Investigation of 1D Compressible Navier-Stokes Equations using COMSOL Multiphysics Equation-Based Modeling

A presentation for the full paper of the same title
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Initiative and Purpose

- gain some in depth experience using the equation-based physics of COMSOL
- recover unique form of conservative compressible Navier-Stokes (NS) equations from PhD dissertation 1992
- validate in 1D using legacy (TWS) and new (FaNS) stabilization methods
- compare results with High Mach Number Flow physics of COMSOL CFD module consistent stabilization
- plan for follow-on research in 2D, 3D, etc.

1D Compressible NS Governing Equations

Assume 1D, ideal gas, constant properties, laminar flow

$$\mathscr{L}(\rho)_{1D} = \frac{\partial \rho}{\partial t} + \frac{\partial m_1}{\partial x} = 0,$$

$$\mathscr{L}(m_1)_{1D} = \frac{\partial m_1}{\partial t} + \frac{\partial (u m_1)}{\partial x} + \mathrm{Eu}\frac{\partial p}{\partial x} - \frac{4\mu}{3\mathrm{Re}}\frac{\partial^2 u}{\partial x^2} = 0,$$

$$\mathscr{L}(g)_{1D} = \frac{\partial g}{\partial t} + \frac{\partial (u g)}{\partial x} + 2\mathrm{EcEu}T\frac{\partial m_1}{\partial x} - \frac{k}{\mathrm{PrRe}}\frac{\partial^2 T}{\partial x^2} - \frac{4\mu\mathrm{Ec}}{3\mathrm{Re}}\frac{\partial}{\partial x}\left[u\frac{\partial u}{\partial x}\right] = 0.$$

closed through fluid properties Cp, Cv, μ, and k; are constant, or evaulated as f(T) or more generally as f(p,T); specific heats, viscosity, thermal conductivity

non-dimensional scaling constants:

- Euler number=Eu={$p_o/(\rho_o a_o^2)$}
- Reynolds number=Re={$\rho_o a_o L_o/\mu_o$}
- Eckert number=Ec={$\rho_o a_o^2/(2g_o)$}
- Prandtl number=Pr={$\mu_o Cp_o/k_o$}

coupling variables:

- momentum $m_1=\rho u$
- total volume-specific enthalpy= $g=\rho H=\rho(e+u^2/2)+p=\rho(CpT+u^2/2)$
- density=ρ,
- velocity=u,
- pressure=p,
- temperature=T, and
- enthalpy=h=e+p/ρ=CpT

Stabilization Methods

Legacy Taylor Weak Statement (TWS) Method - simplest form in 1D

$$TWS_q = \frac{h\beta_q}{2}\hat{u}u\frac{\partial^2 q}{\partial x^2}$$

- q={ρ,m_1,g} state variable solution array
- ûu=abs(u) since û is a unit vector equal to sign(u)
- h=element measure equal element length in 1D
- β_q=state-variable specific tuning parameter

[{6,7,12}/(24√15) typically for this study]

Error Free CFD - the Filtered analytically Navier-Stokes (FaNS) Method simplest form in 1D (dimensionless)

$$\mathrm{FaNS}_{1D}(m1) = \frac{\mathrm{Re}h^2}{12\mu}u^2\frac{d^2m1}{dx^2}$$

$$\mathrm{FaNS}_{1D}(g) = \frac{\mathrm{Re\ Pr}h^2}{12k}u^2\frac{d^2g}{dx^2}$$

TWS ~ O(h^2)
FaNS ~ O(h^4)

see full paper for coding details to implement governing equations and stabilization methods into COMSOL equation-based modeling

Validation: Viscous Burger's Equation

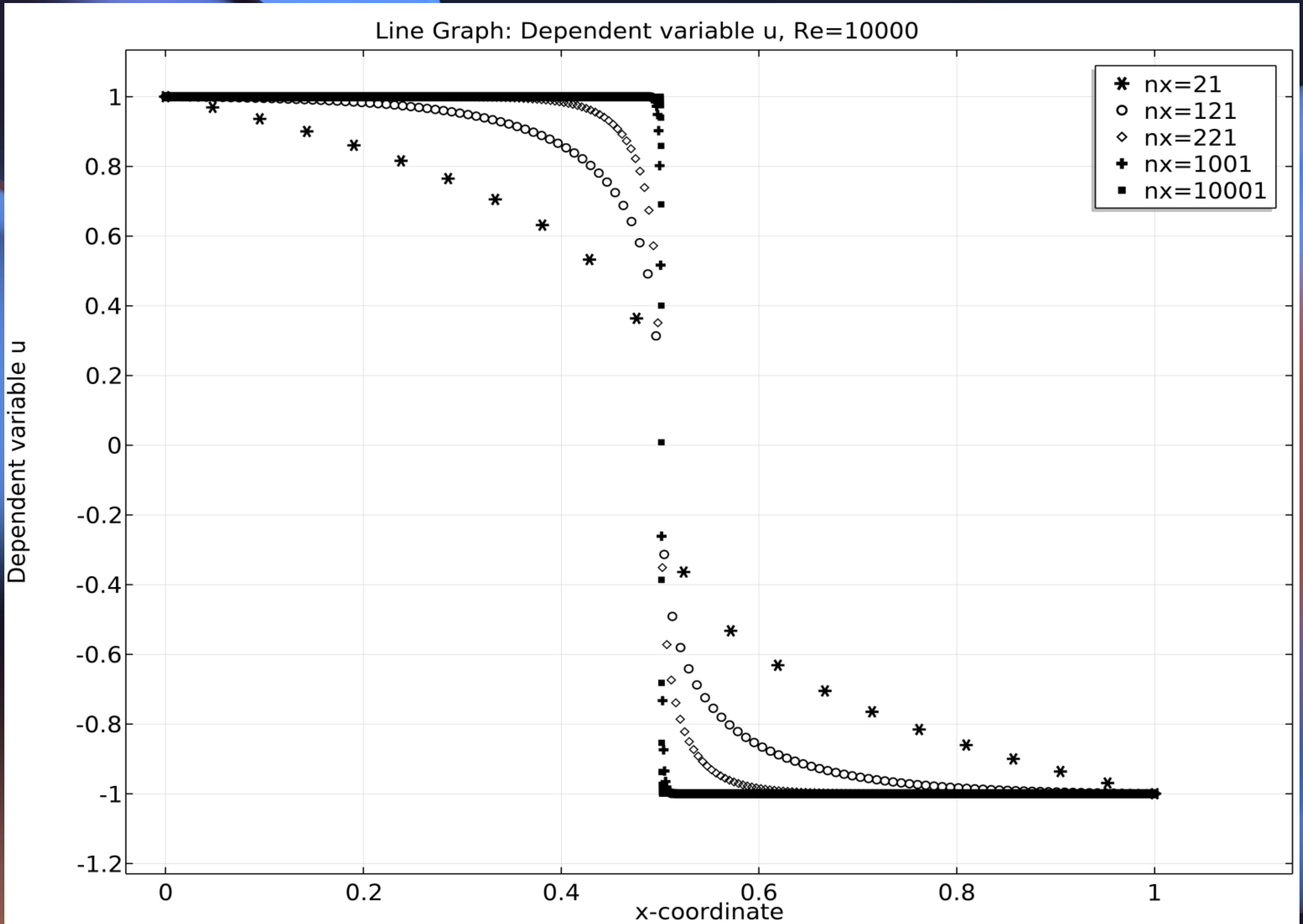
further simplifying assumptions yields:

$u\,u_x - u_{xx}/Re = 0,\ 0 \le x \le 1$
BC: $u(x=0)=1,\ u(x=1)=-1$

known solution:

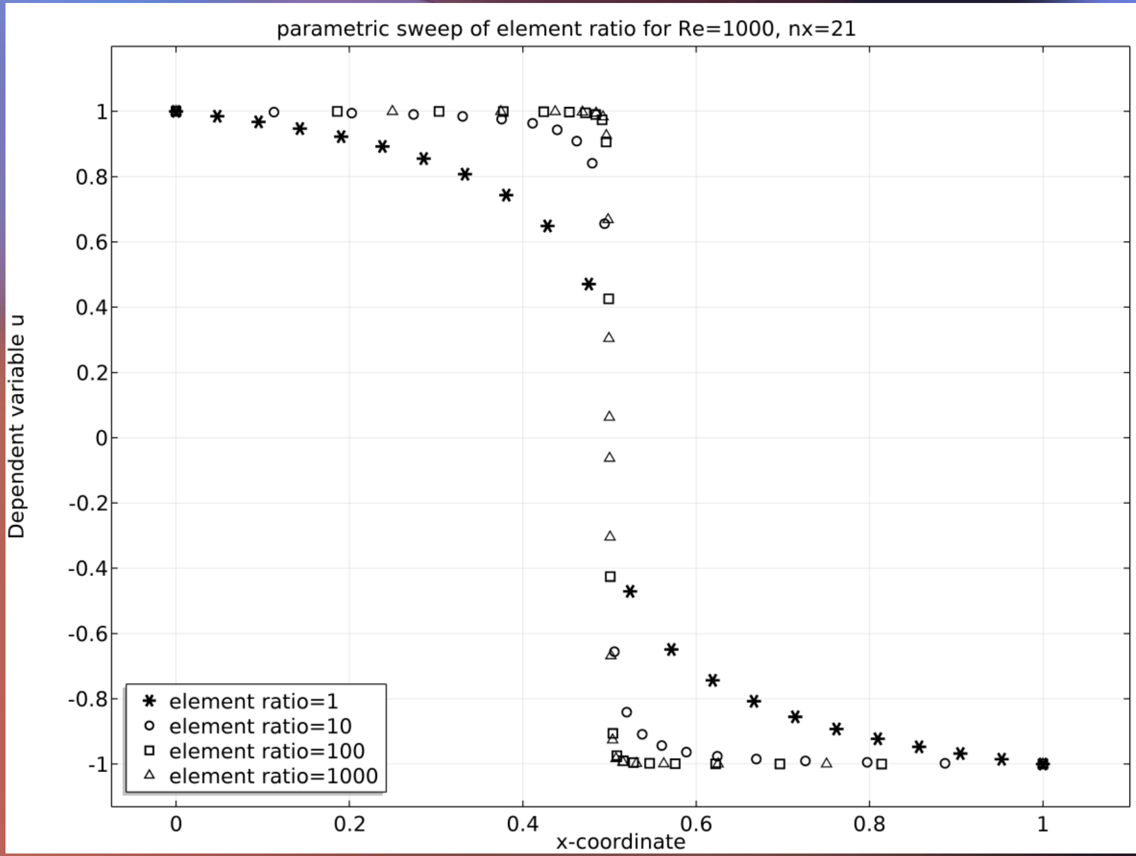
$u=1,\ 0 \le x < 0.5$
$u=0,\ x=0.5$
$u=-1,\ 0.5 < x \le 1$

challenge for stabilization methods: produce the exact solution as Reynolds number approaches infinity

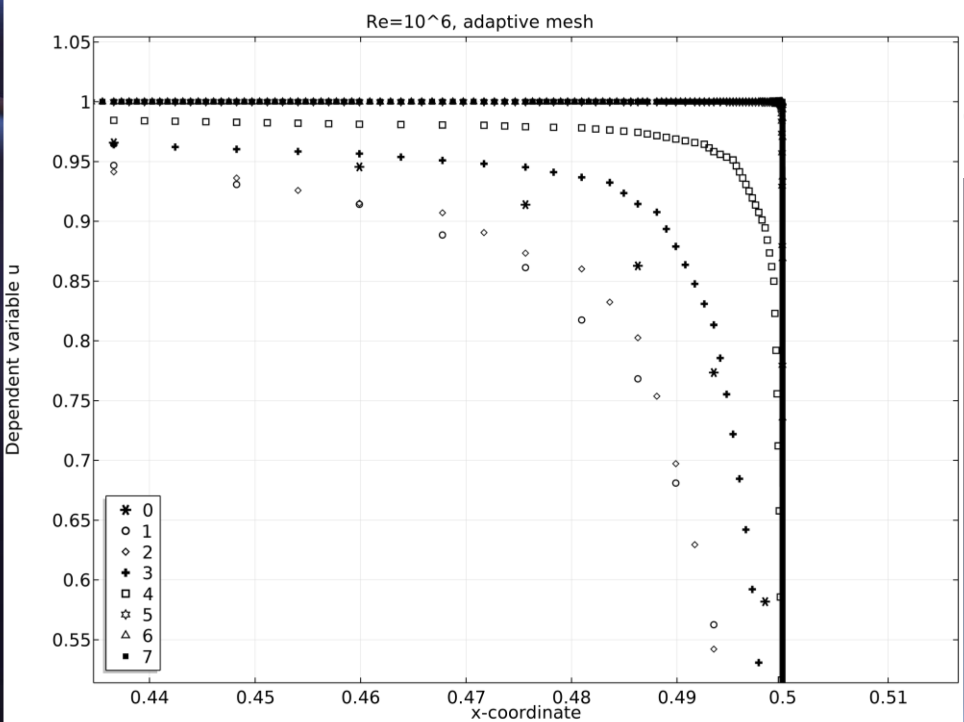


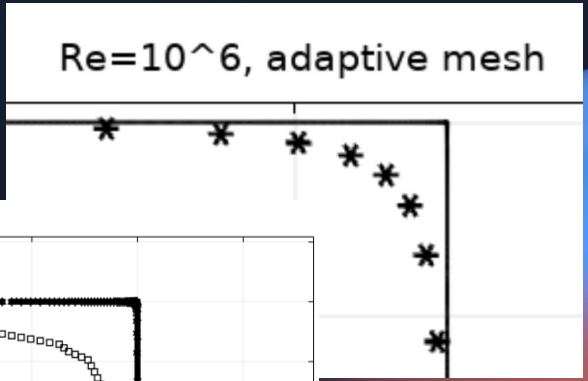
Viscous Burger's Equation Solution using FaNS Stabilization at Re=10^4 for Varying Uniform Mesh Resolution.

Validation: Viscous Burger's Equation (continued)



Viscous Burger's Equation Solution using FaNS Stabilization at $Re=10^3$ for Varying Non-Uniform Mesh Element Ratio at Maximum Number of Elements = 21.



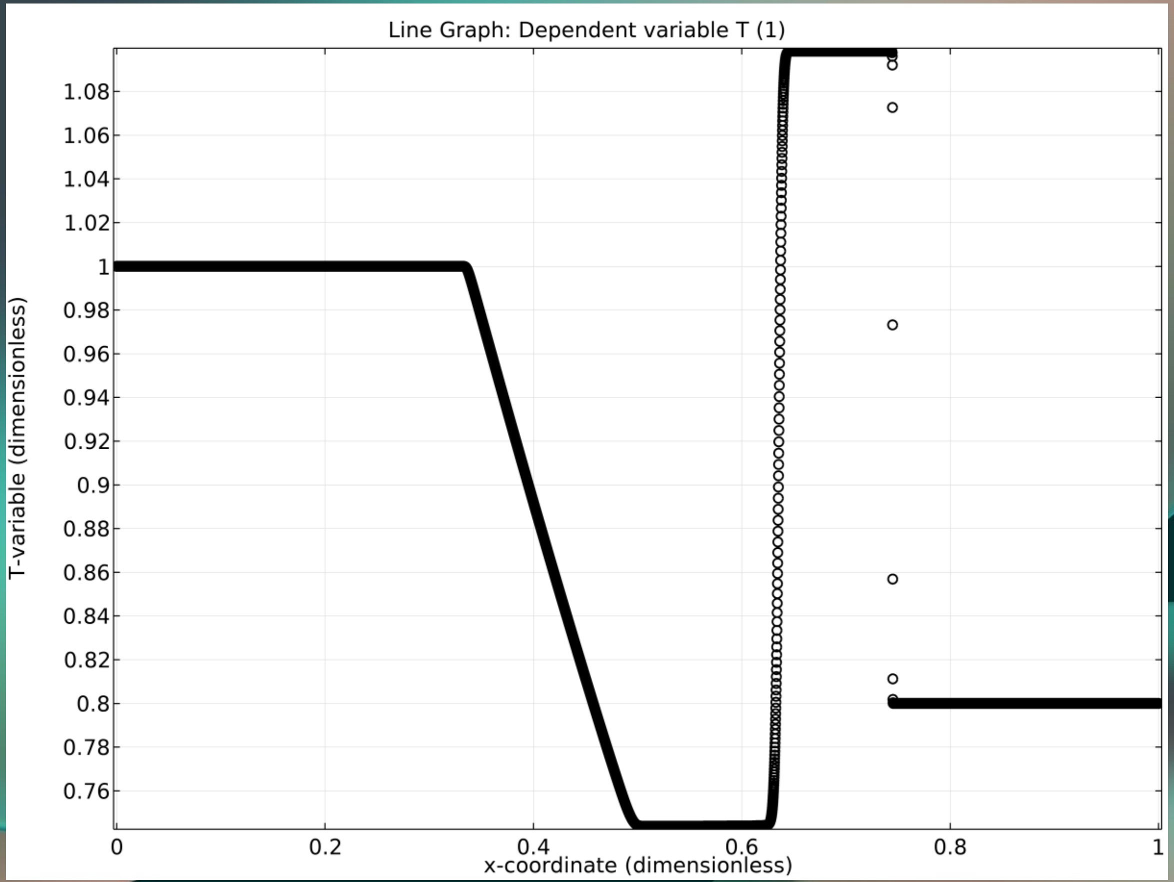


Viscous Burger's Equation Solution using FaNS Stabilization at $Re=10^6$ for Adaptive Mesh yielding 1957 Elements and Element Ratio of 4.883×10^{-6} over 7 Adaptations.

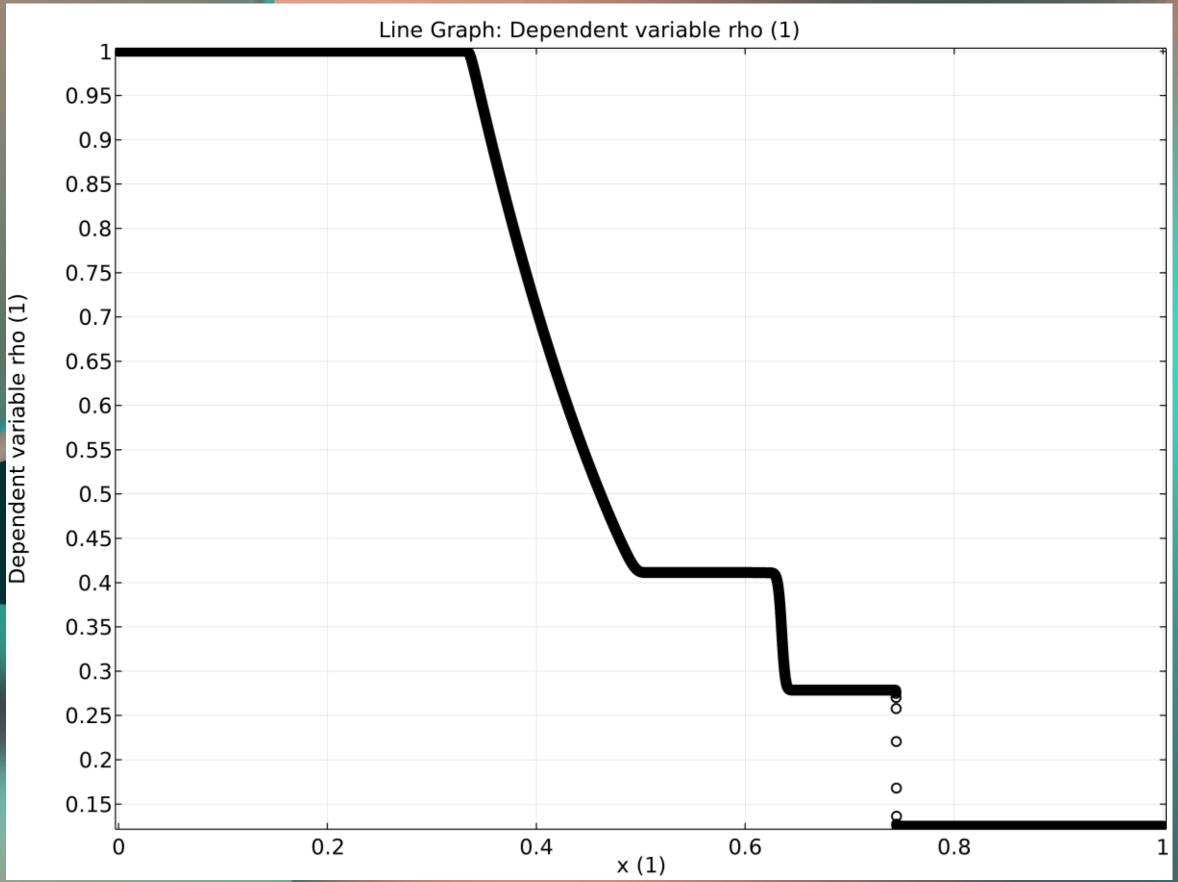
Validation: Riemann Shock Tube by Equation-Based Modeling

A classical benchmark problem in dimensionless form.
full compressible form required {ρ,m_1,g}, unit length ($0 \le x \le 1$), transient f($0 \le t \le 0.14161$s)
initial conditions: $0 \le x < 0.5$, $\rho = p = 1$ & $u = 0$; $0.5 \le x \le 1$, $\rho = 0.125$, $p = 0.1$, & $u = 0$

FaNS method is applied in these results; fixed 10^4 elements, Re=158489
identical results to widely cited references
smooth, monotone, sharp shock (2-4 elements; 0.02-0.04% of span)

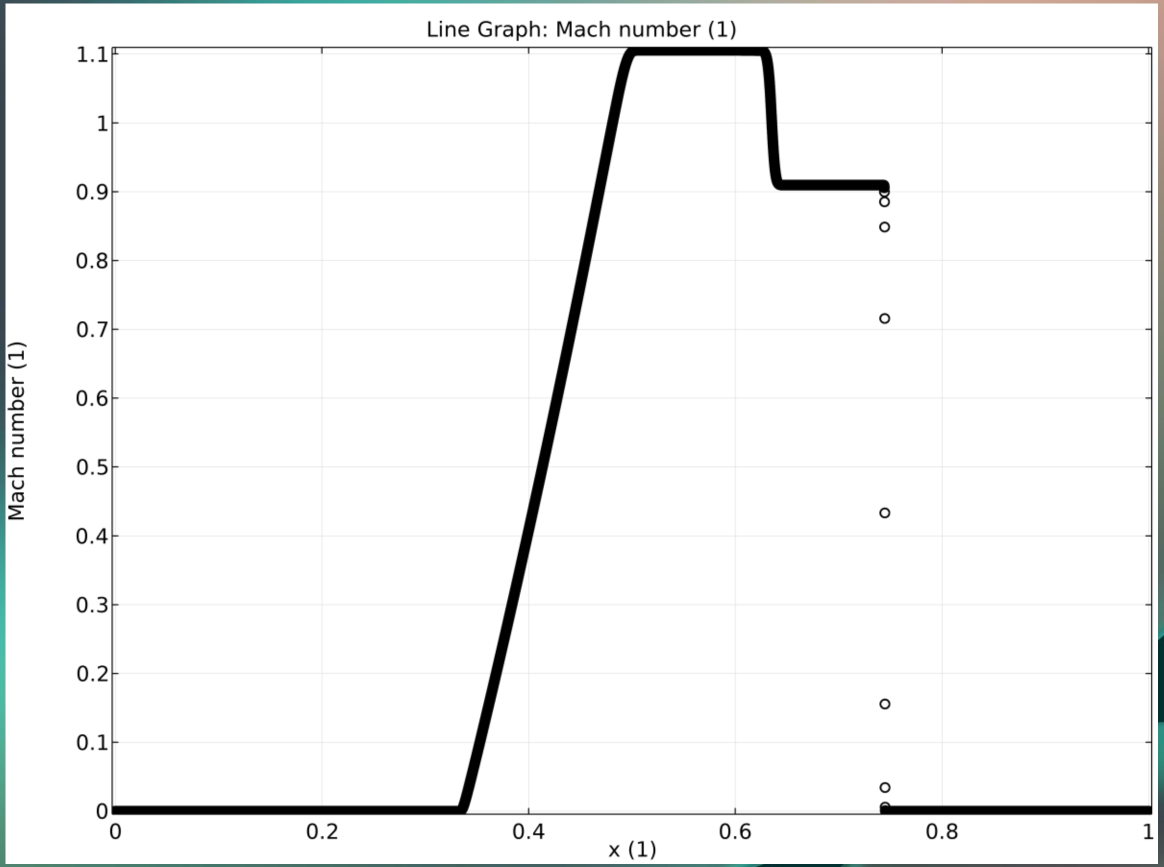


temperature

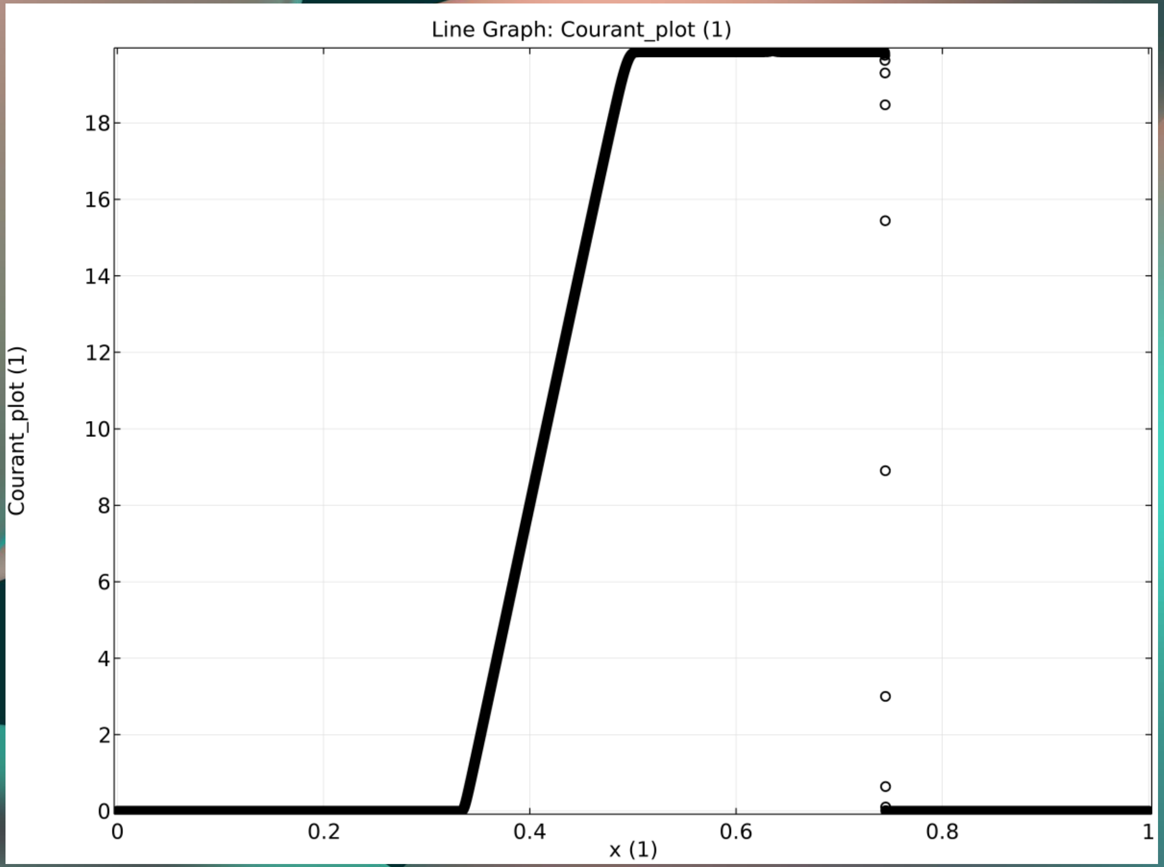


density

Validation: Riemann Shock Tube by Equation-Based Modeling (continued)



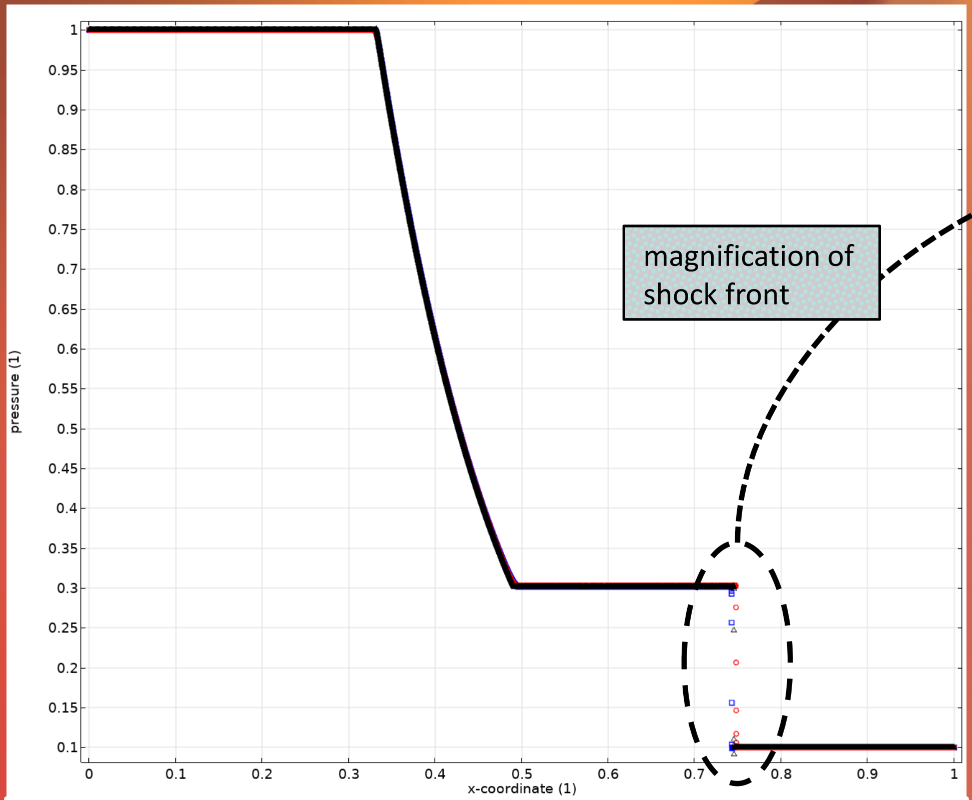
Mach Number



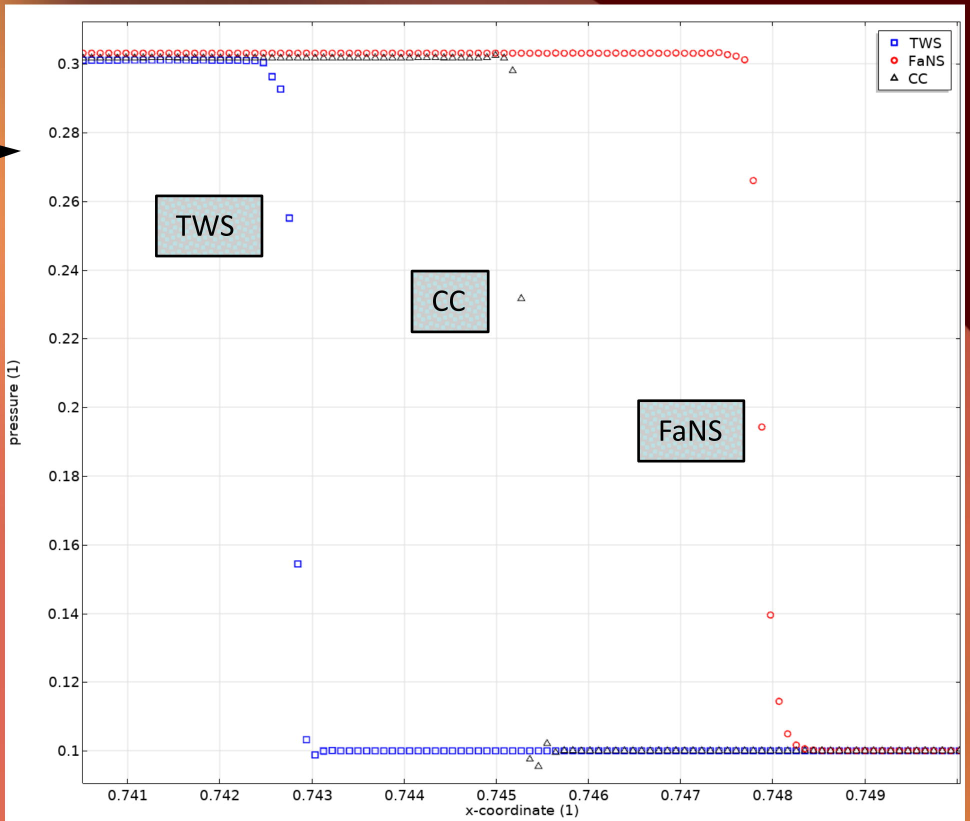
Courant Number

Validation: Riemann Shock Tube by CFD Module

The previous results were obtained using equation-based physics in dimensionless form using either TWS or FaNS stabilization methods.
We now wish to compare results with the High Mach Number Flow physics of the CFD module using COMSOL-consistent (CC) stabilization.
This will require to: (1) mimic 1D geometry with 2D geometry of unit depth/height, (2) build a dimensionless model, and (3) to use laminar flow to avoid turbulence model.

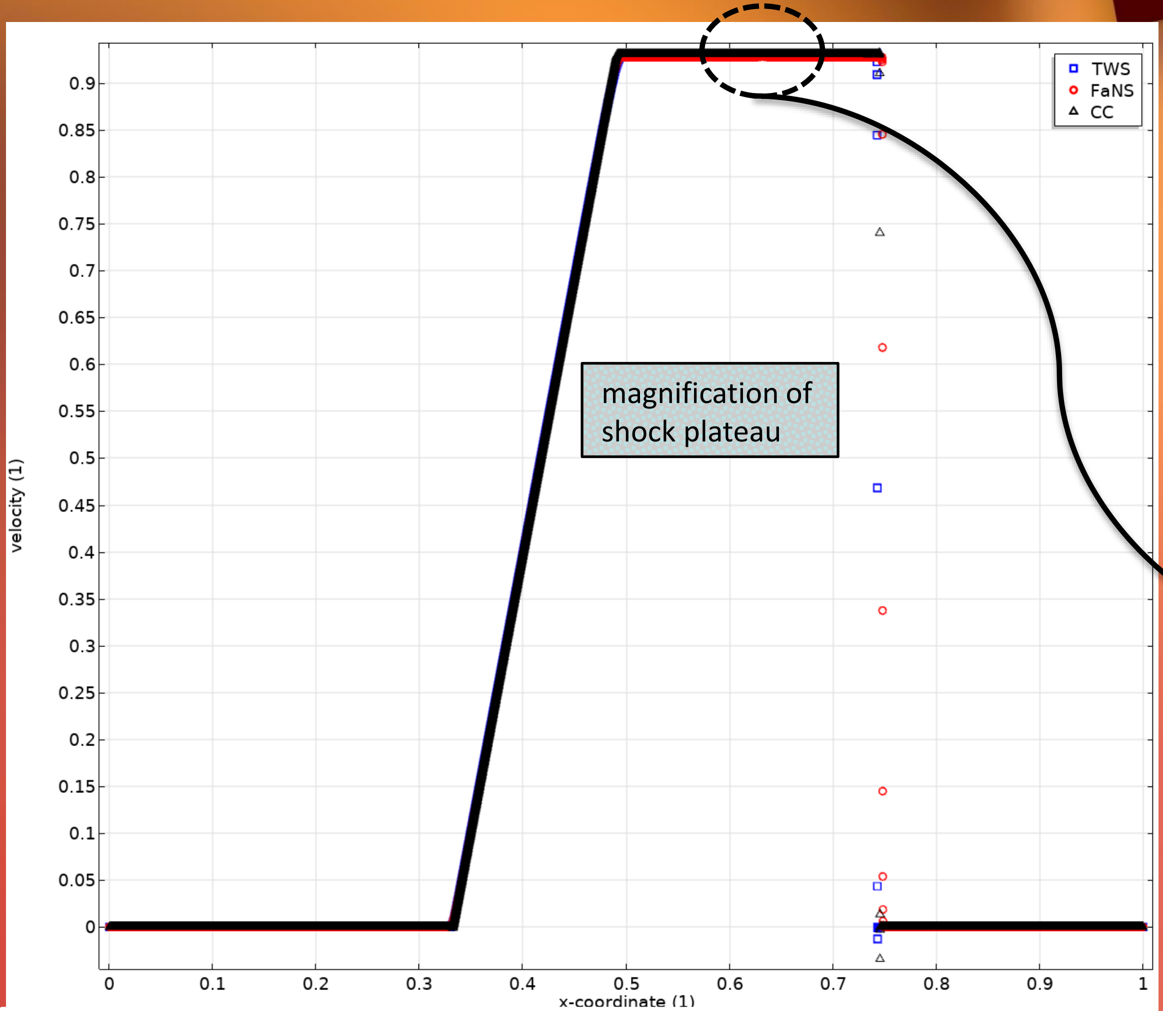


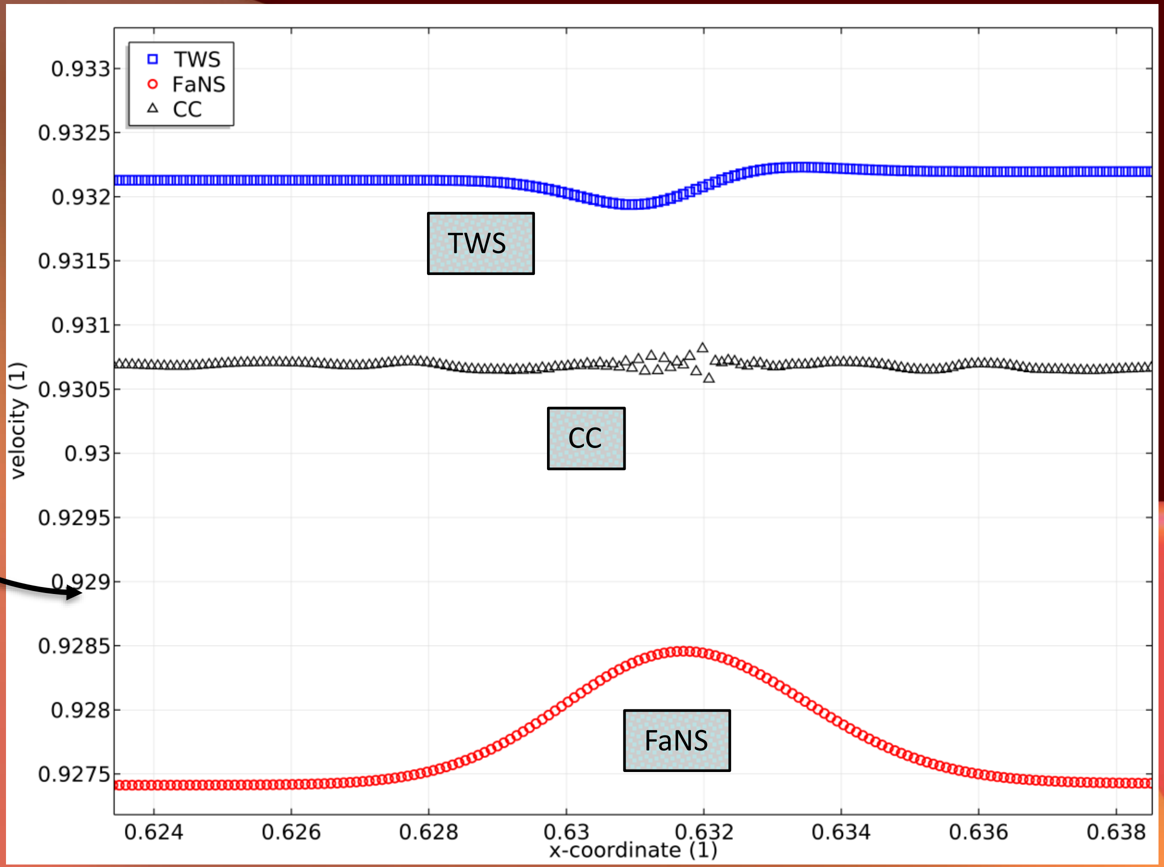
Pressure comparison between CC (black triangle), TWS (blue square), and FaNS (red circle) at final time of 0.14161s, Re=1.0725x10^5 and 10725 elements.



TWS - most diffusive in overshoot, medium undershoot
CC - medium diffusive in overshoot, most undershoot
FaNS - least diffusive in overshoot, small diffusivity in undershoot

Validation: Riemann Shock Tube by CFD Module (continued)





Velocity comparison between CC (black triangle), TWS (blue square), and FaNS (red circle) at final time of 0.14161s, Re=1.0725x10^5 and 10725 elements.

See full paper for detailed discussion on this “hump”, as well as difference in shock front “landing” point, and hypothesized cause for apparent stability issues due to non-conservative form.

Conclusions

- 1D compressible Navier-Stokes equations have been written in dimensionless, conservative form {ρ,m_1,g} using COMSOL equation-based physics modeling.
- A dimensionless, equivalent 1D from the 2D component of the High Mach Number Flow CFD module has been developed in non-conservative form {p,u,T}.
- Two separate stabilization methods, a legacy TWS and new FaNS, have been written and implemented in weak form with both the equation-based and HMN flow CFD module. The stabilization methods are easily enabled/disabled for comparison.
- Both models, equation based and CFD module based, have been validated against known 1D benchmark CFD problems and shown to give expected results for all three stabilization methods.

Suggested Further Research

- Convert the models developed herein into an application for general use and donate to the COMSOL user community.
- Expand the model development to 2D and 3D for conservative-form equation-based models, and external stabilization methods for the CFD module.
- Incorporate turbulence models into the equation-based models, and dimensionless CFD module models. Extend the new stabilization methods into the turbulence models.
- Extend the model development to incorporate a Parabolized Navier-Stokes (PNS) capability; further extend this capability into combined full 3D and PNS switching.