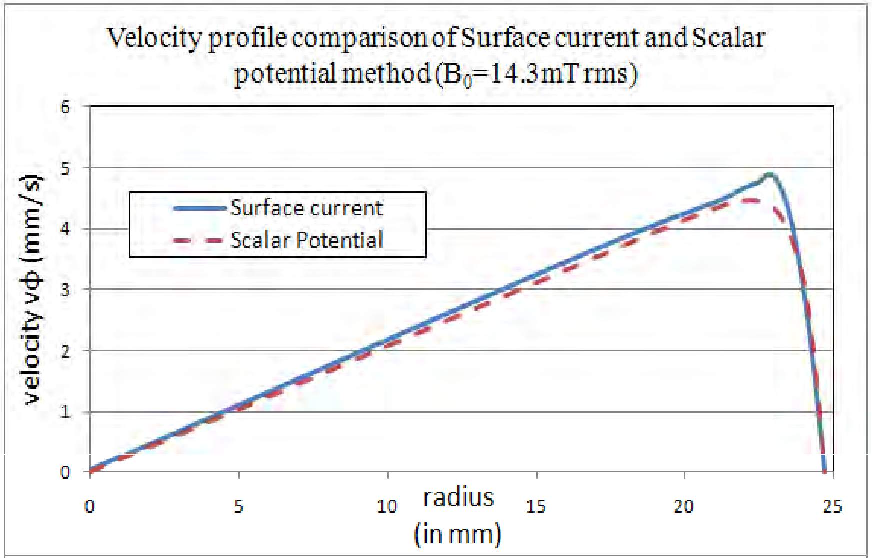
COMSOL 3.5a Results

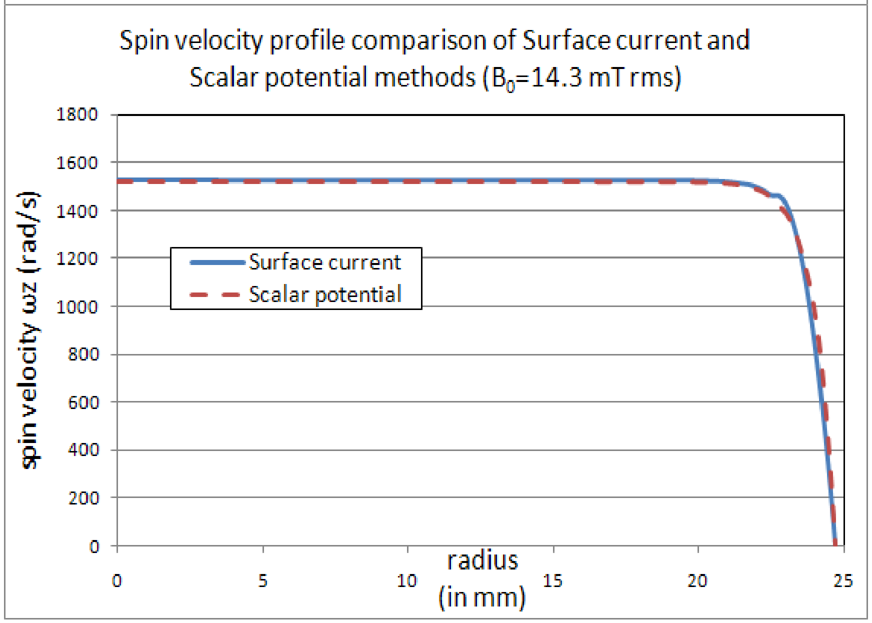


Comparison of COMSOL, Mathematica and Experimental Results



Comparison of scalar potential and surface current method





Subtlety of Scalar Potential Method

$$\frac{\partial \boldsymbol{M}}{\partial t} + \mathbf{v} \bullet \nabla \boldsymbol{M} - \boldsymbol{\omega} \times \boldsymbol{M} + \frac{1}{\tau_{eff}} (\boldsymbol{M} - \boldsymbol{M}_0) = 0$$

$$\frac{\partial \boldsymbol{M}}{\partial t} + \mathbf{v} \bullet \nabla \boldsymbol{M} - \boldsymbol{\omega} \times \boldsymbol{M} + \frac{1}{\tau_{eff}} (\boldsymbol{M} - \chi \boldsymbol{H}_{fluid}) = 0$$

$$\boldsymbol{H}_{fluid} = \boldsymbol{H}_{applied} - \frac{1}{2} \boldsymbol{M} \rightarrow \boldsymbol{H}_{fluid} = \boldsymbol{H}_{applied} - \frac{1}{2} \chi \boldsymbol{H}_{fluid} \rightarrow \boldsymbol{H}_{fluid} = \frac{\boldsymbol{H}_{applied}}{1 + \frac{1}{2} \chi}$$

$$\frac{\partial \boldsymbol{M}}{\partial t} + \mathbf{v} \bullet \nabla \boldsymbol{M} - \boldsymbol{\omega} \times \boldsymbol{M} + \frac{1}{\tau_{eff}} \left(\boldsymbol{M} - \frac{\boldsymbol{H}_{applied}}{1 + \frac{1}{2} \chi}\right) = 0$$

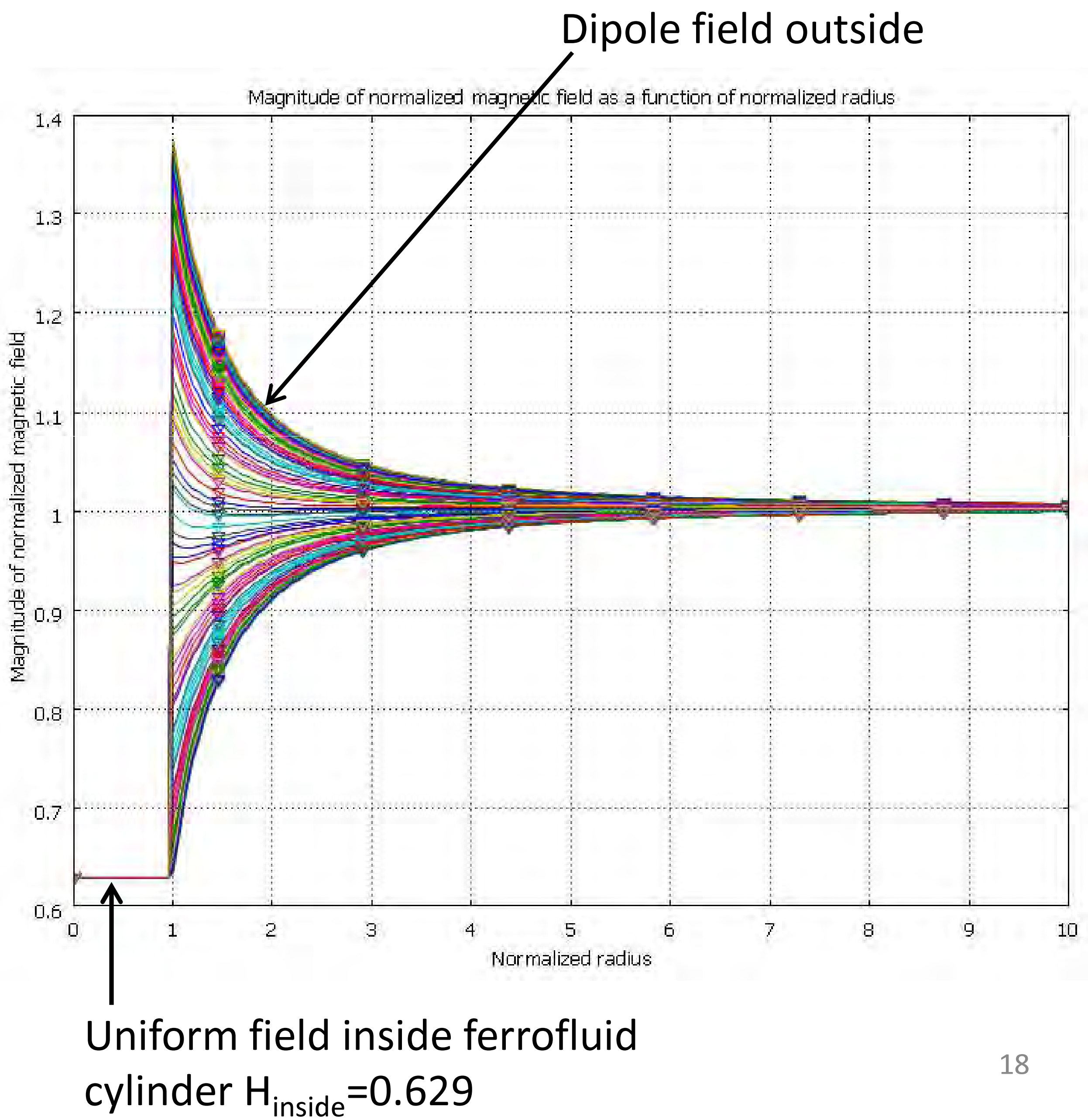
Value of using Surface Current Method

Comparing to Linear Material

$$\boldsymbol{H}_{fluid} = \frac{\boldsymbol{H}_{applied}}{1+\frac{1}{2}\chi}$$

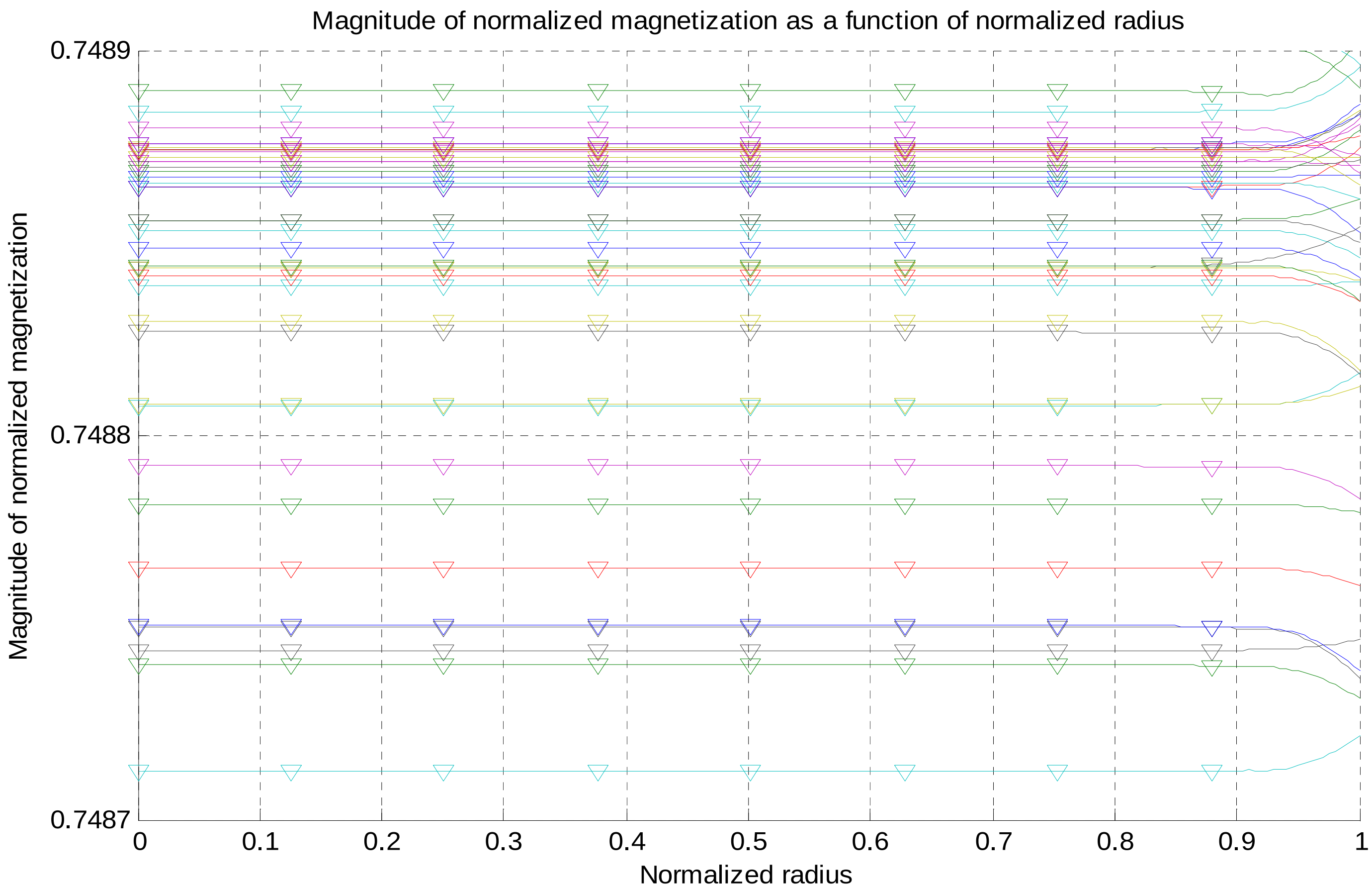
$$\left|\boldsymbol{H}_{applied}\right| = 1, \chi = 1.19,$$

$$\left|\boldsymbol{H}_{fluid}\right| = 0.627, \left|\boldsymbol{M}\right| = \chi\left|\boldsymbol{H}_{fluid}\right| = 0.746$$



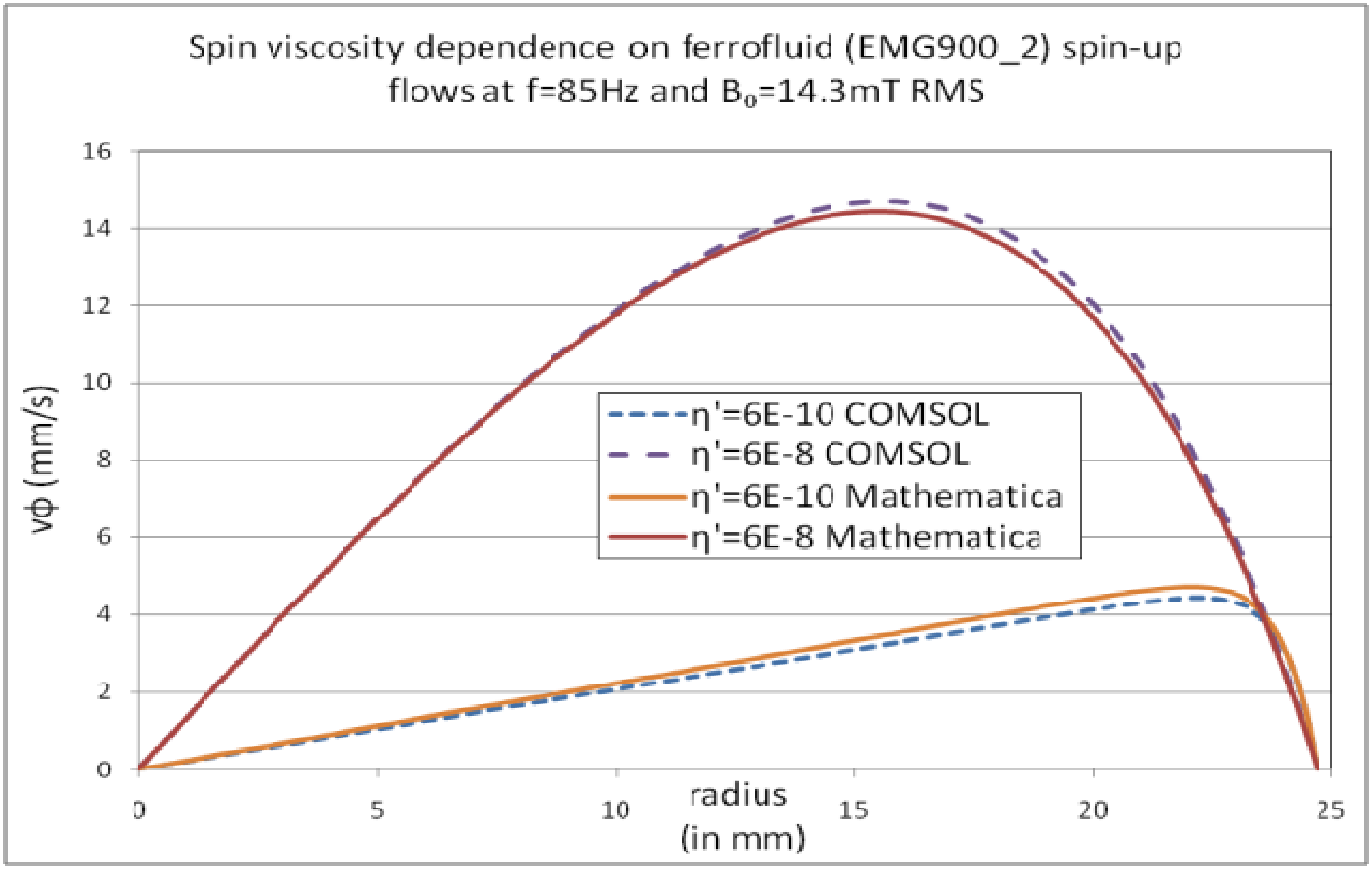
Magnetization

$$\frac{\partial \boldsymbol{M}}{\partial t} + \mathbf{v} \bullet \nabla \boldsymbol{M} - \boldsymbol{\omega} \times \boldsymbol{M} + \frac{1}{\tau_{eff}} (\boldsymbol{M} - \boldsymbol{M}_0) = 0$$



Magnetization is mostly uniform except at the boundary. Solution to Relaxation Equation gives 0.748 almost equal to result obtained using linear magnetic material (0.746)

Dependency of flow profiles on spin viscosity term η’



Conclusions

- COMSOL results compare well with analytical solutions using Mathematica, for spin diffusion dominated ferrofluid flows neglecting demagnetizing effects
- Two domain (Surface current method) is equivalent to single domain (Scalar potential method) for modeling rotating magnetic field
- Care has to be taken to model the magnetic field in single domain method
  - COMSOL takes care of this automatically in 2 domain case
- COMSOL modeling gives deeper understanding of physics (relaxation equation, shape dependency on spin viscosity η') and of subtlety in modeling as one domain problem